PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth

The Mariner, the Ship, and the Sea

High School, Maritime-themed, STEM Mathematics Curriculum

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Curriculum Map to the Common Core Standards

A-CED	**Unit / Lesson Plan**
1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	1. *Equations and Their Uses.* 2. *A Voyage to the Panama Canal.* 3. *The Geometry of the Ship.* 4. *The Naval Battle of Midway.* 5. *A Voyage to the South Seas.* 6. *Nautilus 90 North.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	1. *The Naval Battle of Midway.* 2. *Nautilus 90 North.*
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	1. *The Naval Battle of Midway.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	1. *Equations and Their Uses.* 2. *A Voyage to the Panama Canal.* 3. *The Geometry of the Ship.* 4. *The Naval Battle of Midway.* 5. *A Voyage to the South Seas.* *Nautilus 90 North.*
A-REI	***Unit / Lesson Plan***
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	1. *Equations and Their Uses.* 2. *A Voyage to the Panama Canal.* 3. *The Geometry of the Ship.* 4. *The Naval Battle of Midway.* 5. *A Voyage to the South Seas.* 6. *Nautilus 90 North.*
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	1. *Equations and Their Uses.* 2. *A Voyage to the Panama Canal.* 3. *The Geometry of the Ship.* 4. *The Naval Battle of Midway.*

	5. *A Voyage to the South Seas.* 6. *Nautilus 90 North.*
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	1. *The Naval Battle of Midway.*
8. Represent a system of linear equations as a single matrix equation in a vector variable.	1. *The Naval Battle of Midway.*
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	1. *A Voyage to the Panama Canal.* 2. *Nautilus 90 North.*
F-IF	***Unit / Lesson Plan***
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	1. *Nautilus 90 North.*
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	1. *Nautilus 90 North.* 2. *A Voyage to the South Seas.*
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and	1. *Nautilus 90 North.*

minimums; symmetries; end behavior; and periodicity.	
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.	1. *Nautilus 90 North.*
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	1. *A Voyage to the Panama Canal.* 2. *The Naval Battle of Midway.* 3. *Nautilus 90 North.*
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline,	1. *A Voyage to the Panama Canal.* 2. *The Naval Battle of Midway.* 3. *Nautilus 90 North.*

and amplitude.	
F-BF	***Unit / Lesson Plan***
1. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	1. *Geometry of the Ship.* 2. *Nautilus 90 North.*
3. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x-1) for x ≠ 1. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.	1. *A Voyage to the South Seas.* 2. *Nautilus 90 North.*
4. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems in-volving logarithms and exponents	1. *A Voyage to the South Seas.* 2. *Nautilus 90 North.*
F-LQE	***Unit / Lesson Plan***
1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a	1. *A Voyage to the Panama Canal.* 2. *Nautilus 90 North.*

constant percent rate per unit interval relative to an-other.	
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	1. *A Voyage to the Panama Canal*. 2. *Nautilus 90 North.*
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	1. *Nautilus 90 North.*
4. For exponential models, express as a logarithm the solution to a bct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	1. *A Voyage to the South Seas.* 2. *Nautilus 90 North.*
5. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.	1. *Nautilus 90 North.*
F-TF	***Unit / Lesson Plan***
3. Use special triangles to determine geometrically the values of sine, cosine, tangent for π /3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π-x, π+x, and 2π-x in terms of their values for x, where x is any real number.	1. *The Naval Battle of Midway*. 2. *A Voyage to the South Seas.*
7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	1. *A Voyage to the South Seas.* 2. *Nautilus 90 North.*
8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ),	1. *A Voyage to the South Seas.*

or tan(θ) and the quadrant.	
G-CO	***Unit / Lesson Plan***
1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	*1. Geometry of the Ship.* *2. A Voyage to the South Seas.*
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	*1. Geometry of the Ship.* *2. The Naval Battle of Midway.*
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	*1. Geometry of the Ship.* *2. The Naval Battle of Midway.*
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	*1. Nautilus 90 North.*
12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and	*1. Geometry of the Ship.* *2. The Naval Battle of Midway.* *3. A Voyage to the South Seas.*

constructing a line parallel to a given line through a point not on the line.	
G-SRT	***Unit / Lesson Plan***
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	*1. The Naval Battle of Midway.* *2. Nautilus 90 North.*
10. Prove the Laws of Sines and Cosines and use them to solve problems.	*1. The Naval Battle of Midway.* *2. A Voyage to the South Seas.*
11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	*1. A Voyage to the South Seas.*
G-C	***Unit / Lesson Plan***
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	*1. The Naval Battle of Midway.*
5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	*1. A Voyage to the South Seas.*
G-GMD	***Unit / Lesson Plan***
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	*1. Geometry of the Ship.*
4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	*1. Geometry of the Ship.* *2. The Naval Battle of Midway.*

G-MG	***Unit / Lesson Plan***
1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	*1. Geometry of the Ship.* *2. The Naval Battle of Midway.* *3. A Voyage to the South Seas.*
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	*1. Geometry of the Ship.* *2. The Naval Battle of Midway.*
S-ID	***Unit / Lesson Plan***
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	*1. A Voyage to the Panama Canal.* *2. The Naval Battle of Midway.* *3. A Voyage to the South Seas.* *4. Nautilus 90 North.*
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	*1. A Voyage to the Panama Canal.* *2. The Naval Battle of Midway.* *3. A Voyage to the South Seas.* *4. Nautilus 90 North.*
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	*1. A Voyage to the Panama Canal.* *2. The Naval Battle of Midway.* *3. A Voyage to the South Seas.* *4. Nautilus 90 North.*
S-IC	***Unit / Lesson Plan***
2. Decide if a specified model is consistent with results from a given	*1. The Naval Battle of Midway.*

data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?	

Glossary of Mathematics Terms[1]

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (- 3/4) = (- 3/4) + 3/4 = 0.

Associative property of addition. See table below.

Associative property of multiplication. See table below.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See table below.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

[1] http://www.corestandards.org/Math/Content/mathematics-glossary/glossary/

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See*: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See table below.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form a or -a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 Ã· 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 Ã— 4/3 = 4/3 Ã— 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See table below.

Properties of equality. See table below.

Properties of inequality. See table below.

Properties of operations. See table below.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:*terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:*median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, …

Associative property of addition	(a +b) + c = a + (b+c)
Commutative property of addition	a + b = b + a
Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) x c = a x (b x c)
Commutative property of multiplication	a x b = b x a
Multiplicative identity property 1	a x 1 = 1 x a = a
Existence of multiplicative inverses	For every *a* = 0 there exists 1/*a* so that *a* x 1/*a* = 1/*a* x *a* = 1
Distributive property of multiplication over additions	a x (b + c) = a x b + a x c

Reflexive property of equality	a = a.
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c , then a = c.
Addition property of equality	If a = b, then a +c = b + c.
Subtraction property of equality	If a = b then a – c = b – c.
Multiplication property of equality	If a = b, then a x c = b x c.
Division property of equality	If a = b and c ≠ 0, then a + c = b + c.
Substitution property of equality	

$$Exactly\ one\ of\ the\ following\ is\ true: a < b, a = b, a > b.$$
$$If\ a > b\ and\ b > c\ then\ a > c.$$
$$If\ a > b, b < a.$$
$$If\ a > b, then\ -a < -b.$$
$$If\ a > b, then\ a \pm c > b \pm c.$$
$$If\ a > b\ and\ c > 0, then\ a\, x\, c > b \times c.$$
$$If\ a > b\ and\ c < 0, then\ a\, x\, c < b \times c.$$
$$If\ a > b\ and\ c > 0, then\ a \div c > b \div c.$$
$$If\ a > b\ and\ c < 0, then\ a \div c < b \div c.$$

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Seeing with Sound

Equations and Their Uses

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Introduction to SONAR

Many ships use SONAR (short for ***SO***und ***N***avigation and ***R***anging) to find things under water. Scientific ships use it to map the bottom of the ocean and find objects that are very deep. Warships use SONAR to find enemy submarines. Because one cannot see underwater easily, sound is used to hear things that are underwater.

Listening with SONAR is much like listening with your ears. Imagine yourself at a party where music is playing and people are talking and laughing. Your friend is standing near you talking to you. Your ability to hear him is based on many factors. Can you think of what those factors are? List as many as you can below:

____

____

____

____

Let's give each of these a name.

1. Detection Threshold (DT): ____
2. Source level (SL): ____
3. Noise level (NL): ____
4. Transmission Loss (TL): ____
5. Target Strength (TS): ____

You could put these terms together in a sentence:

The sound level at which you can just barely hear your friend equals how loudly your friend speaks minus how far away from you your friend is minus how much noise is in the background.

Now, let's rewrite this sentence, putting in the abbreviations for each term:

The sound level (DT) at which you can just barely hear your friend equals how loudly your friend speaks (SL) plus how well someone could hear him standing one foot from him (TS) minus twice how far away from you your friend is (2TL) minus how much noise is in the background (NL).

Now, let's translate this into an equation:

$$DT = SL + TS - 2TL - NL$$

This is the basic SONAR equation we will use in this unit.

We can write this equation in terms of each variable. For example we can rewrite the equation in terms of SL by solving for SL in the basic SONAR equation above:

Source level (SL) equals detection threshold (DT) plus twice the transmission loss (2TL) plus noise level (NL)

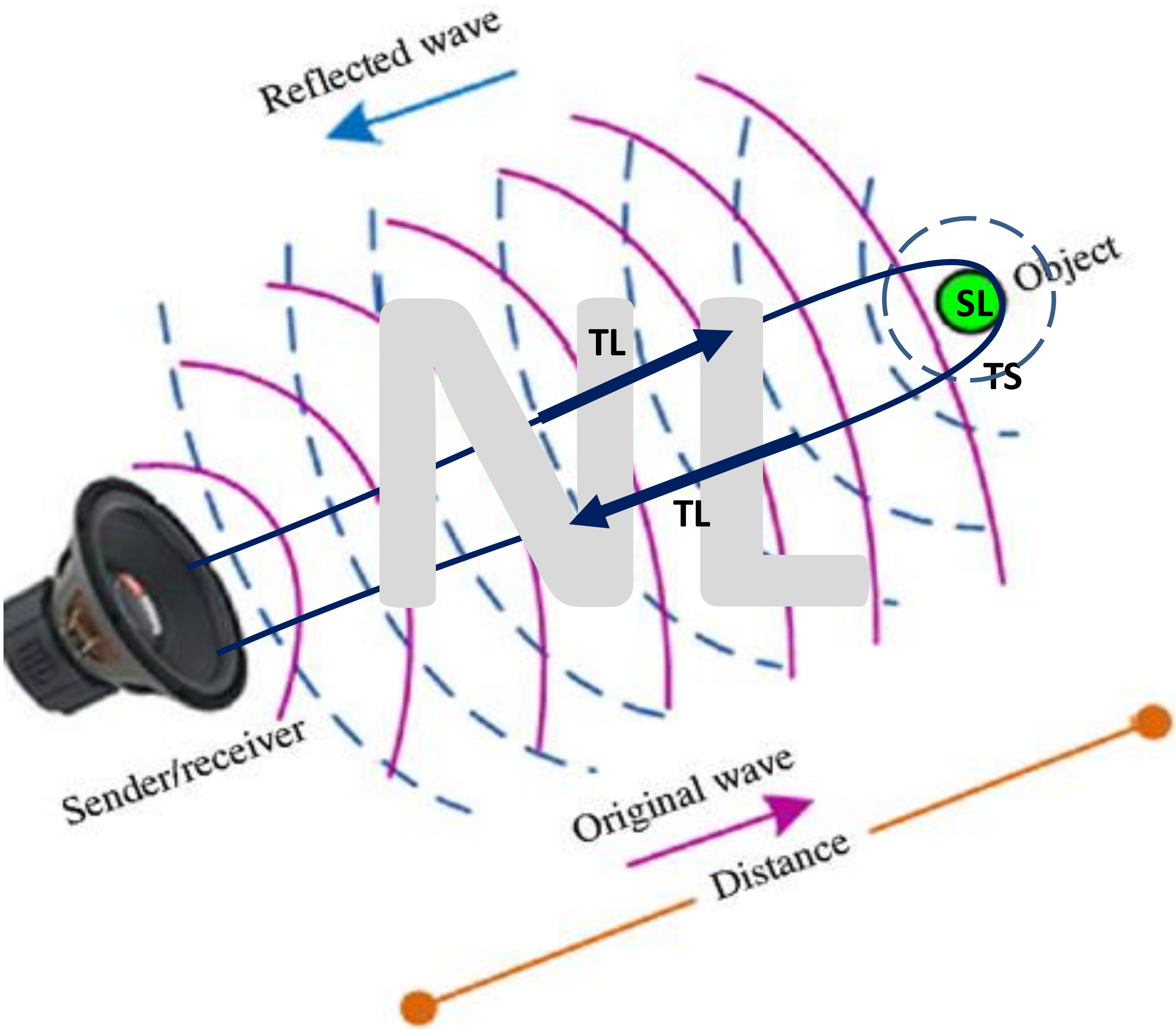
$$SL = DT + 2TL + NL - TS$$

Now try to rewrite the equation in terms of transmission loss (TL) and noise level (NL):

$TL =$ ____

$NL =$ ____

How SONAR Works



Name: ___________________________
Date: _____________________________
Advisor: __________________________

Solving Equations of One Variable – I

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

1. **How to solve equations of one variable:**

   a. $$\begin{array}{r} 3x + 4 = 15 \\ \underline{\quad -4 \quad -4} \\ 3x + 0 = 11 \end{array}$$

   b. $\frac{3x}{3} = \frac{11}{3}$

   c. $x = \frac{11}{3}$

2. **Problem Set:**

a) $5x + 9 = 39$

b) $9p + 11 = -7$

c) $6 - 2d = 42$

d) $9 - c = -13$

e) $2m + 5 = 17$

f) $9p + 20 = -7$

g) $5x + 9 = 54$

h) $3 + 2x = 21$

i) $6 - 8d = -42$

j) $127 = 2x + 17$

j) $5.2 + 1.3x = -1.3$

k) $13 = \frac{a}{3} - 2$

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Date: ______________________________
Advisor: ___________________________

Solving Equations of One Variable – II

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

I. **SONAR Equation.** All problems will use the following basic SONAR equation we have already discussed:

$$DT = TS + SL - 2TL - NL$$

II. **Variables** (all variables express sound levels in decibels [db]):

A. DT = Detection Threshold
B. TS = Target Strength
C. SL = Source Level
D. TL = Transmission Loss
E. NL = Noise Level

III. **Problems:**

A. $10db = TS + 4db - 2(5db) - 12db$

1. Add given numbers on right-hand side together: $4db - 2(5db) - 12db =$

$$4db - 10db - 12db = -18db$$

2. Substitute the sum for the given numbers: $10db = TS - 18db$

3. Add 18db to both sides:

$$\begin{array}{r} 10db = TS - 18db \\ \underline{+18db \qquad\quad + 18db} \\ 28db = TS \;\; - 0db \end{array}$$

4. And the answer is..... $TS = 28db$

"2 Level" Problems	"3 Level" Problems	"4 Level" Problems
B. $30db = 10db - NL$	J. $30db = 5db - NL + 4db$	R. $30db = 5db - NL + SL$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
C. $46db = TS - 41db$	K. $46db = TS + 65db - 41db$	S. $46db = TS + 65db - NL$
$TS =$ ______________	$TS =$ ______________	$TS =$ ______________
D. $80db = 44db + SL$	L. $80db = 37db + SL - 21db$	T. $80db = TS + SL - 21db$
$SL =$ ______________	$SL =$ ______________	$SL =$ ______________
E. $12db = 87db - 2TL$	M. $12db = 35db - 2TL - 15db$	U. $12db = TS - 2TL - 15db$
$2TL =$ ______________	$2TL =$ ______________	$2TL =$ ______________
$TL =$ ______________	$TL =$ ______________	$TL =$ ______________
F. $55db = 21db - NL$	N. $55db = 81db - NL + 24db$	V. $55db = TS - NL + 24db$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
G. $8.2db = TS - 4.1db$	O. $8.2db = 15db - TS - 4.1db$	W. $8.2db = SL - TS - 4.1db$
$TS =$ ______________	$TS =$ ______________	$TS =$ ______________
H. $9.7db = 6.9db - NL$	P. $9.7db = 2.3d + 6.9db - NL$	X. $9.7db = SL + 6.9db - NL$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
I. $56db = 33.3db + SL$	Q. $0db = 33.3db + SL - 56db$	Y. $0db = 33.3db + SL - NL$
$SL =$ ______________	$SL =$ ______________	$SL =$ ______________

Name: ____________________________
Date: ______________________________
Advisor: ___________________________

Solving Equations of One Variable – III

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

I. **SONAR Equation.** All problems will use the following basic SONAR equation we have already discussed:

$$DT = f(TS, SL, TL, NL) = TS + SL - 2TL - NL$$

II. **Variables** (all variables represent sound levels in decibels [db]):

A. DT = Detection Threshold
B. TS = Target Strength
C. SL = Source Level
D. TL = Transmission Loss
E. NL = Noise Level

If the detection threshold (DT) is less than 1.0db, it will be impossible to detect the target. If DT is less than 10db, it is highly unlikely that the target will be detected.

III. **Problems:**

1. A nuclear-powered submarine uses pumps to cool its nuclear reactor. These pumps make a great deal of noise. A destroyer searching for such a submarine uses a SONAR to listen for the sound of those pumps. If the source level (SL) of the submarine is 40db, the target strength (TS) of the submarine is 34db, the transmission loss (TL) is 20db and the noise level (NL) is 20db, will the destroyer detect the submarine?

2. Will the destroyer detect the submarine if the noise level (NL) increases to 44db?

3. A frigate's SONAR has a detection threshold (DT) for a submarine of 10db. If the noise level (NL) is 20db, the transmission loss (TL) is 10db, and the submarine's target strength (TS) is 28db, what is the source level (SL) of the submarine?

4. What will the new detection threshold (DT) be if the submarine's source level (SL) increases by 5db over the level that is the answer to problem "C"?

5. A National Ocean Survey ship uses SONAR to detect underwater natural gas emissions. If the SONAR has a detection threshold (DT) of 15db with a noise level (NL) of 8db, a transmission loss (TL) of 12db, and a source level (SL) of 0db, what is the target strength of the natural gas emission?

6. What would the new detection threshold (DT) be if the noise level (NL) decreased by 4db?

Name: ____________________________
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Solving Multi-Step Equations – I

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

1. **How to solve multi-step equations:**

   a. $4x - 2 = 3x + 4$

   $\underline{-3x \qquad -3x}$

   $1x - 2 = 0 \quad + 4$

   b. $x - 2 = 4$

   $\underline{\quad +2 \ \ +2}$

   $x + 0 = 6$

   c. $x = 6$

2. **Problem Set:**

   a. **Basic Skills Practice.** Solve each equation:

1) $2x - 2 = 4x + 6$ 2) $3x + 5 = 2x + 2$ 3) $4x + 3 = 5x - 4$

4) $2x - 5 = 4x - 1$ 5) $5x + 24 = 2x + 15$ 6) $5y - 10 = 14 - 3y$

7) $12 - 6z = 10 - 5z$ 8) $5m - 7 = -6m - 29$ 9) $-10x + 3 = -3x + 12 - 4x$

10) $6p - 12 = -4p + 18$ 11) $\frac{w}{2} + 7 = \frac{w}{3} + 9$ 12) $6 - \frac{t}{4} = 8 + \frac{t}{2}$

Name: ___________________________
Date: ____________________________
Advisor: __________________________

Solving Multi-Step Equations – II

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

I. **SONAR Equation.** All problems will use the following basic SONAR equation we have already discussed:

$$DT = TS + SL - 2TL - NL$$

II. **Variables** (all variables express sound levels in decibels [db]):

A. DT = Detection Threshold
B. TS = Target Strength
C. SL = Source Level
D. TL = Transmission Loss
E. NL = Noise Level

III. **Problems:**

A. $10db + 2TS = 3TS + 4db - 2(5db) - 12db$

1. Add given numbers on right-hand side together: $4db - 2(5db) - 12db =$

$$4db - 10db - 12db = -18db$$

2. Substitute the sum for the given numbers: $10db + 2TS = 3TS - 18db$

3. Add 18db to both sides:

$$\begin{array}{r} 10db + 2TS = 3TS - 18db \\ \underline{+18db \qquad\qquad\qquad + 18db} \\ 28db + 2TS = 3TS \;\; - 0db \end{array}$$

4. Subtract 2TS from both sides:

$$\begin{array}{r} 28db + 2TS = 3TS \;\; - 0db \\ \underline{-2TS \;\; - 2TS \qquad\quad} \end{array}$$

5. And the answer is..... $TS = 28db$

"2 Level" Problems	"3 Level" Problems	"4 Level" Problems
B. $30db - 2NL = 10db - NL$	J. $30db - 2NL = 5db - NL + 4db$	R. $30db - 6SL = 5db - NL - 5SL$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
C. $46db + 3TS = 4TS - 41db$	K. $46db + 2TS = 3TS + 65db - 41db$	S. $-2NL + 46db + 2TS = TS + 65db - NL$
$TS =$ ______________	$TS =$ ______________	$TS =$ ______________
D. $80db - 4SL = 44db - 3SL$	L. $80db - 6SL = 37db + SL - 21db - 6SL$	T. $80db - 2SL + TS = TS - SL - 21db$
$SL =$ ______________	$SL =$ ______________	$SL =$ ______________
E. $12db = 4TL + 87db - 2TL$	M. $12db - 5TL = 35db - 3TL - 15db$	U. $18.7TS + 12db - 6TS = 12.7TS - 2TL - 15db$
$2TL =$ ______________	$2TL =$ ______________	$2TL =$ ______________
$TL =$ ______________	$TL =$ ______________	$TL =$ ______________
F. $55db - 17NL = 21db - 16NL$	N. $55db + 5NL = 81db - NL + 24db + 5NL$	V. $2(TS - NL) + 55db = TS + 2NL + 24db$
$NL =$ ______________	$NL =$ ______________	$TS =$ ______________
G. $8.2db + 12TS = 13TS - 4.1db$	O. $4TS + 8.2db - TS = 15db + 4TS - 4.1db$	W. $8.2db - 4(SL + TS) = -3SL - 3TS - 4.1db$
$TS =$ ______________	$TS =$ ______________	$SL =$ ______________
H. $9.7db + 4NL = 6NL + 6.9db - NL$	P. $9.7db + 3NL = 2.3db + 5NL + 6.9db - NL$	X. $6(2SL - 3NL) + 9.7db = 2(6SL + 6.9db) - 17NL$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
I. $56db + 12SL = 33.3db - 11SL$	Q. $0db + 12SL = 33.3db + SL - 56db + 12SL$	Y. $7(-3NL + 0db) = 33.3db + SL - 21NL$
$SL =$ ______________	$SL =$ ______________	$SL =$ ______________

Name: ____________________________
Date: _____________________________
Advisor: ___________________________

Solving Equations with Distributive Property – I

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

1. Solving equations using the distributive property **(show work on separate sheet of paper)**:

$$3(x-4) = 48$$

$$3x - 12 = 48$$

$$\underline{\quad +12 \;+\; 12}$$

$$3x + 0 = 60$$

$$\frac{3x}{3} = \frac{60}{3}$$

$$x = 20$$

$$4x - 8(x+1) = 8$$

$$4x - 8x - 8 = 8$$

$$-4x - 8 = 8$$

$$\underline{\quad +8 \;+8}$$

$$-4x \quad = 16$$

$$-\frac{4x}{-4} = \frac{16}{-4}$$

$$x = -4$$

2. **Problem Set.**

   a. **Basic Skills Practice**.

1) $4n - 2 + 7n = 20$
2) $3(r-4) = 9$
3) $6x - 4(2x+1) = 12$
4) $3y - 2(3y+2) = 8$
5) $3(x+1) = 2x + 7$
6) $3w - 1 - 4w = 4 - 2w$
7) $8x - 5 = 4x + 4 - 2x$
8) $15 - 3y = y + 13 + y$
9) $4a - 4 = -2a + 14$
10) $4m - 5 = 3m + 7$
11) $2(x+1) = 3x - 3$
12) $2m - 4 = 2(6 - 7m)$

Name: ____________________________
Date: ____________________________
Advisor: __________________________

Solving Equations with Distributive Property – II

Calculating Fuel Consumption

1. **Background.** In order to find the sunken submarine, our ships need to remain at sea for days at a time, burning fuel to move and maintain the light, power, and air conditioning necessary to operate the ship and sustain the crew. Being very careful, a destroyer-type ship with a full load of fuel can remain at sea without refueling for approximately three weeks.

   It is the job of the ship's Chief Engineer (CHENG) and his engineers to monitor fuel use and report to the ship's Captain daily on how much has been burned, how fast it was burned and how much is left. With this information, the Captain can decide if he will keep his ship on its mission or take time out to refuel from a tanker at sea.

   A typical destroyer is pictured below. The

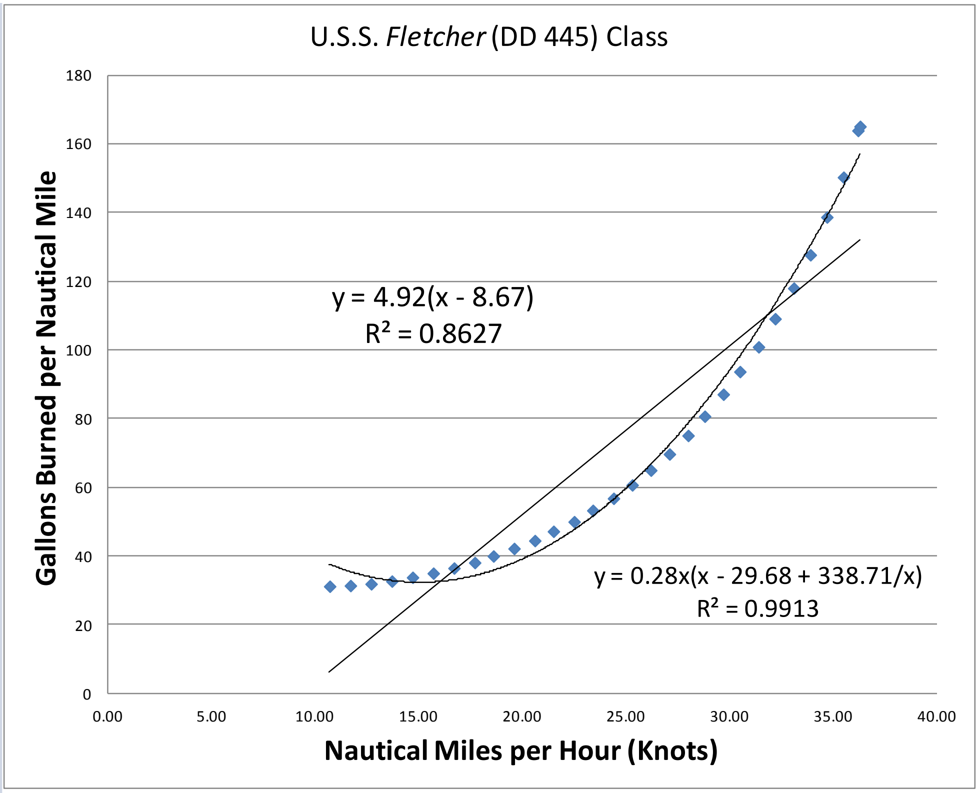
U.S.S. *Fletcher* (DD 445)

   The following table provides the data that the Chief Engineer and his engineers use to make their recommendations to the Captain.[2] The table shows that at a given speed in nautical miles per hour (knots), the ship will burn a certain number of gallons. This information is graphically depicted in the scatter plot next to the table.

---

[2] *From War Service Fuel Consumption of U.S. Naval Surface Vessels* (FTP 218), UNITED STATES FLEET, Headquarters of the Commander in Chief, 1945. Online at: http://www.ibiblio.org/hyperwar/USN/ref/Fuel/.

Knots	Gallons per NM
10.70	31.20
11.70	31.40
12.70	31.90
13.70	32.70
14.70	33.80
15.70	35.00
16.70	36.50
17.70	38.10
18.60	40.00
19.60	42.20
20.60	44.50
21.50	47.20
22.50	50.00
23.40	53.30
24.40	56.80
25.30	60.70
26.20	65.00
27.10	69.70
28.00	75.10
28.80	80.70
29.70	87.10
30.50	93.70
31.40	100.90
32.20	109.10
33.10	118.00
33.90	127.70
34.70	138.70
35.50	150.30
36.20	163.90
36.30	165.10



One line and one polynomial curve are fitted to the graph. Note that both have an R-squared value greater than 0.5. What this means is that either the linear equation or the curve's equation could be used to predict gallons burned for this class of destroyer. We will use both in the following exercises.

Linear Equation: $y = 4.92(x - 8.67)$

Polynomial Equation: $y = 0.28x(x - 29.68 + \frac{338.71}{x})$

2. **Problem Set.**

a. Complete the table below by predicting the gallons burned per nautical mile for the given speeds using each equation.

Example: For $x = 10\ knots$ using the linear equation:

$$y_1 = 4.92(10 - 8.67) = 4.92(1.33) = 6.54\ gallons\ per\ nm.$$

$x = Knots$	$y_1 = \frac{Gallons}{NM} = 4.92(x - 8.67)$	$y_2 = \frac{Gallons}{NM} = 0.28x(x - 29.68 + \frac{338.71}{x})$	**Difference**
10			
15			
20			
25			
30			
35			
40			

b. Which equation provides a better predictor of fuel use (particularly at lower and high speeds? Why?

Name: ___________________________
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The Bedford Incident – Questions

Place all answers on this sheet. 7 – 13 correct answers = 2.
14 – 20 correct answers = 3. 21 correct answers = 4.

1. What are the medical personnel in the ship's sick bay testing when the new medical officer arrives?

   Red cabbage.

2. Where does the item they are testing come from?

   A Russian submarine

3. What is the name of the Captain of U.S.S. *Bedford*?

   Captain Verlander

4. What does Mr. Munsford do for a living?

   Journalist

5. What did the German Commodore waiting in the Captain's Cabin do during the Second World War?

   Command a German U-boat

6. What is *Bedford* doing in the Denmark Strait?

   Hunting for a Russian submarine

7. What is the code name of the Russian submarine?

   Big Red

8. What angers the Captain about the Russian trawler they see?

   It dumps garbage in front of Bedford

9. What is the name of the Ensign whom the Captain yells at on the bridge?

   Ensign Ralston

10. What did the unidentified air contact turn out to be?

   A weather balloon

11. In what country's territorial waters does *Bedford* find the Russian submarine?

   Greenland

12. How many hours can the Russian submarine remain underwater?

   24 hours

13. What was Mr. Munsford doing when *Bedford* found the Russian submarine?

   Taking a shower

14. What does Commander, NATO, North Atlantic tell *Bedford* to do?

   Track the Russian submarine and take no other action

15. What does the second message from COMNATONORTH tell *Bedford* to do?

   Wait

16. What does the Russian submarine try to do to escape?

   Find a narrow path through the icebergs that Bedford *cannot get through*

17. What is the name of the *Bedford's* SONAR operator?
    Queffel

18. Why do the doctor and the Captain get into an argument?

    The Captain and the doctor disagree about what to do about Queffel when Queffel breaks down.

19. What does *Bedford* do when the Russian submarine finally surfaces?

    Tries to run the submarine down

20. Why does Mr. Ralston fire the rocket torpedo at the Russian submarine?

    He mistakes a casual remark from the Captain as an order to fire

21. What happens to *Bedford* at the end of the movie?

    She is destroyed by torpedoes fired from the Russian submarine

Name: ___________________________
Date: _____________________________
Advisor: __________________________

The Bedford Incident – Questions

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Game Rules

I. Set-up.

A. **Game Board:** The exercise will be played on the classroom game board. Each grid square is 50 nautical miles by 50 nautical miles in area.

B. **Scenario:** A United States *Los Angeles* class nuclear – powered submarine has sunk in the Central Pacific Ocean. It's last known position (known as the DATUM) was 350 nautical miles northwest of the Hawaiian Islands. This would put it in grid square 01036 on the game board. Because of ocean currents, however, the submarine could be in any one of the surrounding grid squares.

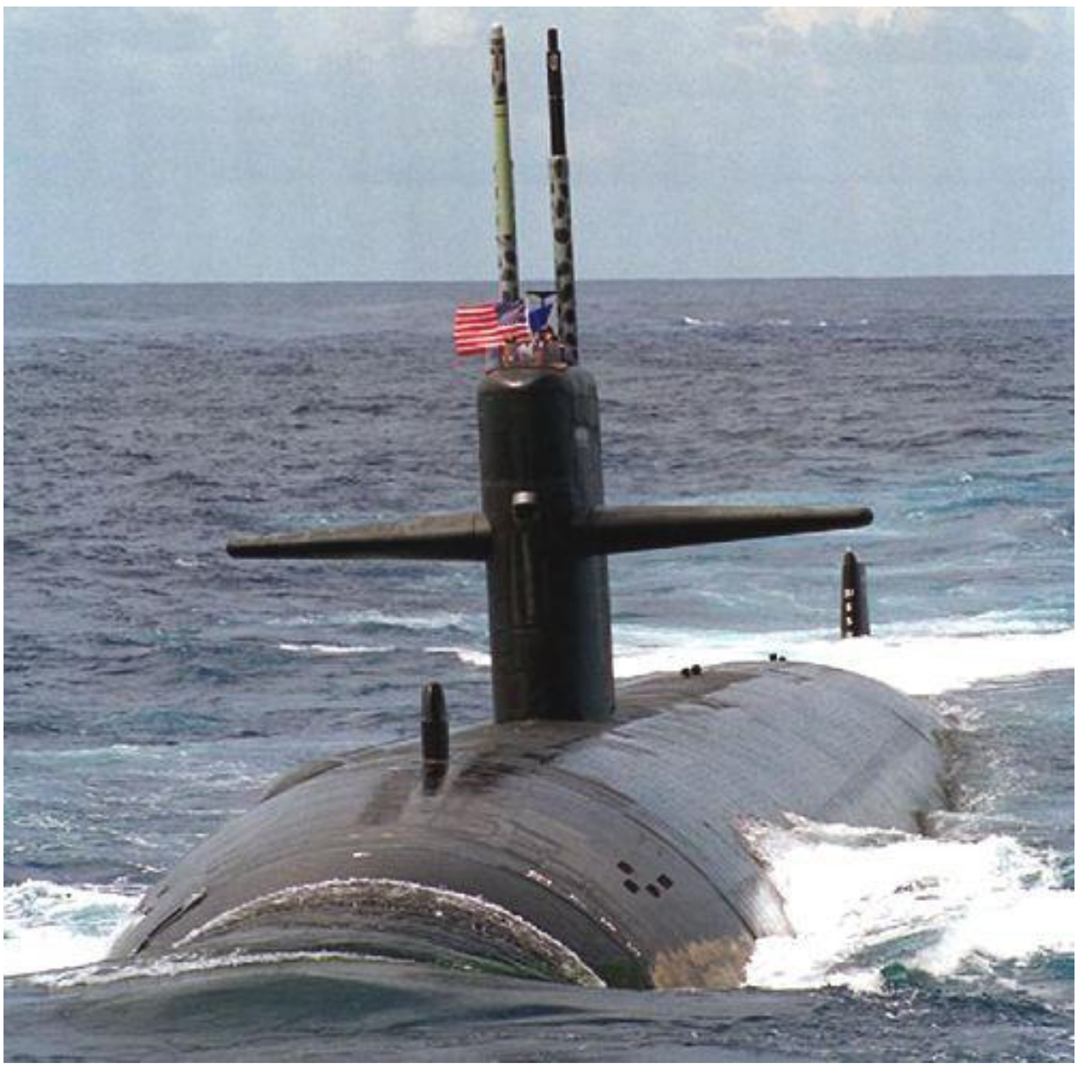


Figure 1: U.S. *Los Angeles* Class Nuclear-powered Submarine

Game Rules

Of the 141 Sailors onboard, there are 75 survivors who must be rescued within 72 hours or their power will fail and they will suffocate. The mission of your team is to find the submarine within 60 hours so that a deep sea rescue vehicle (DSRV) can descend to the submarine and rescue the survivors. **TS of the submarine is 60db and its SL is 30db.**

Figure 2: Deep Sea Rescue Vehicle (DSRV)

C. **Detection Probabilities and Ocean Environment:** The probabilities that the submarine is in each of the grid squares as well as the environmental impacts on SONAR and payoff points in each grid square are as follows:

Grid Square	Probability of Detection at 10 kts.	Time to Search at 10 kts.	Probability of Detection at 20 kts.	Time to Search at 20 kts.	Noise Level	Payoff Points
01036	1.00	20 hours	0.75	10 hours	30db	10
01037	0.50	20 hours	0.40	10 hours	30db	20
01024	0.50	20 hours	0.40	10 hours	10db	20
01048	0.50	20 hours	0.40	10 hours	10db	20
01025	0.40	20 hours	0.30	10 hours	10db	30
01049	0.40	20 hours	0.30	10 hours	10db	30
01035	0.30	20 hours	0.15	10 hours	30db	40
01023	0.20	20 hours	0.10	10 hours	10db	50
01047	0.20	20 hours	0.10	10 hours	10db	50

Figure 3: Location Probabilities and Ocean Environment

Game Rules

II. Teams.

A. **Composition.** Each class will be divided into teams of three.

B. **Team Leader.** Each team leader will be responsible for his or her teams decisions. The team leader will grade the members of his / her team. The team leader will be graded by the classroom teacher.

C. **Procedure.** Each team will be assigned a ship to conduct the search. The Transmission Loss (TL) associated with that ship's SONAR will be established based on the average of the team member's scores on the unit problem sets. The higher the (TL), lower the chances of detecting the sunken submarine. Therefore, the better the students performed on the problem sets, the better their chances of detecting the submarine.

Average Problem Set Grades	Transmission Loss (TL)
4	**10db**
3	**20db**
2	**30db**
1	**40db**

III. Game Play. The object of the game is to score the most points. Teams will be awarded points for selecting the grid square in which the submarine rests and for accurately detecting the submarine. Each team has 60 hours to search for and detect the sunken submarine.

A. Turns. A turn will be a team's search of a grid square. Each team will continue play until it runs out of hours of search.

B. Grid Selection: Each team will select a grid square to search and a search speed. Each grid square is assigned payoff points with higher points assigned to grid squares which have lower probabilities that the submarine is in that grid square. Teams will be awarded that grid square's payoff points if:

1. The team accurately detects the sunken submarine in that grid square.

2. The team completes a search of that grid square, failing to detect the submarine but it is subsequently determined that the submarine is in that grid square.

C. Speed Selection. Each team will select a search speed, either 10 knots or 20 knots. Detection probability drops at the higher speed but a team can complete a search more quickly.

D. SONAR Equation Solution. Each team will solve its SONAR equation for each grid square searched by finding its detection threshold (DT). Since DT is a measure of probability of detection, DT's will be assigned probabilities of detection, given the submarine is in the grid square, as follows:

Detection Threshold (DT)	**Conditional Probability of Detection $P(DT = \_\_\_\_)$**
0 – 9 db	0.20
10 – 19 db	0.50
20 – 29 db	0.75
30 db and greater	1.00

Correctly solving the SONAR equation results in the award of 10 points. For example:

1. Team 1 decides to search grid square 01025 at 10 knots. Team 1 achieved an average grade of "2" on the unit problem sets so the team's TL is 30db. The NL of 01025 is 10db.

2. The SONAR equation, therefore, is:

$$DT = TS + SL - 2TL - NL$$

$$DT = 60db + 30db - 2(30db) - 10db = 20db$$

E. Determination of Overall Detection Probability.

1. The classroom teacher, acting as umpire, will determine the team's overall detection probability. In the previous example, if the submarine is in the grid square, it would be:

$$P(overall) = P(DT = 20db) \times P(at\ speed\ of\ 20\ kts.) = 0.75 \times 0.40 = 0.30$$

2. Using a random number generator, the classroom teacher will determine the outcome of the search.

   a. If the random number is greater than $P(overall)$, then the classroom teacher will inform the team that they have not detected the submarine but that the submarine may still be in the grid square.

   b. If the random number is less than or equal to $P(overall)$, then the classroom teacher will inform the team that they have found the submarine if it is in the grid square or that it is definitely not in the grid square if, in fact, it is not. If it is not, the team will know that it need not search the square again.

F. Points Award. In the above example, points would be awarded as follows:

1. If the submarine is in grid square 01025:

$$100\ points\ (for\ finding\ the\ submarine) + 30\ points\ (payoff\ points) = 130\ points$$

2. If the submarine is in grid square 01025 but is not detected:

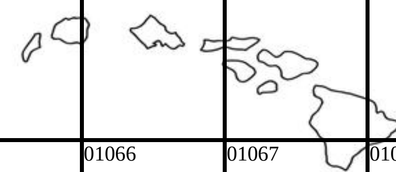
$$30\ points\ (awarded\ after\ the\ submarine\ is\ discovered\ by\ that\ team\ or\ someone\ else)$$

3. If the submarine is not in the grid square:

$$0\ points$$

Game Board

01021	01022	01023	01024	01025	01026	01027	01028	01029	01030	01031	01032
01033	01034	01035	01036 DATUM	01037	01038	01039	01040	01041	01042	01043	01044
01045	01046	01047	01048	01049	01050	01051	01052	01053	01054	01055	01056
01057	01058	01059	01060	01061	01062	01063	01064	01065	01066	01067	01068



Submarine Search Worksheet

Team: __________

Turn: __________

1. Team Grade Average: __________________________

   Team Transmission Loss (TL): __________________________

2. Search Speed Selection (10 or 20 kts.): __________________________

3. Grid Square Selection:

   01036
   01037
   01024
   01048
   01025
   01049
   01035
   01023
   01047

4. Grid Square Noise Level (NL):

   a. __________________________

   b. __________________________

5. Sonar Equation Solution:

DT =	TS +	SL -	2TL -	NL
a. _____ b. _____	a. _____ b. _____	a. _____ b. _____	a. _____ b. _____	a. _____ b. _____

6. Overall Probability of Detection:

   a. __________________________

   b. __________________________

$$P(overall) = P(DT = \_\_\_\_) \times P(at\ speed\ of\ \_\_\_\ kts.)$$

7. Search Outcome: __________________________

8. Points Awarded: __________________________

Name: ____________________________
Date: ______________________________
Advisor: ___________________________

Algebra I Practice Assessment – Equations

a. Solve for the variable in each equation:

a. $x + 7 = 10$

b. $-9 + x = 5$

c. $k + 6.8 = -4.2$

d. $8x = 72$

e. $\frac{q}{7} = 8$

f. $-\frac{x}{8} = 44$

g. $66 = 2f - 4$

h. $-4 = \frac{x}{2} - 5$

i. $n - 6 = 2n - 14$

b. Solve for the variable in each equation:

a. $8x - 1.5 = 2x$

b. $6p - 12 = -4p + 18$

c. $-10x + 3 = -3x + 12 - 4x$

d. $5(x - 3) = 10$

e. $7x - 2(x + 6) = -2$

f. $3 - 4x = 5(x + 6)$

g. $x + \frac{5}{8} + \frac{3x}{4} = \frac{2}{3} + 5x$

h. $2x + 3 = 3x + 5$

i. $5m - 3(2m - 3) = 2(m + 3)$

Name: ____________________________
Date: ______________________________
Advisor: ___________________________

Algebra I Practice Assessment II – Equations

Show all work on separate sheets of paper stapled to this one. Put answers on this sheet.

1. Solve for the variable in each equation:

a. $x + 9 = 15$ b. $11x = 132$ c. $k + 4.4 = -5.6$

e. $p/12 = 10$ e. $-\frac{x}{15} = 40$ f. $2n - 8 = 3n - 16$

2. Solve for the variable in each equation:

a. $10x - 4.8 = 2x$ b. $6p - 15 = -5p + 18$ c. $7(x - 4) = 14$

e. $9x - 4(x + 6) = -4$ e. $38 - 11x = 9(x + 2)$ f. $x + \frac{5}{9} + \frac{3x}{18} = \frac{2}{6} + 5x$

3. Solve each equation for the indicated variable:

a. $p + b = d, for\ b$ b. $y = zx + 12, for\ z$ c. $15t = 7r, for\ t$

d. $\frac{ma}{q} = q, for\ a$ e. $4a + 4b = 28c, for\ b$ f. $6y = 2x + 3b, for\ b$

Name: ___________________________
Date: ____________________________
Advisor: __________________________

Algebra I Assessment – Equations

Show all work on separate sheets of paper stapled to this one. Put answers on this sheet.

1. Solve for the variable in each equation:

   a. $x + 14.5 = 21.5$
   b. $\frac{12x}{15} = 4$
   c. $15n - 6 = 9n - 42$

   d. $9(x - 2) = 27$
   e. $38 - 12x = 4(2x + 2)$
   f. $2x + \frac{1}{4} + \frac{4x}{6} = \frac{2}{9} + x$

   g. $14x - 5.5 = 3x$
   h. $37.68\ in. = 2\pi r\ in.$
   i. $100°C = \frac{5}{9}(x°F - 32)$

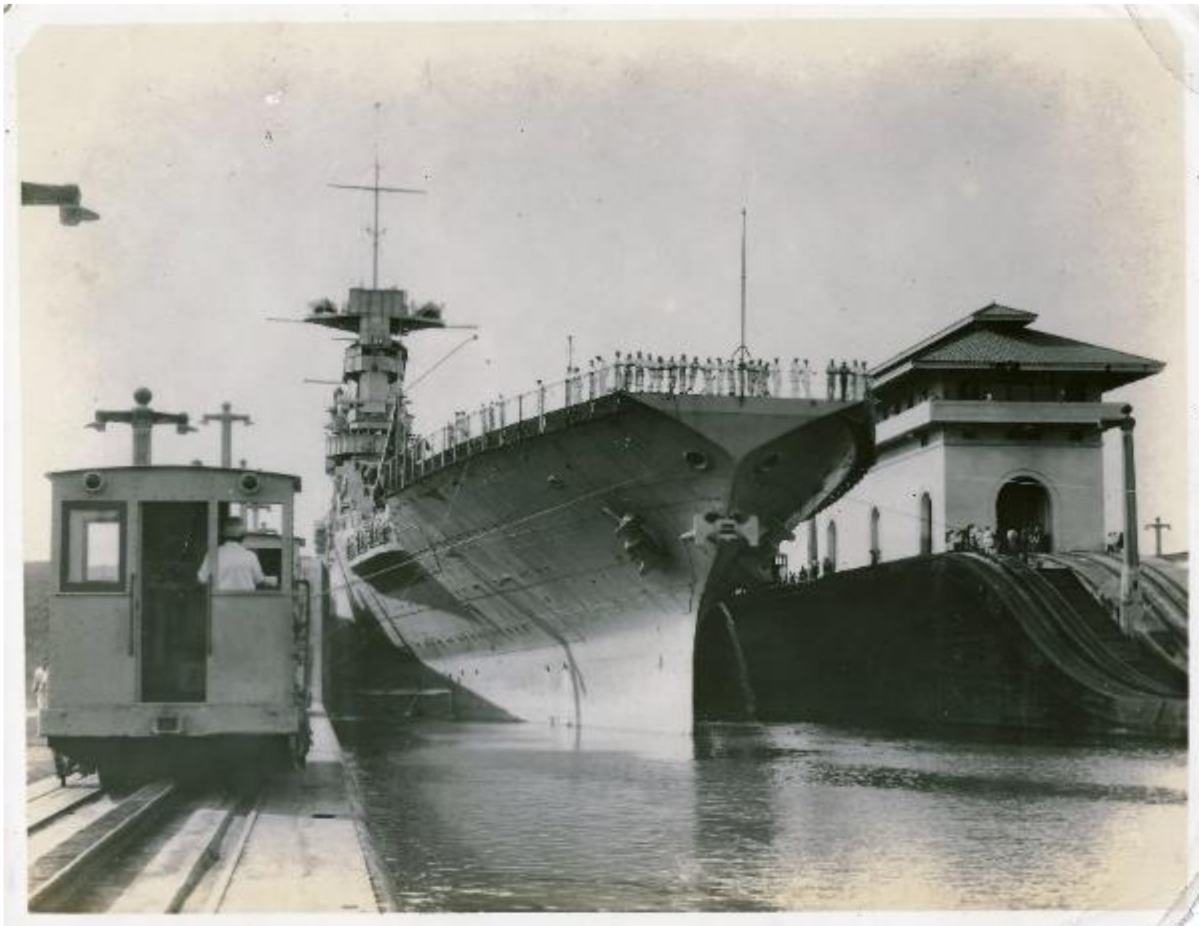
2. Solve each equation for the indicated variable:

   a. $e - f = g, for\ g$
   b. $6\alpha = 2\beta + 3\Phi, for\ \Phi$
   c. $v = \frac{P}{m}, for\ m$

3. Use the formula $A = \frac{1}{2}h(b_1 + b_2)$ to find the length of a missing base for a trapezoid with the following dimensions. Round your answers to the nearest hundredth.

   A. A = 200, b_1 = 24, h = 10

PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth



"A Voyage to the Panama Canal"

Basic Navigation and Ship Operation

Thomas R. Beall
Captain, U. S. Navy (Ret.)

2013 – 2014 School Year
Revision 3

Name: ________________________

Date: ________________________

Advisor: ________________________

A Voyage to the Panama Canal

1. **Background of the Panama Canal.**[3] The 50 mile – long international waterway known as the Panama Canal allows ships to pass between the Atlantic Ocean and Pacific Ocean, saving about 8000 miles (12,875 km) from a journey around the southern tip of South America, Cape Horn. Construction of the canal was completed in 1914.

   The canal makes the trip from the east coast to the west coast of the U.S. much shorter than the route taken around the tip of South America prior to 1914. Though traffic continues to increase through the canal, many oil supertankers and large warships cannot fit through the canal. There is even a class of ships known as "Panamax," those built to the maximum capacity of the Panama Canal and its locks.

   It can take anywhere from 8 to 25 hours to traverse the canal through its three sets of locks (including waiting time due to traffic). Ships passing through the canal from the Atlantic Ocean to the Pacific Ocean actually move from the northwest to the southeast, due to the east-west orientation of the Isthmus of Panama.

2. **Locks.**[4] The canal's locks are large basins which raise or lower ships to the level of the seaway the ship is entering by raising or lowering the water level under a ship. Each lock is 110 feet wide by 1000 feet long. The amount of concrete used for all locks was 3,440,488 cubic meters. The entrance and exit to each lock is through large gates are 65 feet wide and 7 feet thick. The heights vary from 47 to 82 feet depending on its position.

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[3] Rosenberg, Matt. *Panama Canal*. Accessed online Nov. 28, 2011 at: http://geography.about.com/od/specificplacesofinterest/a/panamacanal.htm.

[4] Source: Panama Canal Authority. Accessed online Nov. 28, 2011 at: http://www.pancanal.com/eng/noticiero/canal-faqs/physical/physical-faqframeset-v2.html

Water enters the lock chamber through a system of main culverts, which are the same size as the Hudson River tubes of the Penn Central Station Railroad and are "large enough to drive a train through." From these main culverts, 10 sets of lateral culverts extend under the lock chamber from the side wall and 10 sets from the center wall. Each lateral culvert has a set of 5 openings, each measuring 4-1/2 feet in diameter. As the water is released into the main culverts, by way of a gravity flow system, by opening upper end valves and closing lower end ones, it is diverted into the 20 lateral culverts and distributed through 100 openings in the chamber floor.

An average of 52 million gallons of fresh water is used each time a lock is filled. A lock chamber takes approximately eight (8) minutes to be filled. All water used in any lock chamber comes from Gatun Lake. This lake covers 163.38 square miles and was created when Madden Dam was built. At one time, Gatun Lake was the largest artificial lake in the world.


N 8°57'11.52"
N 8°56'45.6"
N 8°56'19.68"
W 79°34'40.8"
W 79°34'14.88"
W 79°33'48.96"
W 79°33'23.04"
Calle Tabernilla
Owen St.
Calle Balboa
Calle Plank
Calle La Boca
Calle Martinida
Calle Empire
Calle Akee
Calle Mango
Calle Croton
Calle Perlas
Calle College
Av Amador
Puente de las Americas
Carretera Panama-Arraijan
Acceso a Veracruz
Calle Cedar
Calle Miller
Image © 2011 DigitalGlobe
© 2011 Google
Imagery Date: 12/25/2009
2004
8°57'01.72" N 79°33'46.18" W elev 75 ft

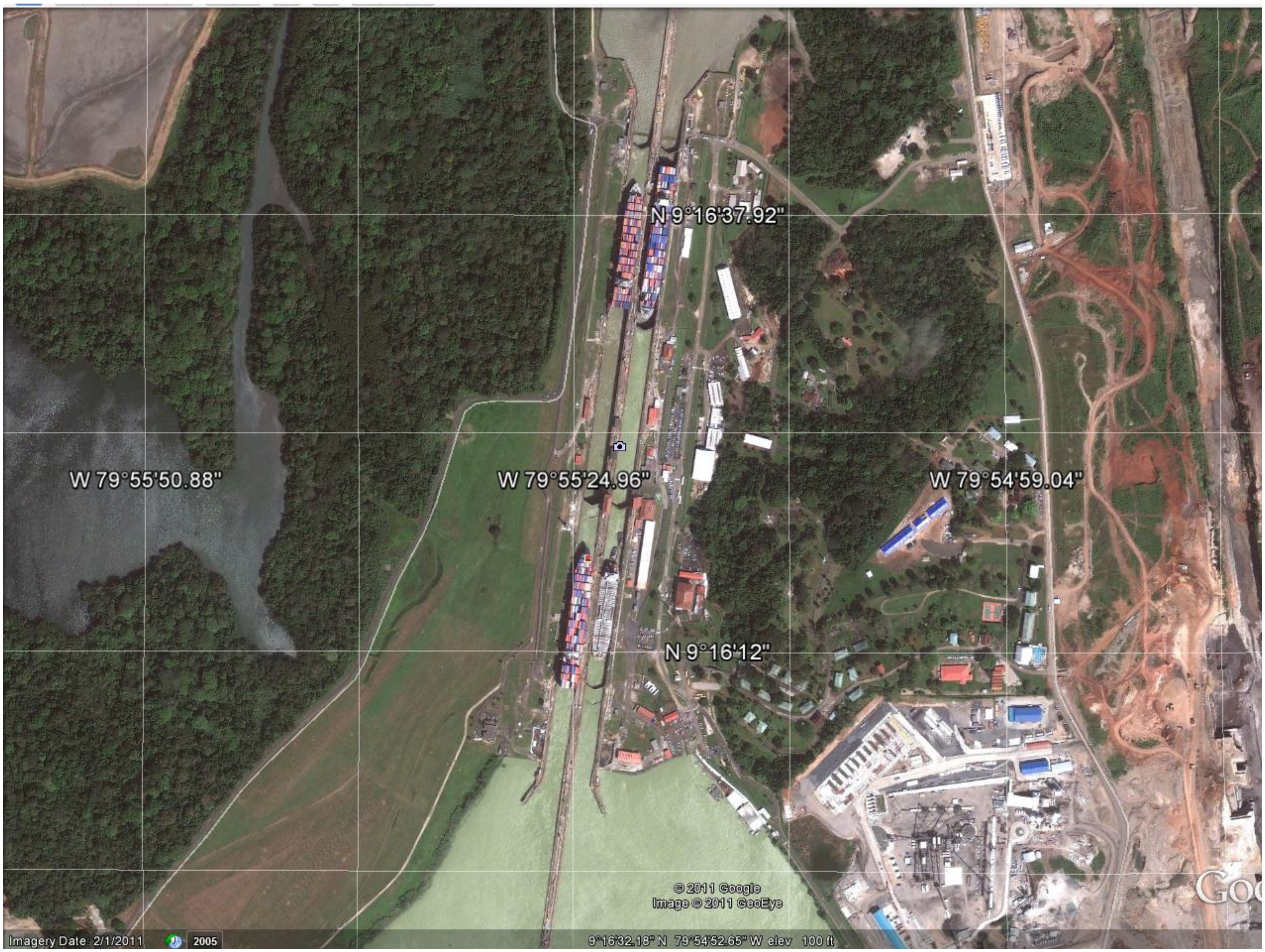
N 8°59'47.04"
56"
W 79°35'32.64"
W 79°35'6.72"
N 8°59'21.12"
Image © 2011 DigitalGlobe
© 2011 Google
Calle Morse
Cidade-do-Saber
Via-Vieja-De-Gaillard
Calle-Landrick
Calle-Hamilton
Calle-Salleman


N 9°01'56.64"
W 79°38'8.16"
W 79°37'42.24"
N 9°01'30.72"
Puente Centenario
Via Centenario
Calle Natá
Calle Sigri
Calle Espalaba
Calle Otoque
Paraiso Rd.
Calle Jaraba
Calle Guiana
Calle Hamilton
Calle Palo Alto
Calle Chiriquí
Image © 2011 DigitalGlobe
© 2011 Google
Imagery Date: 12/25/2009
2002
9°01'43.42" N 79°37'37.48" W elev 138 ft


W 79°39'25.92"
N 9°03'14.4"
W 79°39'
Image © 2011 DigitalGlobe
© 2011 Google
Imagery Date: 1/11/2008
2002
9°03'04.66" N 79°38'53.27" W elev 246 ft


N 9°07'7.68"
W 79°45'2.88"
W 79°44'36.96"
W 79°44'11.04"
N 9°06'41.76"
© 2011 Google
Image © 2011 GeoEye
2010
9°07'20.26" N 79°44'01.92" W elev 93 ft
Google earth
Eye alt 6974 ft


W 79°46'20.64"
© 2011 Google
Image © 2011 GeoEye
Imagery Date 2/20/2011
2010
9°06'41.87" N 79°46'15.80" W elev 92 ft


N 9°16'37.92"
W 79°55'50.88"
W 79°55'24.96"
W 79°54'59.04"
N 9°16'12"
© 2011 Google
Image © 2011 GeoEye
Imagery Date: 2/1/2011
2005
9°16'32.18" N 79°54'52.65" W elev 100 ft


N 9°23'32.64"
W 79°55'24.96"
N 9°23'6.72"
W 79°54'59.04"
W 79°54'33.12"
© 2011 Europa Technologies
© 2011 Google
Image © 2011 GeoEye
2/1/2011
2004
9°23'12.93" N 79°54'52.86" W elev -12 ft
Google

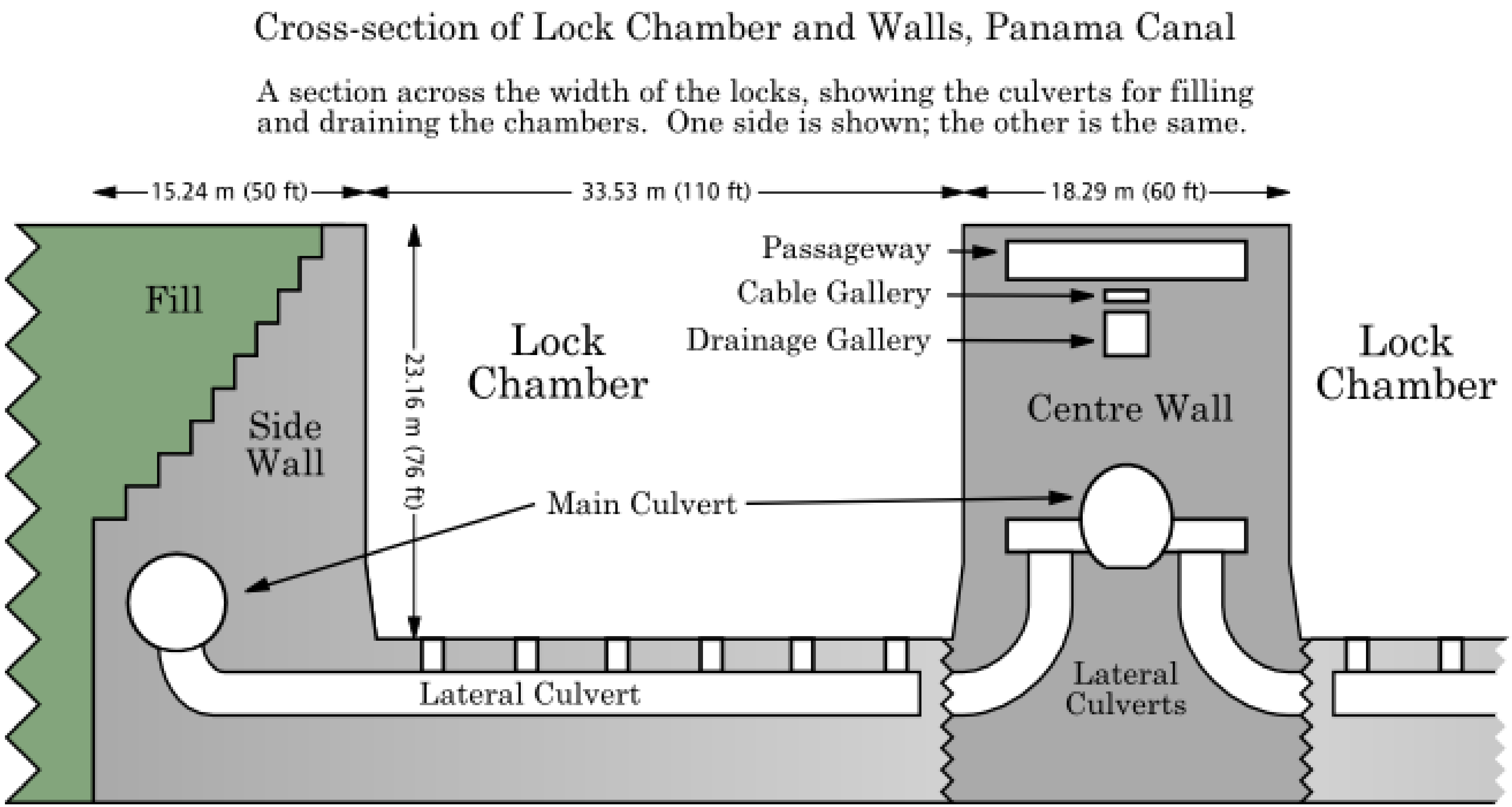


Figure 1: Lock Cross-section

3. **Transit Planning.** We will plan a voyage from the U. S. Naval Station at San Diego, California (32° 40.75’ N, 117° 07.64’ W) to the Panama Canal (9° 23’ 19” N, 79° 55’ 10” W).

Figure 2: Naval Station San Diego in the 1940’s

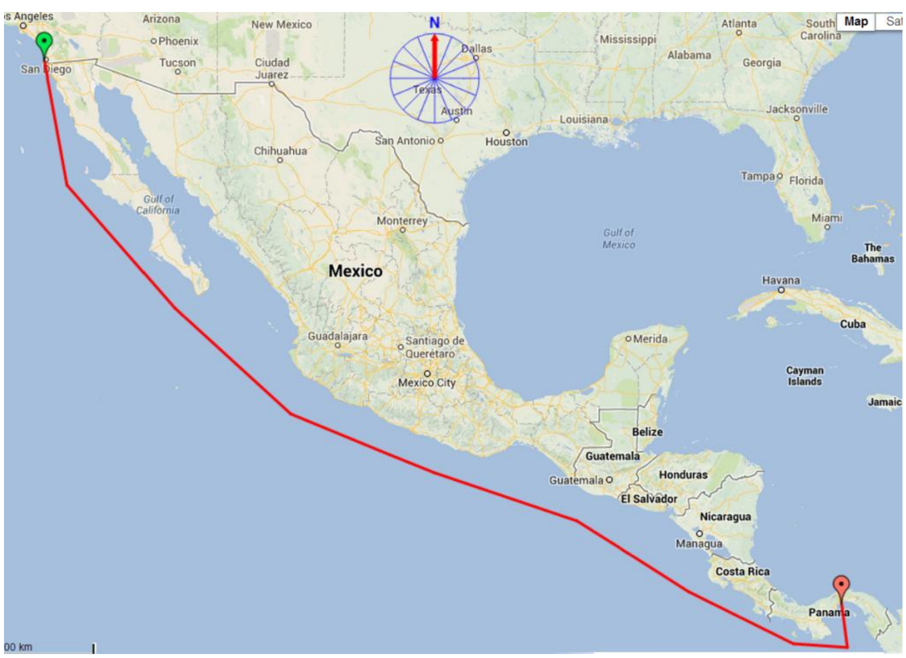
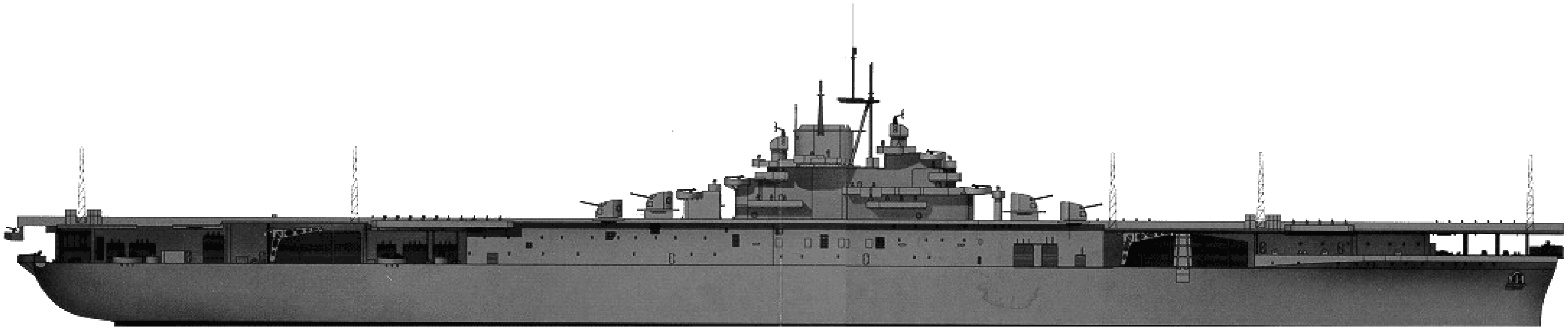
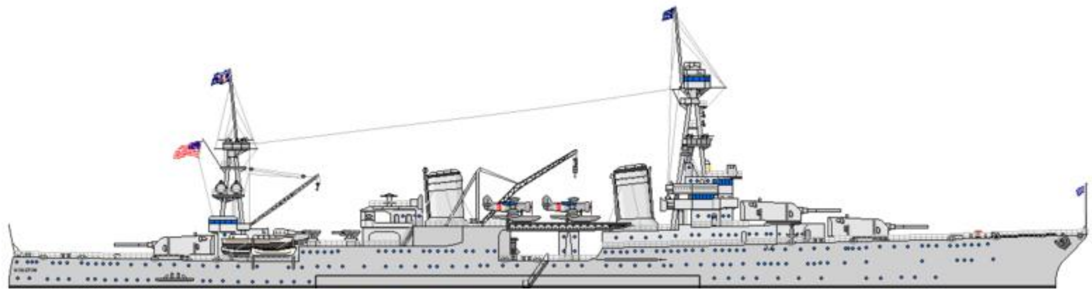


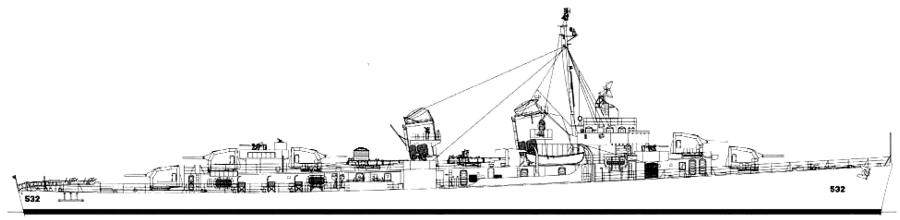
Figure 3: Transit Track



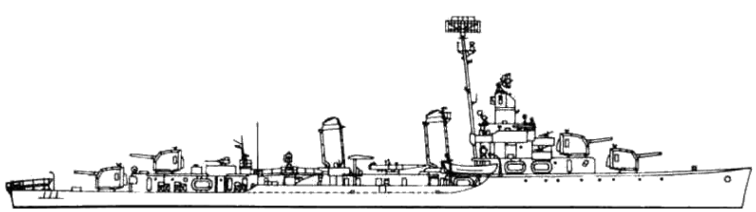
U.S.S. *Essex* (CV – 9)



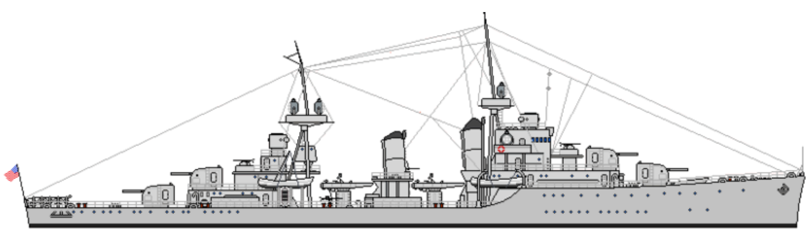
U.S.S. *Augusta* (CA – 31)



U.S.S. *Fletcher* (DD – 445)



U.S.S. *Benson* (DD – 421)



U.S.S. *Conyngham* (DD – 371)

Panama Canal Transit Planning – Task I
Due at the Beginning of Class on Friday, November 8th, 2013

1. **Background.** A U. S. Navy Task Force is traveling from San Diego, CA to the Panama Canal. Before the Task Force arrives at the Canal, each ship must provide the Panama Canal Authority (ACP) with information about that ship. You will use the attached sheets, online resources, and the models of your ships provided to assemble the data. The Task Force is organized as follows:

Task Force 5
Commander (CTF FIVE): Captain T. R. Beall, USN (Ret.)

Ship	**Billet**	**Period 3**	**Period 5**	**Period 6**
U.S.S. *Essex*	Captain			
(CV 9)	Navigator			
Aircraft Carrier	Quartermaster			
	Chief Engineer			
	M.P.A.			
U.S.S. *Augusta*	Captain			
(CA 31)	Navigator			
Heavy Cruiser	Quartermaster			
	Chief Engineer			
U.S.S. *Fletcher*	Captain			
(DD 445)	Navigator			
Destroyer	Quartermaster			
	Chief Engineer			
U.S.S. *Benson*	Captain			
(DD 421)	Navigator			
Destroyer	Quartermaster			
	Chief Engineer			
U.S.S. *Conyngham*	Captain			
(DD 371)	Navigator			
Destroyer	Quartermaster			
	Chief Engineer			

Table 1: Navigation Unit Assignments

2. **Instructions.** Each person will complete the attached form, attaching the required graph and arithmetic work. The team "Captain" will approve and sign each person's submission. **No signature, no credit.**

Name: ________________________

Date:________________________

Advisor:________________________

Panama Canal Ship Data

1. Ship's Name: ____________________
2. Captain's Name: ____________________
3. Ship's Measurements (*Dictionary of American Naval Fighting Ships*[5]):
    a. Displacement (dp): ____________________
    b. Length Overall (l): ____________________
    c. Beam (b): ____________________
    d. Draft (d): ____________________
4. Blind Distances:
    a. Forward: ____________________
    b. Aft: ____________________
5. Fuel Statistics (FTP 218[6]):
    a. Full Load (gallons): ____________________
    b. Fuel Rate at 200 R.P.M. (Table 1):
        i. Gallons / hour (column 2): ____________________
        ii. Speed (column 9): ____________________
        iii. Gallons / engine mile (column 11): ____________________

[5] http://www.history.navy.mil/danfs/
[6] http://www.ibiblio.org/hyperwar/USN/ref/Fuel/

c. From Table 1, use column 1 and column 11 to construct a table and a graph of gallons burned per engine mile for each propeller RPM listed. Fit a linear model and a polynomial model to the data and display the equations (see example on next page).

i. Attach the graph to this sheet.

ii. Write the equations below:

Linear Model: $y =$ ______________________________

Polynomial Model: $y =$ ______________________________

____

Signature of Commanding Officer

Example of a graph of propeller shaft R.P.M. vs. gallons burned per engine mile – U.S.S. *Massachusetts* (BB – 59).

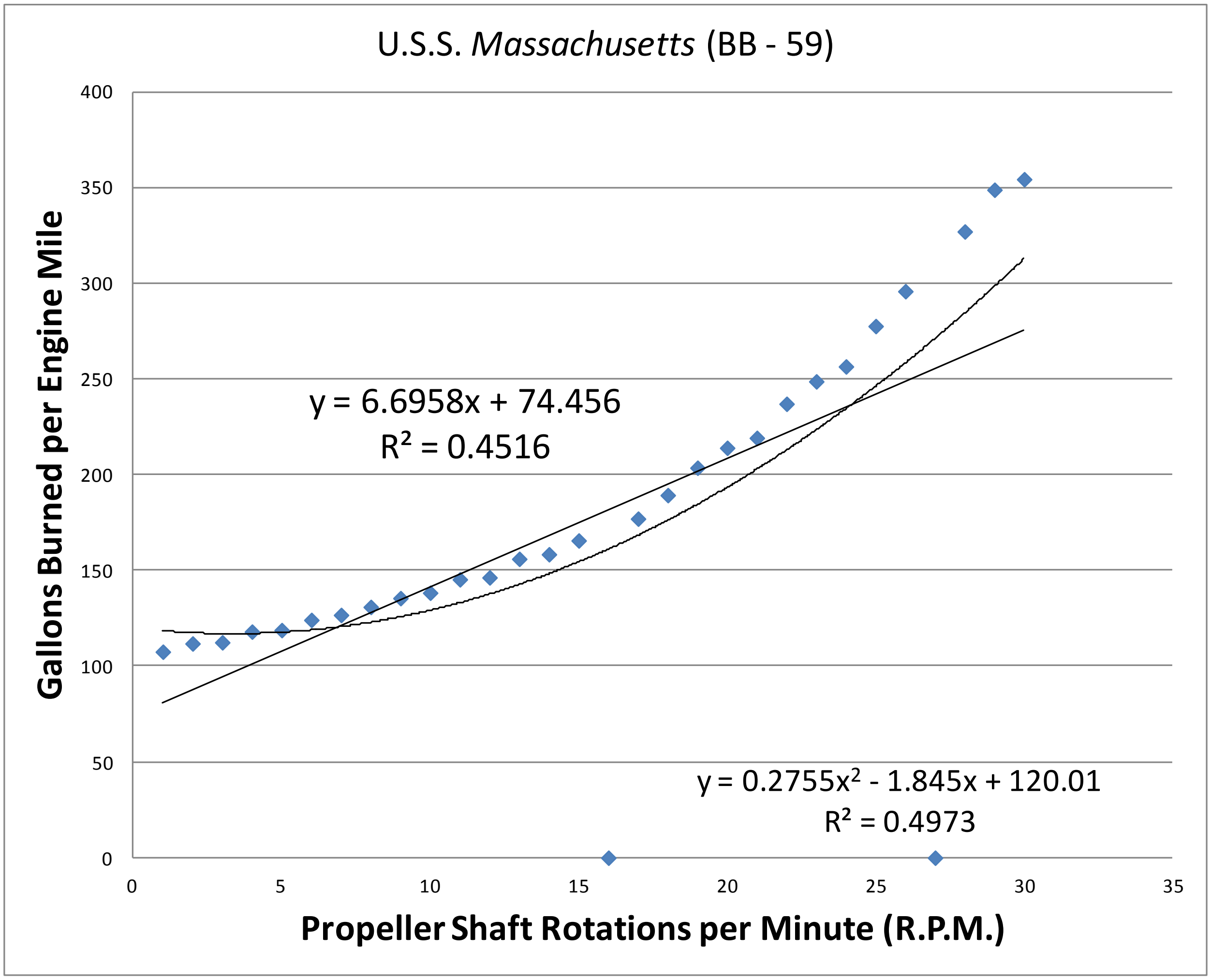


Figure 4: U.S.S. *Massachusetts* (BB – 59) R.PM. vs. Gallons Burned per Engine Mile

Finding "Blind Distance"

1. **Background.** In order to proceed through the canal, you must provide the ACP with a lot of information about your ship. One key piece of information is the ship's "blind distance". This is the distance from the front of the ship (the bow) or the back of the ship (the stern) over which you cannot see the water because it is hidden by the ship. The following diagrams depict the blind distances for an officer standing the ship's bridge (Conning Position No. 1) from the bow and the stern.

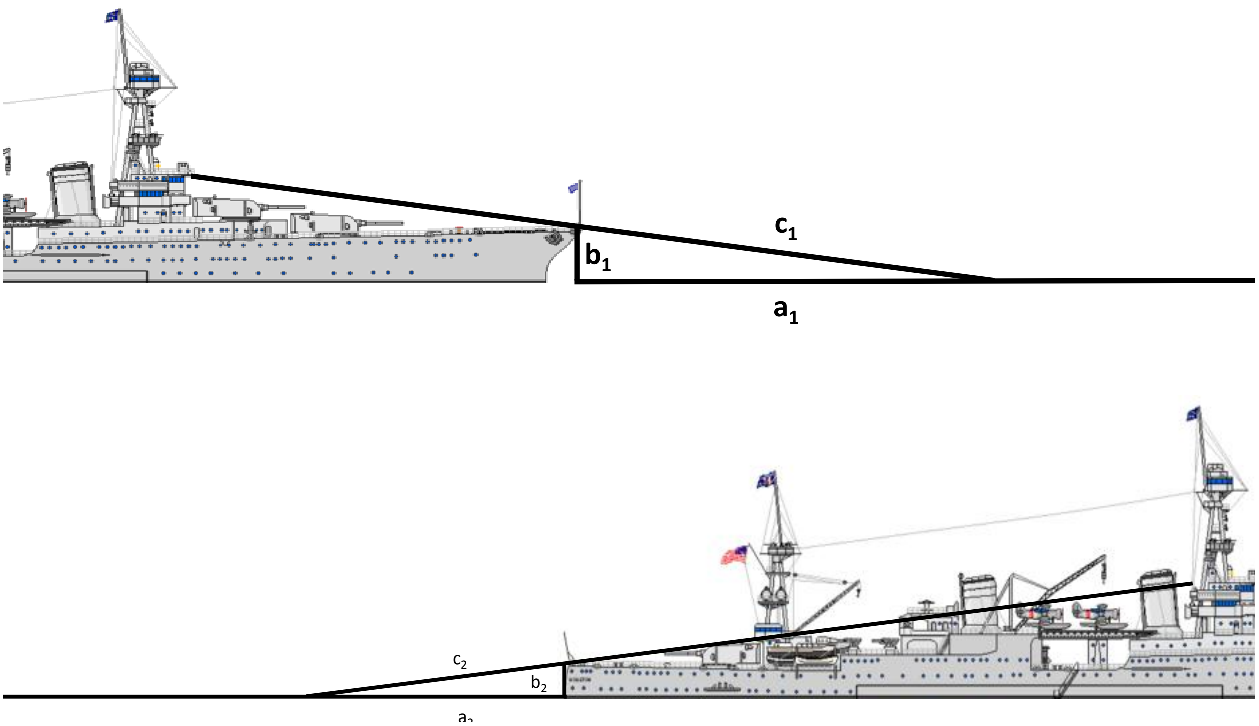
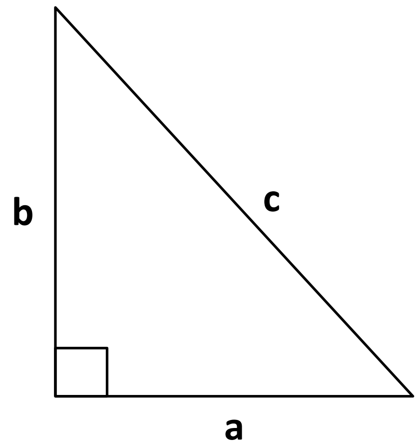


Figure 5: Blind Distances for U.S.S. *Augusta* (CA – 31)

2. To find the "blind distance" from the bow and the stern, you can use a relationship known as the Pythagorean Theorem which states that for any right triangle with sides *a* and *b* and hypotenuse *c*, the following relationship is true:



$$a^2 + b^2 = c^2$$

3. **Task.** To complete the data forms, you must determine the forward and aft "Blind Distances" for your ship. You will use a small scale model of your ship to complete the computations.

    a. Determine the scale factor to convert the model's length in inches to the actual length of the ship in inches.

    b. Measure the distance from the waterline to the deck of the model and use the scale factor to find the actual distance.

    c. Measure the distance from the bridge, through the forepeak, to the waterline of the model and use the scale factor to find the actual distance.

    d. You now have c_i and b_i. Use the Pythagorean Theorem to find a_i. Convert inches to feet and inches for the final answer.

    Ensure you show all work on separate sheets of paper, attached to the data form.

Name: ________________________

Date:_________________________

Advisor:_________________________

Panama Canal Transit Planning – Task II
Due at the Beginning of Class on Monday, November 18th, 2013

1. The drawing on the next page is a cross – section view of the canal locks, showing maximum clearances. Your first task will be to determine if your ship can move safely through the locks. Reproduce the lock cross-section drawing on the next page using a scale factor of 0.6.

    a. Multiply each dimension by 0.6 to convert the measurement in meters into centimeters.

    b. Tape four pieces of graph paper together.

    c. Using a ruler and pencil and the centimeter measurements, reproduce the lock cross – section drawing.

2. Draw a cross-section of your ship hull using the same scale factor and place it in the center of the lock to determine if your ship will fit into the lock.

    a. Convert the beam and draft of your ship from feet and inches to meters.

    b. Multiply the new beam and draft measures by 0.6 to convert to centimeters.

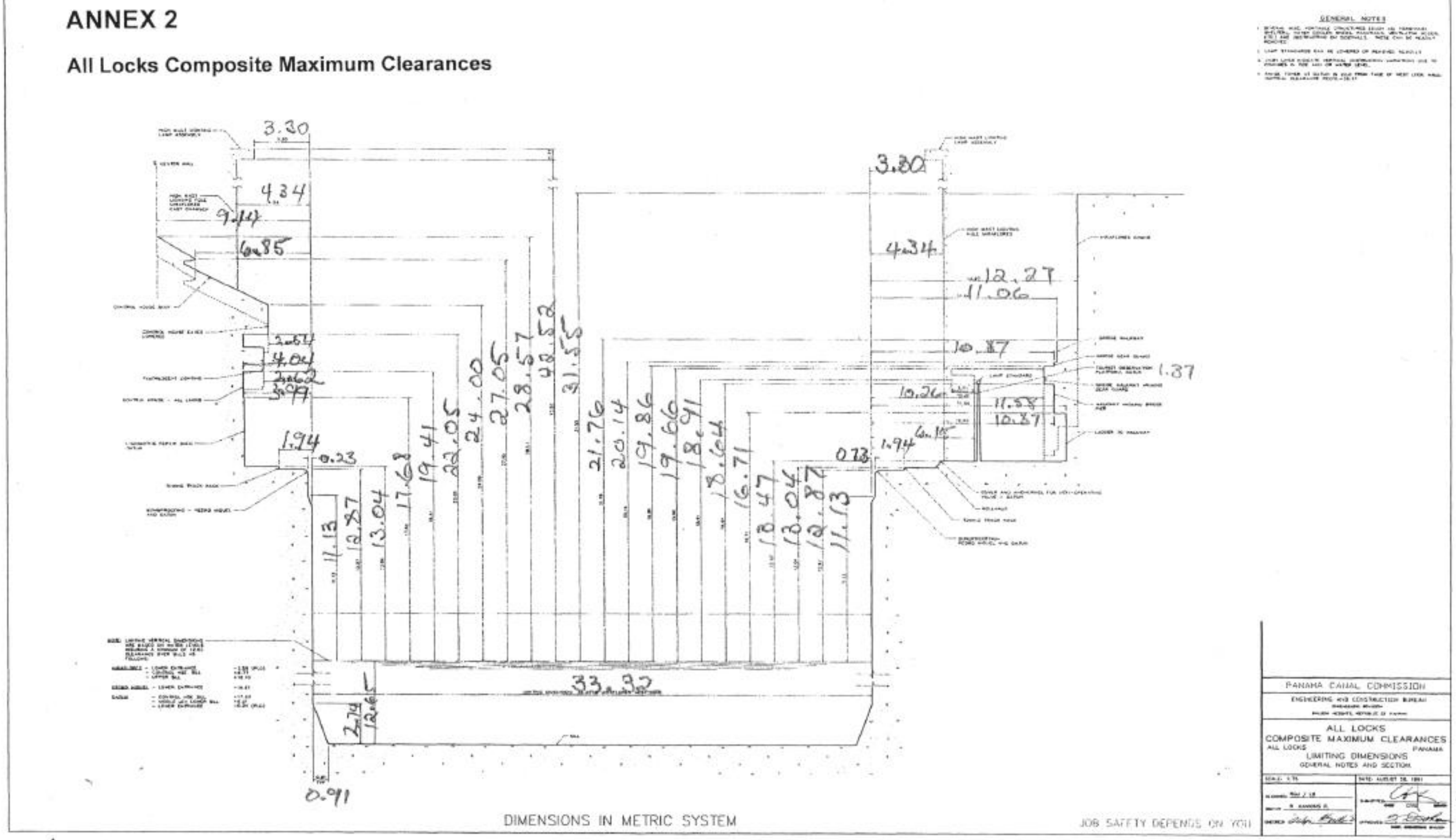
    c. Draw the cross-section of the ship into the center of the lock.

    d. Determine the distance in meters from each side of the ship to the lock wall and the bottom of the ship to the lock floor. Label those distances on your lock drawing.

    e. Your drawing will look like figure 2 on the page following the lock drawing.

ANNEX 2

All Locks Composite Maximum Clearances



SUPERSEDS SAME DWG. 5044-424, DATED 10-17-63

5044-424 SHEET 1 OF 2

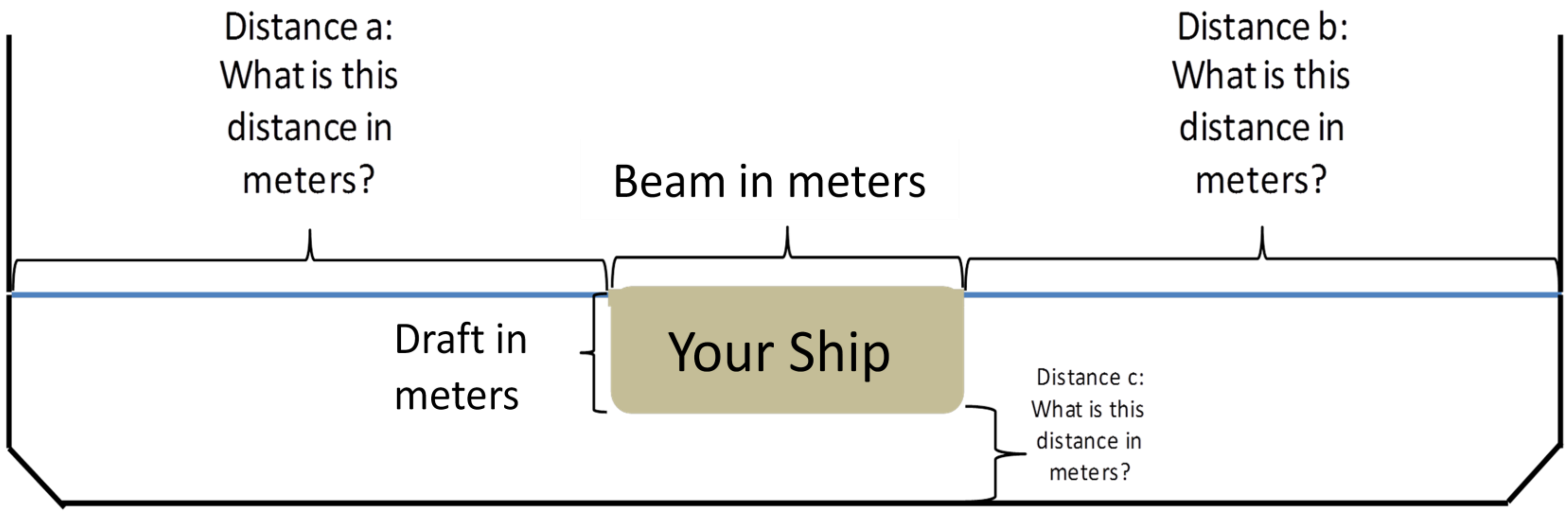


Figure 6: Cross-section of Your Ship in Your Lock Drawing

Measurement Conversions

(Use this page to make your measurement conversions)

$$\mathbf{0.6 \times m = c}$$

Where: $m = the\ actual\ measurement\ in\ meters$

$c = the\ scaled\ measurement\ in\ centimeters$

m	$\times\ scale\ factor =$	c
	$\times 0.6 =$	
	$\times 0.6 =$	
	$\times 0.6 =$	
	$\times 0.6 =$	
	$\times 0.6 =$	
	$\times 0.6 =$	
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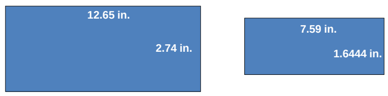
Lock Drawing Slides

Slide 1

Two Geometry Concepts

Congruence: The relationship between figures having the same shape and the same size; congruent segments are segments that match exactly, congruent angles have the same measure.

Similarity: The relationship between figures having congruent angles and proportional sides.



Are these rectangles similar?

Slide 2

Beginning Your Lock Drawing

Step 1: Number Conversions. Multiply each measurement by 0.6 and then convert to centimeters.

m	scale factor	d
12.65	0.6	7.59
2.74	0.6	1.64
0.91	0.6	0.55
11.13	0.6	6.68

Slide 3

Beginning Your Lock Drawing

Step 2: Limiting Dimension line. Find the "Limiting Dimension" line at the bottom of the drawing handed out in class. It measures 33.32 meters so, on your drawing, it will measure:

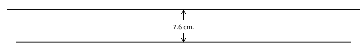
$$33.32 \times 0.6 = 20\ cm.$$

Draw this line near the bottom of your graph paper.

Slide 4

Beginning Your Lock Drawing

Step 3: Lock Sill. Next, measure 7.6 cm. below the "Limiting Dimension" line and draw a 2nd line, centered on the first, that is 1.8 cm. shorter. This line is the "sill" of the lock:

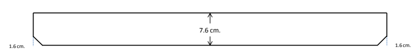


Slide 5

Beginning Your Lock Drawing

Step 4: Bottom of Lock.

- Draw 2 diagonal line segments from the sill such that the highest point of each segment is 1.6 cm. above the level of the sill and directly below the endpoint of the "Limiting Dimension" line.
- Connect the highest point of each segment to the "Limiting Dimension" line.

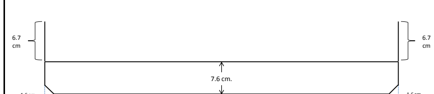


Slide 6

Beginning Your Lock Drawing

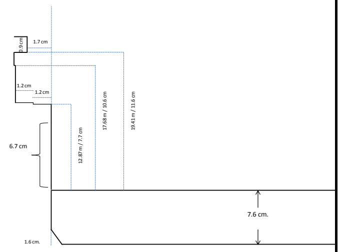
Step 5: Basin Walls. From the "Limiting Dimension" line, draw two vertical lines at each end, 6.7 cm. in length. You have now drawn the lock basin (the part that fills with water).

Now, complete the drawing.



Slide 7

Drawing the Control Tower



Slide 8

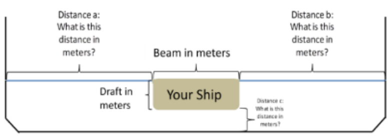
Drawing the Control Tower

If you want to shoot for a "5" or greater:

- **Use large poster board**
- **Use a scale factor of 0.8**
- **Decorate your drawing**
  - **Buildings and walls illustrated with colors**
  - **Water illustrated**
  - **Pumps and culverts per diagram near front of packet**
  - **Ship drawn / illustrated and colored**

Slide 9

Drawing your ship in the locks



Name: ________________________

Date: ________________________

Advisor: ________________________

Task III – Panama Canal Transit Fuel Consumption
Due at the Beginning of Class on Wednesday, November 20th, 2013.
Show all work on separate sheets of paper attached to this one.

3. **Background.** On her journey from San Diego to the Panama Canal, your ship will burn a lot of fuel. It will be important for you to keep track of how much your ship burns and how much is left so that you can decide if and when it is time to refuel your ship. How fast a ship is going and how much fuel is burned over a given mile is a function of how fast the propeller shaft is turning per minute (R.P.M.). The table below provides the propeller shaft R.P.M. for a given speed for all five ships.

	R.P.M.				
Knots	***Essex***	***Augusta***	***Fletcher***	***Benson***	***Conyngham***
10	77.47	96.12	94.75	81.67	189.99
15	109.36	140.24	142.55	123.14	239.84
20	146.19	189.15	193.36	164.60	289.69
25	187.97	242.85	247.17	206.06	339.55
30	234.70	301.34	303.99	247.53	389.40
35	286.36	364.62	363.82	288.99	439.25
40	342.98	432.69	426.65	330.46	489.10

Table 2: Propeller Shaft R.P.M. vs. Ship's Speed

4. **Step 1.** Using table 2, fill in the shaft R.PM. for your ship in column 2 of table 3 for each speed in column 1.

5. **Step 2.** Using the graph and equations you completed for your Panama Canal Ship Data Submission, complete the following table. Using your ship's R.P.M. data shown in the second table. Show all work on separate sheets of paper attached to this one.

$\boldsymbol{Knots}$	$\boldsymbol{x = R.P.M.}$	**Gallons Burned per nm using Linear Equation** $\boldsymbol{y_1 = \frac{Gallons}{nm} =}$	**Distance Traveled in 24 Hours** $\boldsymbol{y_2\ nm = 24\ hours \times x \frac{nm}{hour}}$	**Gallons Burned in 24 Hours**
10				
15				
20				
25				
30				
35				
40				

Table 3: Gallons Burned per Nautical Mile vs. Propeller Shaft R.P.M.

Example: If your linear equation is $4.92x - 42.6564$, then, for a speed of 10 knots:

a. R.P.M. for 10 knots is 75.15. Gallons burned per nautical mile at 75.15 R.P.M. equals:

$$1.53 \times (75.15) - 42.6564 = 114.9795 \frac{gallons}{nm}$$

b. Distance traveled in 24 hours equals:

$$24\ hours \times 10 \frac{nm}{hour} = 240\ nm$$

c. Gallons burned in 24 hours equals:

$$114.9795 \frac{gallons}{nm} \times 240\ nm = 27595.08\ gallons$$

6. **Step 3.** Next complete the following table. Show all work on separate sheets of paper attached to this one.

$x = Knots$	y_1 = Gallons Burned in 24 Hours (from previous table)	y_2 = Full Load of Fuel (from 5. a. of Panama Canal Ship Data Submission)	Gallons Remaining after 24 Hours $= y_2 - y_1$	Percent of Full Load Burned after 24 Hours $= \frac{y_1}{y_2}$
10				
15				
20				
25				
30				
35				
40				

Table 4: Fuel Remaining After 24 Hours

Example: If, at 10 knots, you burn 27595.08 gallons, and your full load of fuel is 588,625 gallons:

a. Gallons remaining after 24 hours:

$$= 588625\ gallons - 27595.08\ = 561029.92\ gallons$$

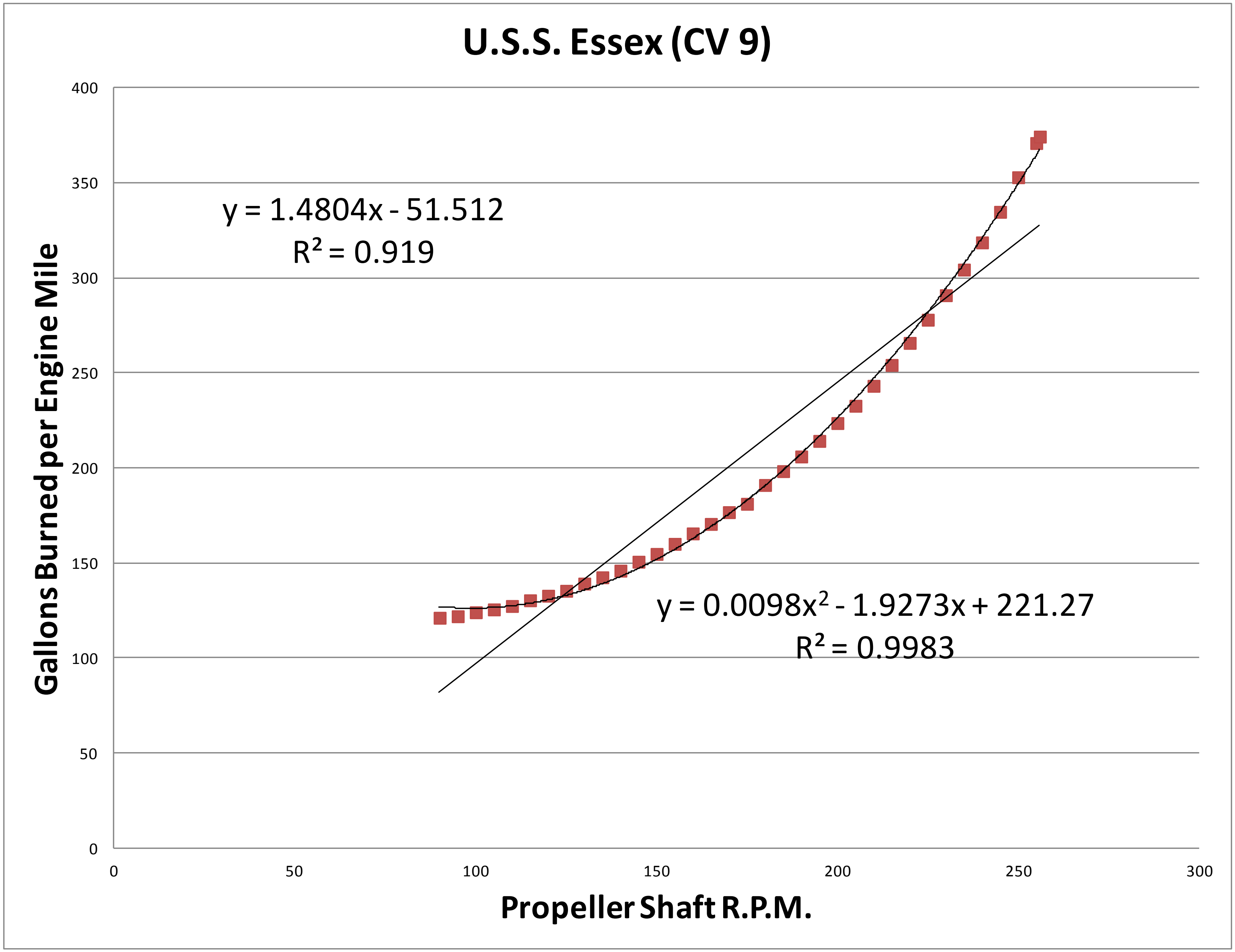
b. Percent of full load burned after 24 hours:

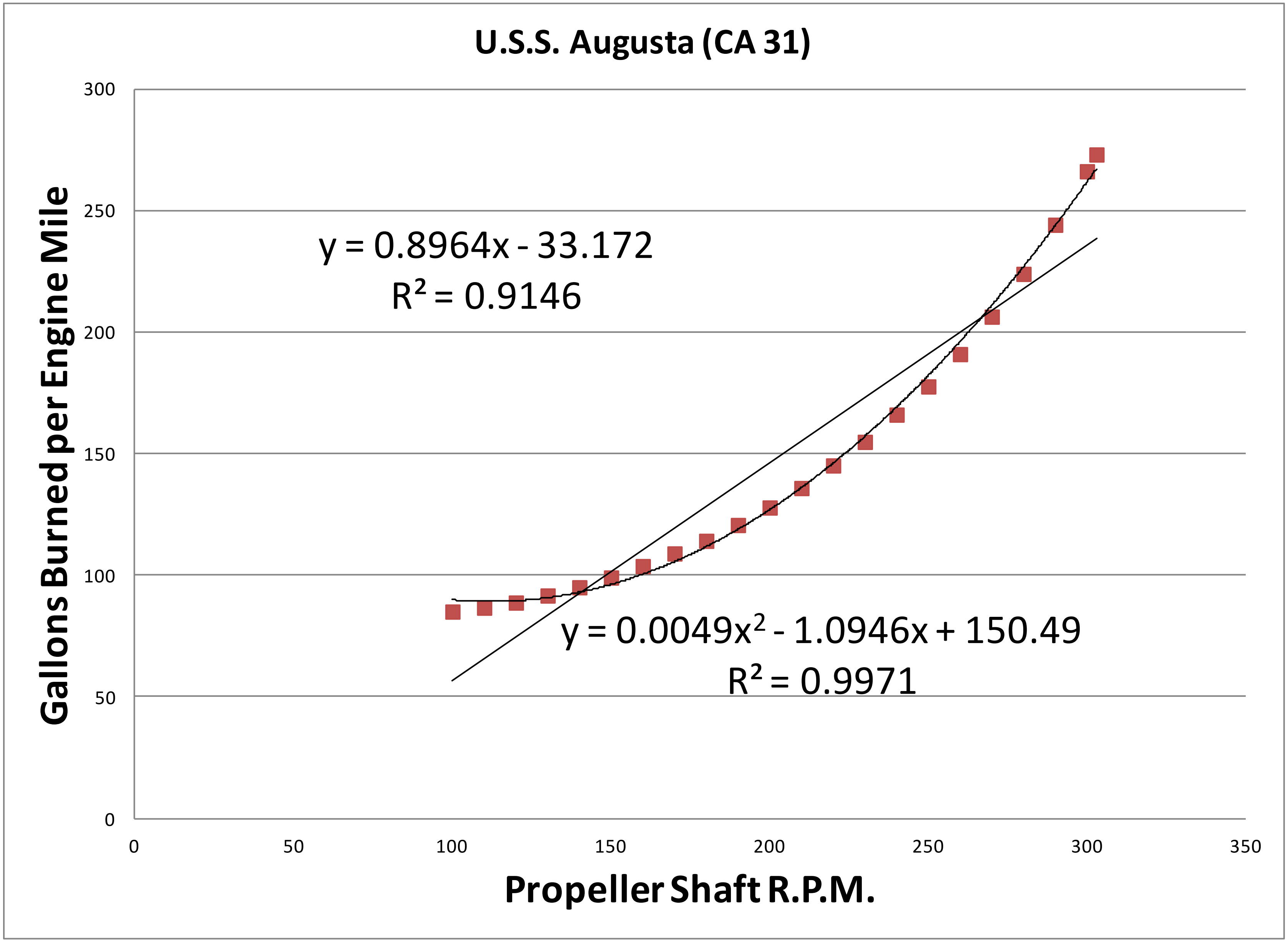
$$\frac{27595.08\ gallons}{588625\ gallons} = 4.69\%$$

Panama Canal Ship Data Answers

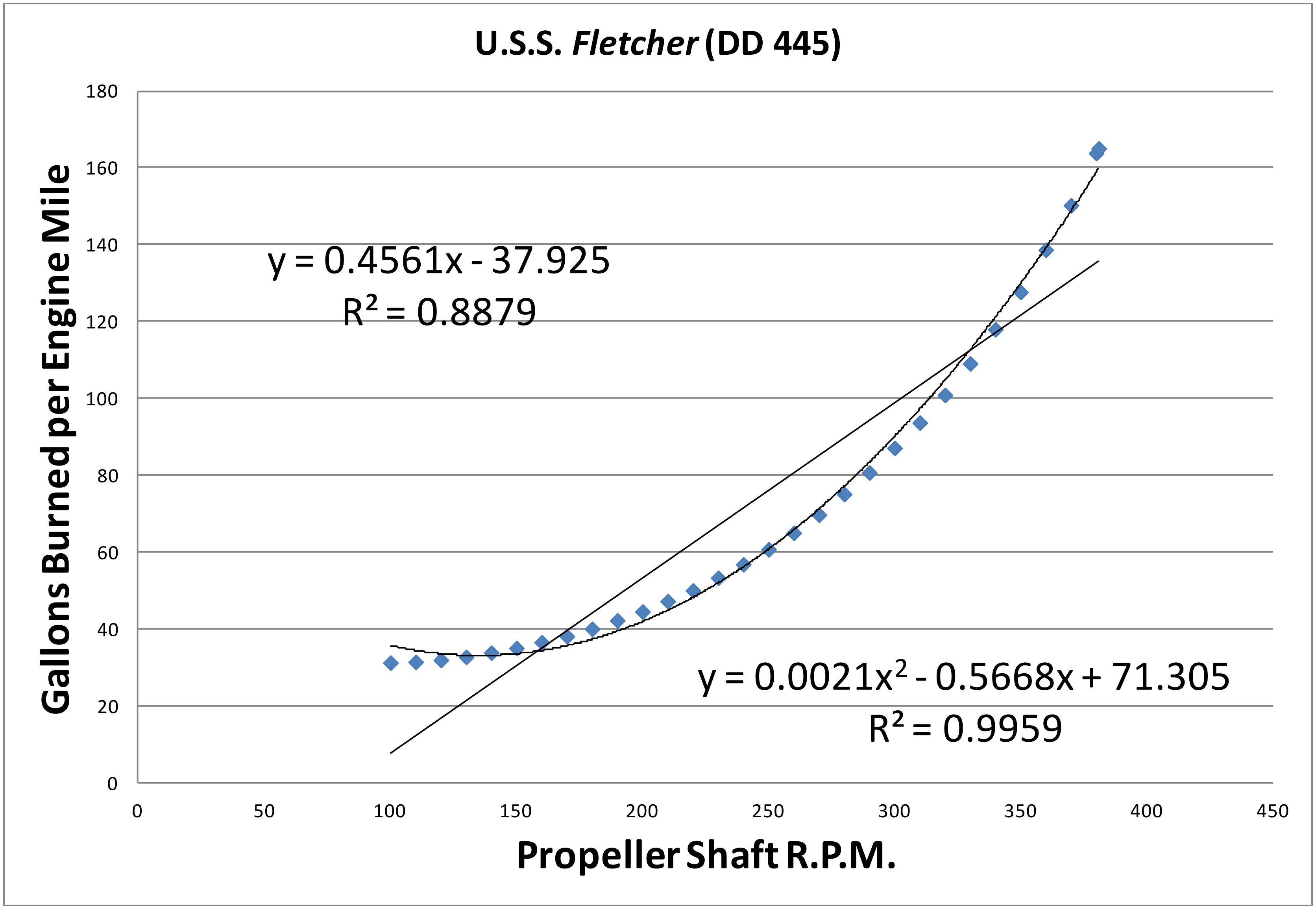
	Essex	*Augusta*	*Fletcher*	*Benson*	*Conyngham*
Displacement	**27,100**	**9,050**	**2,100**	**1,620**	**1,500**
Length	**872’**	**600’ 3”**	**376’ 3”**	**348’ 2”**	**341’ 4”**
Beam	**93’**	**66’ 1”**	**39’ 8”**	**36’ 1”**	**35’**
Draft	**28’ 7”**	**16’ 4”**	**13’**	**17’ 6”**	**9’ 10”**
Blind Dist. F.	**573.03’**	**216.24’**	**127.12’**	**126.36’**	**126.35’**
Blind Dist. A.	**633.36’**	**160.63’**	**168.16’**	**193.00’**	**193.00’**
Full Load Fuel	**1,753,410**	**588,625**	**142,655**	**129,082**	**146,256**
Gal./hr. (200)	**5,837**	**2,661**	**886**	**810**	**783**
Speed (200)	**26.1±0.3**	**20.8±0.1**	**19.9±0.2**	**20.9±0.2**	**20.6±0.2**
Gal./eng. mi.	**223.6**	**127.9**	**44.5**	**38.8**	**38.0**

Fuel Consumption Graphs for All Ships

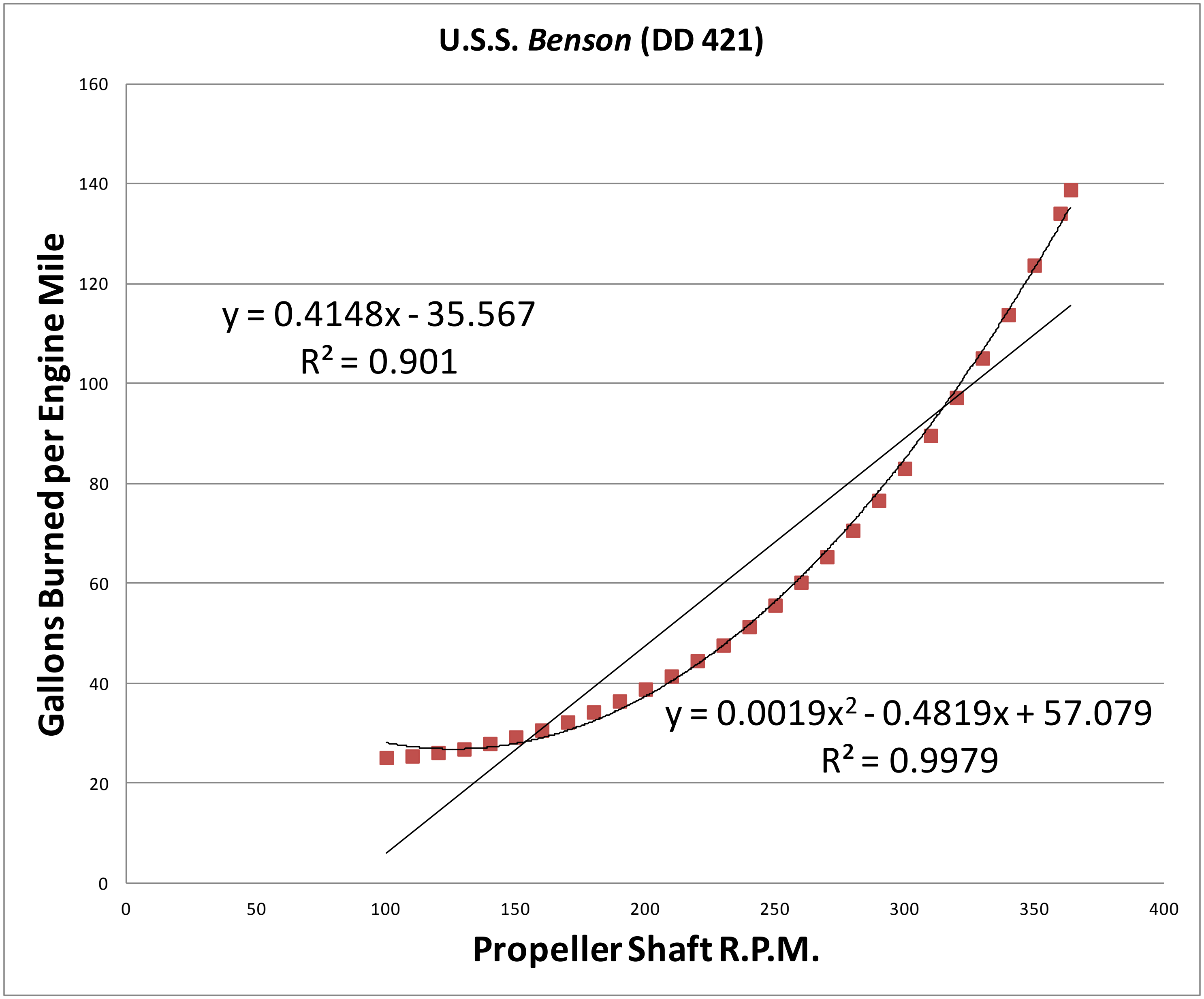



U.S.S. Augusta (CA 31)
y = 0.8964x - 33.172
R² = 0.9146
y = 0.0049x² - 1.0946x + 150.49
R² = 0.9971
Gallons Burned per Engine Mile
Propeller Shaft R.P.M.
0
50
100
150
200
250
300
350

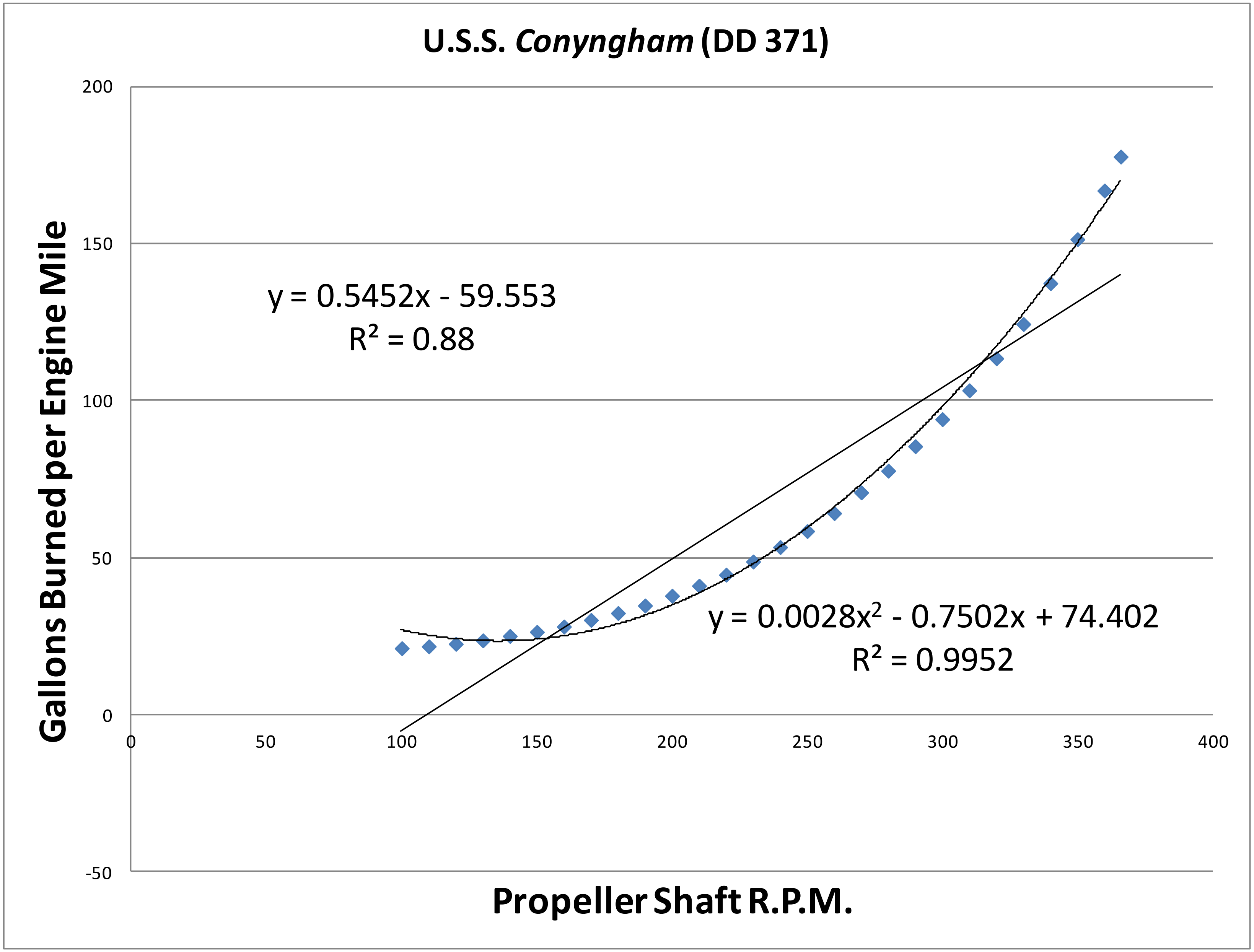
Fuel Consumption Graphs for All Ships



Fuel Consumption Graphs for All Ships



Fuel Consumption Graphs for All Ships



War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Essex* (CV - 9)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

    a. Blind distance submitted does not agree with information on file.

    b. Ship's data is incomplete or inaccurate.

    c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

4. The grades for your group are as follows;

    a. Captain: **4 / 3 / 2 / 1**

    b. Navigator: **4 / 3 / 2 / 1**

    c. Quartermaster: **4 / 3 / 2 / 1**

    d. Chief Engineer: **4 / 3 / 2 / 1**

    e. M.P.A. **4 / 3 / 2 / 1**

For the Commission,

Copy to:
Commander, 15th Naval District
Commander, Task Force 5

War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Augusta* (CA - 31)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

    a. Blind distance submitted does not agree with information on file.

    b. Ship's data is incomplete or inaccurate.

    c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

4. The grades for your group are as follows;

a. Captain:	**4 / 3 / 2 / 1**
b. Navigator:	**4 / 3 / 2 / 1**
c. Quartermaster:	**4 / 3 / 2 / 1**
d. Chief Engineer:	**4 / 3 / 2 / 1**
e. M.P.A.	**4 / 3 / 2 / 1**

For the Commission,

Copy to:
Commander, 15th Naval District
Commander, Task Force 5

War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Fletcher* (DD - 445)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

   a. Blind distance submitted does not agree with information on file.

   b. Ship's data is incomplete or inaccurate.

   c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

4. The grades for your group are as follows;

   a. Captain: **4 / 3 / 2 / 1**

   b. Navigator: **4 / 3 / 2 / 1**

   c. Quartermaster: **4 / 3 / 2 / 1**

   d. Chief Engineer: **4 / 3 / 2 / 1**

   e. M.P.A. **4 / 3 / 2 / 1**

For the Commission,

Copy to:
Commander, 15^{th} Naval District
Commander, Task Force 5

War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Benson* (DD - 421)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

    a. Blind distance submitted does not agree with information on file.

    b. Ship's data is incomplete or inaccurate.

    c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

4. The grades for your group are as follows;

    a. Captain: **4 / 3 / 2 / 1**

    b. Navigator: **4 / 3 / 2 / 1**

    c. Quartermaster: **4 / 3 / 2 / 1**

    d. Chief Engineer: **4 / 3 / 2 / 1**

    e. M.P.A. **4 / 3 / 2 / 1**

For the Commission,

Copy to:
Commander, 15^{th} Naval District
Commander, Task Force 5

War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Conyngham* (DD - 371)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

    a. Blind distance submitted does not agree with information on file.

    b. Ship's data is incomplete or inaccurate.

    c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

4. The grades for your group are as follows;

    a. Captain: **4 / 3 / 2 / 1**

    b. Navigator: **4 / 3 / 2 / 1**

    c. Quartermaster: **4 / 3 / 2 / 1**

    d. Chief Engineer: **4 / 3 / 2 / 1**

    e. M.P.A. **4 / 3 / 2 / 1**

For the Commission,

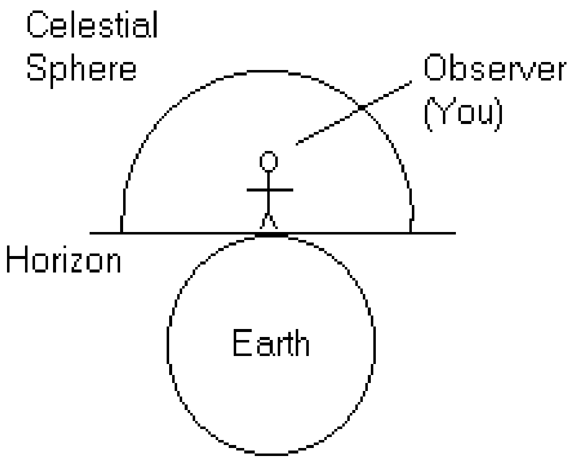
Copy to:
Commander, 15th Naval District
Commander, Task Force 5

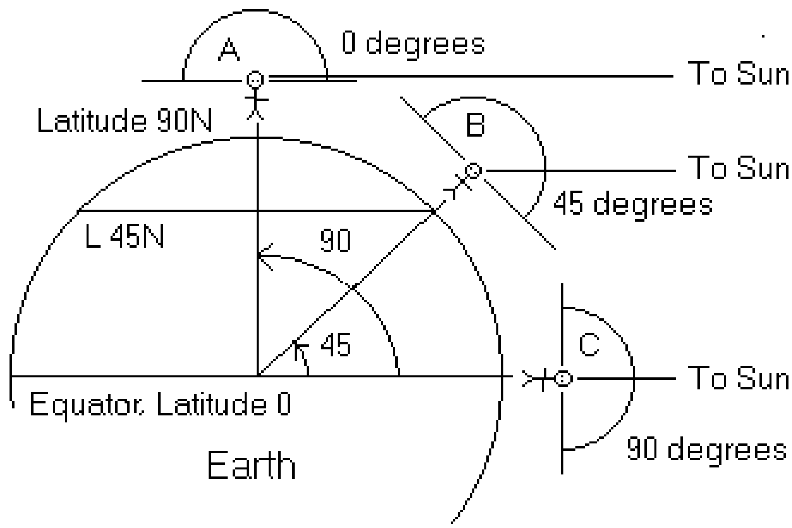
Name: ___________________________

Date:____________________________

Advisor:_________________________

Navigating by Latitude in the Northern Hemisphere[7]





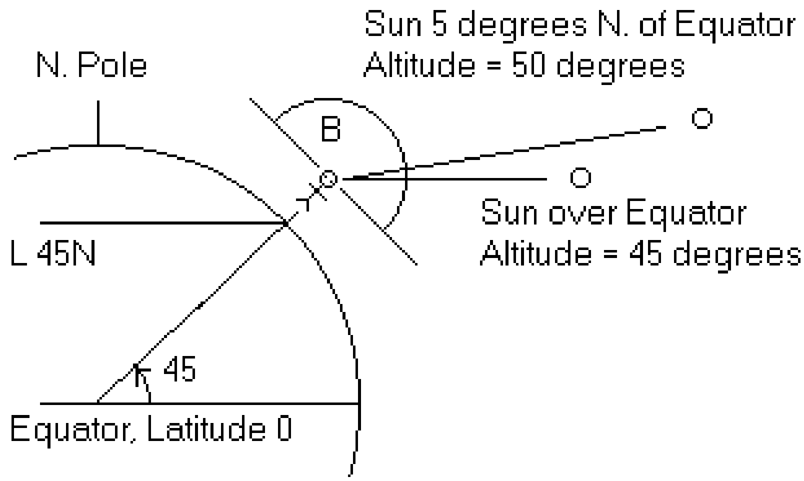
1. If the Sun is directly over the Equator, you can determine your latitude by observing the angular elevation of the sun over the horizon and then using the following formula:

$$Latitude = 90° - Sun's\ altitude$$

Find your latitude if the Sun's altitude is as follows:

Sun's Altitude	Your Latitude
15°	
27°	
45°	
70°	
90°	

[7] From Sammons, James I.: Navigating Around the World by Observing the Sun, accessed online at http://www.pbs.org/wgbh/nova/teachers/ideas/sammons/packet.html.



2. If you know how high above or below the Equator the Sun is on a given day (called the Sun's declination), you can find your latitude using the following formula:

$Latitude = 90° - Sun's\ altitude + Sun's\ declination$ (Sun in Northern Hemisphere)

$Latitude = 90° - Sun's\ altitude - Sun's\ declination$ (Sun in Southern Hemisphere)

Find your latitude if the Sun's altitude and declination are as follows:

Sun's Declination	**Sun's Altitude**	**Latitude**
15° N	42° N	
17° S	17° N	
23° N	68° N	

3. In the Northern Hemisphere, you can determine your latitude at night by observing the altitude of Polaris (the North Star)"

$$Latitude = Altitude\ of\ Polaris$$

Telling time at sea.

After 12 noon was determined by the Sun’s altitude,

1. A sailor would watch and turn a one-half-hour glass.
2. At each turning, he would ring the ship’s bell:

1 bell	12:30 AM / PM	4:30 AM / PM	8:30 AM / PM
2 bells	1:00	5:00	9:00
3 bells	1:30	5:30	9:30
4 bells	2:00	6:00	10:00
5 bells	2:30	6:30	10:30
6 bells	3:00	7:00	11:00
7 bells	3:30	7:30	11:30
8 bells	4:00	8:00	12:00

Name: ____________________________

Date:____________________________

Advisor:____________________________

Navigating by Latitude in the Northern Hemisphere

1. You are travelling at sea in the Northern Hemisphere. At 12 noon:

   a. You observe the Sun at an altitude of 49° above the horizon.
   b. You know that the Sun's declination today is 9° N.

   Therefore, your latitude is:

$$90° - 49° + 9° = 50°N$$

2. At midnight, you observe Polaris at 50° 21' above the horizon. How many miles north of your noon latitude have you moved?

$$21' = 21\ nm$$

3. At 12 noon the next day:

   a. You observe the Sun at an altitude of 48° above the horizon.
   b. You know the Sun's declination for today is 8° N.

   Therefore, your latitude is:

$$90° - 48° + 8° = 50°N$$

   If one degree of latitude equals 60 nautical miles, how many nautical miles north or south of yesterday's noon latitude are you today?

$$50° - 50° = 0°$$

Name: ________________________

Date:_________________________

Advisor:_________________________

Navigating by Latitude in the Northern Hemisphere

1. You are travelling at sea in the Northern Hemisphere. At 12 noon:

   a. You observe the Sun at an altitude of 49° above the horizon.
   b. You know that the Sun's declination today is 9° N.

   Therefore, your latitude is: ____

2. At midnight, you observe Polaris at 50° 21' above the horizon. If one minute of latitude equals one nautical mile, how many miles north of your noon latitude have you moved?

   ____

3. At 12 noon the next day:

   a. You observe the Sun at an altitude of 48° above the horizon.
   b. You know the Sun's declination for today is 8° N.

   Therefore, your latitude is: ____

   If one degree of latitude equals 60 nautical miles, how many nautical miles north or south of yesterday's noon latitude are you today?

   ____

Name: ______________________

Date: ______________________

Advisor: ______________________

Navigating by Latitude in the Northern Hemisphere II

Introduction: You are the navigator of His Britannic Majesty's Ship *Centurion*, a 60 gun sailing battle ship sent to the Pacific Ocean in the year 1740 to seize a great treasure ship that sails each year from Manila in the Philippines to Acapulco in Mexico. You have begun to sail west from Acapulco to Manila.

1. **Basic Skills:**

    a. Determine your ship's noon latitude and record it in the log table below:

Date	Sun's Altitude	Sun's Declination	Latitude
Nov 05	55°55' N	17°15' S	16°50' N
Nov 06	55°43' N	17°32' S	16°45' N
Nov 07	55°32' N	17°48' S	16°40' N
Nov 08	55°21' N	18°04' S	16°35' N
Nov 09	55°10' N	18°20' S	16°30' N
Nov 10	55°00' N	18°35' S	16°25' N
Nov 11	54°50' N	18°50' S	16°20' N

    b. Why are the declinations of the Sun "south" at this time of year?

    Because it is after the autumnal equinox so the Sun is south of the Equator.

    c. Has your latitude increased or decreased between November 11th and November 17th? Does this mean *Centurion* is moving north or south?

d. Your log indicates the following distances travelled over a 30 – second interval at the following times during November 16th. Compute your ship's average speed for each watch using the logged data in the table below. **Show all work on separate sheets of paper.**

Time	**Noon – 4 pm**	**4 – 8 pm**	**8 pm – 12 am**	**12 – 4 am**	**4 – 8 am**	**8 am - Noon**
1 bell	69 yards	99 yards	**83.3 yards**	84.2 yards	62.7 yards	68 yards
2 bells	71.2 yards	94.5 yards	**116 yards**	100 yards	62 yards	69.2 yards
3 bells	73 yards	94.5 yards	**83.3 yards**	85.3 yards	64.3 yards	69 yards
4 bells	75.3 yards	90.2 yards	**66.67 yards**	68 yards	61 yards	67.8 yards
5 bells	**80 yards**	90 yards	**66.67 yards**	64.3 yards	61.8 yards	65.4 yards
6 bells	88.2 yards	89.4 yards	**83.3 yards**	83.3 yards	61.2 yards	68 yards
7 bells	95 yards	86 yards	**83.3 yards**	83.3 yards	66 yards	66.2 yards
8 bells	94.3 yards	84.2 yards	**66.67 yards**	60 yards	70 yards	68 yards
Average Speed						

Example: For the 8 pm to 12 am watch, average speed (A) can be computed as follows:

$$A = \frac{\left(\frac{83.3 + 116 + 83.3 + 66.67 + 66.67 + 83.3 + 83.3 + 66.67}{8}\right) \times 2 \times 60}{2000} =$$

e. Notice that your average speed increases on the Noon – 4 pm watch and decreases again on the 8 pm – 12 am watch. What do you think accounts for this?

Wind speed often increases in the afternoon when air temperature rises.

Name: ____________________________

Date: ____________________________

Advisor: ____________________________

Navigating by Latitude in the Northern Hemisphere II

Introduction: You are the navigator of His Britannic Majesty's Ship *Centurion*, a 60 gun sailing battle ship sent to the Pacific Ocean in the year 1740 to seize a great treasure ship that sails each year from Manila in the Philippines to Acapulco in Mexico. You have begun to sail west from Acapulco to Manila.

2. **Basic Skills:**

   a. Determine your ship's noon latitude and record it in the log table below:

Date	Sun's Altitude	Sun's Declination	Latitude
Nov 05	55°55' N	17°15' S	
Nov 06	55°43' N	17°32' S	
Nov 07	55°32' N	17°48' S	
Nov 08	55°21' N	18°04' S	
Nov 09	55°10' N	18°20' S	
Nov 10	55°00' N	18°35' S	
Nov 11	54°50' N	18°50' S	

   b. Why are the declinations of the Sun "south" at this time of year?

   c. Has your latitude increased or decreased between November 11th and November 17th? Does this mean *Centurion* is moving north or south?

d. Your log indicates the following distances travelled over a 30 – second interval at the following times during November 16th. Compute your ship's average speed for each watch using the logged data in the table below. **Show all work on separate sheets of paper.**

Time	**Noon – 4 pm**	**4 – 8 pm**	**8 pm – 12 am**	**12 – 4 am**	**4 – 8 am**	**8 am - Noon**
1 bell	69 yards	99 yards	**83.3 yards**	84.2 yards	62.7 yards	68 yards
2 bells	71.2 yards	94.5 yards	**116 yards**	100 yards	62 yards	69.2 yards
3 bells	73 yards	94.5 yards	**83.3 yards**	85.3 yards	64.3 yards	69 yards
4 bells	75.3 yards	90.2 yards	**66.67 yards**	68 yards	61 yards	67.8 yards
5 bells	**80 yards**	90 yards	**66.67 yards**	64.3 yards	61.8 yards	65.4 yards
6 bells	88.2 yards	89.4 yards	**83.3 yards**	83.3 yards	61.2 yards	68 yards
7 bells	95 yards	86 yards	**83.3 yards**	83.3 yards	66 yards	66.2 yards
8 bells	94.3 yards	84.2 yards	**66.67 yards**	60 yards	70 yards	68 yards
Average Speed						

Example: For the 8 pm to 12 am watch, average speed (A) can be computed as follows:

$$A = \frac{\left(\frac{83.3 + 116 + 83.3 + 66.67 + 66.67 + 83.3 + 83.3 + 66.67}{8}\right) \times 2 \times 60}{2000} =$$

e. Notice that your average speed increases on the Noon – 4 pm watch and decreases again on the 8 pm – 12 am watch. What do you think accounts for this?

Name: ________________________

Date:_________________________

Advisor:_________________________

Navigating by Latitude and Longitude[8]

1. To find your **latitude at sea**, all you need is to measure how far the Sun is above the horizon (*Sun's altitude*) and know what degree of latitude the Sun is over at this time of year (*Sun's declination*). We then apply the formulas we have already learned:

$$Latitude = 90° - Sun's\ altitude + Sun's\ declination$$

(Sun over Northern Hemisphere)

$$Latitude = 90° - Sun's\ altitude - Sun's\ declination$$

(Sun over Southern Hemisphere)

2. To find your **longitude at sea,** you need to determine your local time. To do this, you need an accurate time piece and you need to know how far east or west of the Prime Meridian you are.

   a. The Earth rotates on its axis and completes one revolution (on average) every 24 hours. Since the Earth is a sphere, it rotates 360° (on average) every 24 hours. Therefore, it rotates 15° every hour because:

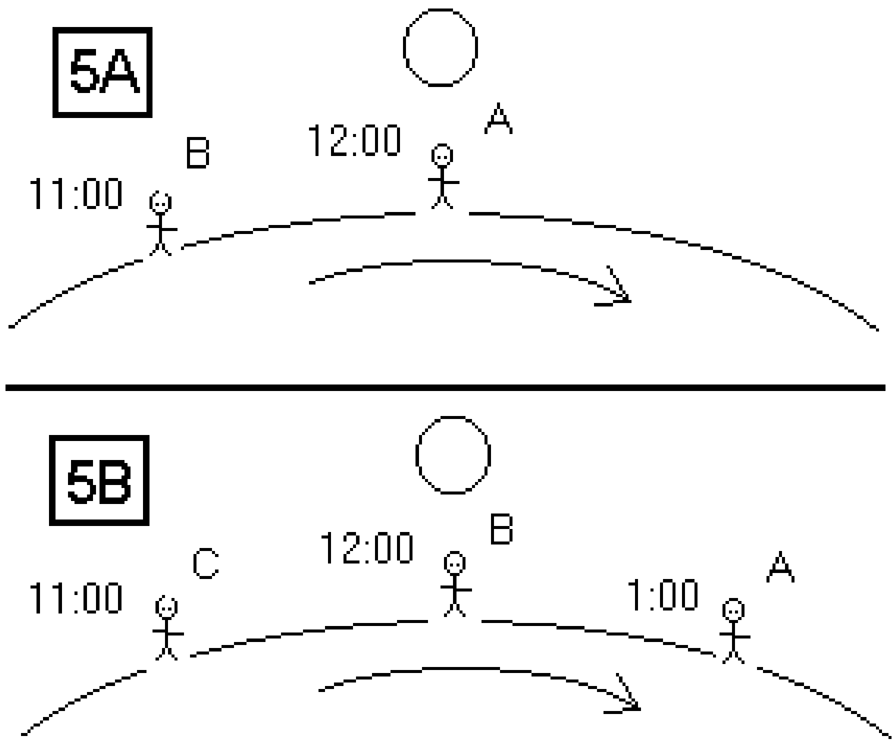
   $$360° \div 24\ hours = 15°\ per\ hour = \frac{15°}{hr}.$$

   b. Taking this one step further, we can say that the Earth rotates 1° in four minutes because:

   $$1\ hour \div 15° = 60\ minutes \div 15° = 4\ minutes\ per\ degree = \frac{4'}{1°}.$$

[8] From Sammons, James I.: Navigating Around the World by Observing the Sun, accessed online at http://www.pbs.org/wgbh/nova/teachers/ideas/sammons/packet.html.

c. Finding your local time is easy, just look at your watch. To determine your longitude, however, you have to compare that time with the time at the Prime Meridian (called Greenwich Mean Time or GMT). To do that, you have to consider how time differs across the Earth. Look at the following diagram:



i. In picture 5A, two people are standing on the Earth. Person B is 15° west of Person A. The Sun is directly above Person A so he is observing his Local Apparent Noon (12:00 PM). Person B, 15° west, will not observe his Local Apparent Noon for one hour (because it will take the Earth one hour to turn the 15° necessary to put Person B directly under the Sun). Person B's local time, therefore, is 11:00 AM.

ii. In picture 5B, as the Earth turns, Person A moves away from the Sun (so that the Sun appears to be moving toward the western horizon) and Person B moves closer to the Sun (so that the Sun appears to be moving directly overhead). When the Sun is directly overhead Person B, it is his Local Apparent Noon (12:00 PM). Person A, 15° east of person B, is at a local time of 1:00 PM, while Person C, 15° east of Person B, is at a local

time of 11:00 AM. This is where the idea of "time zones" comes from. Persons A, B, and C are each in a different time zone.

iii. When we compare the times of Persons A, B, and C, we find that Person C has the earliest time because he is furthest west. Person A has the latest time because he is furthest east. This leads to the following rule:

Local time earlier, position is westward.

Local time later, position is eastward.

d. Once an accurate time piece that could be taken to sea was invented, sailors determined their longitude by setting their time pieces (called chronometers) to Greenwich Mean Time (GMT – the time at the Prime Meridian) and then, each day, comparing the time of Local Apparent Noon to GMT.

(1) $$12{:}00 - 8{:}24 = 3{:}36$$

(2) $$3\ hours \times \frac{15°}{hour} = 45°$$

(3) $$36' \div \frac{4'}{1°} = 9°$$

(4) $$45° + 9° = 54°$$

Since LAN is later than GMT, we use the rule above and determine "local time later, position is eastward" so our longitude is 54° E.

Name: ________________________

Date:_________________________

Advisor:_________________________

Navigating by Latitude and Longitude

Show all work on a separate sheet of paper.

1. You are aboard a ship navigating the Pacific Ocean. Each day at LAN (12:00 PM local time) you observe the Sun to determine your latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 21st	20:00	40° 13'	19° 47' S		
November 22nd	20:04	41° 50'	20° 00' S		
November 23rd	20:08	41° 27'	20° 13' S		
November 24th	20:12	41° 04'	20° 26' S		

Name: ________________________

Date: ________________________

Advisor: ________________________

Navigating by Latitude and Longitude

Show all work on a separate sheet of paper.

2. You are aboard a ship navigating the Pacific Ocean. Each day at LAN (12:00 PM local time) you observe the Sun to determine your latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 21st	20:00	40° 13'	19° 47' S	30° 00'N	120° 00' W
November 22nd	20:04	41° 50'	20° 00' S	28° 10' N	121° 00' W
November 23rd	20:08	41° 27'	20° 13' S	28° 20' N	122° 00' W
November 24th	20:12	41° 04'	20° 26' S	28° 30' N	123° 00' W

Name: ________________________

Date:_________________________

Advisor:_________________________

Voyage to the Panama Canal – Review Nr. 1

Show all work on separate sheets of paper.

In this unit so far we have explored a number of Algebra and Geometry concepts in the context of planning a voyage to the Panama Canal. Specifically, we have:

1. **Learned the following equations for determining a ship's latitude at sea:**

$$Latitude\ (at\ noon) = 90° - Sun's\ altitude + Sun's\ declination$$

(Sun over Northern Hemisphere)

$$Latitude\ (at\ noon) = 90° - Sun's\ altitude - Sun's\ declination$$

(Sun over Southern Hemisphere)

$$Latitude\ (at\ night\ in\ Northern\ Hemisphere) = Altitude\ of\ Polaris$$

Review problems: Compute ship's latitude at noon for the following:

Date	Sun's Altitude	Sun's Declination	Latitude
Dec 8	55°55' N	22°46' S	
Dec 9	55°43' N	22°52' S	
Dec 10	55°32' N	22°57' S	
Dec 12	55°21' N	23°02' S	

2. **Learned how to tell time at sea:**

Telling time at sea.

After 12 noon was determined by the Sun's altitude,

1. A sailor would watch and turn a one-half-hour glass.
2. At each turning, he would ring the ship's bell:

1 bell	12:30 AM / PM	4:30 AM / PM	8:30 AM / PM
2 bells	1:00	5:00	9:00
3 bells	1:30	5:30	9:30
4 bells	2:00	6:00	10:00
5 bells	2:30	6:30	10:30
6 bells	3:00	7:00	11:00
7 bells	3:30	7:30	11:30
8 bells	4:00	8:00	12:00

3. **Learned an equation for determining ship's speed taken every half – hour:**

$$A = \frac{\left(\frac{sum\ of\ the\ half - hourly\ distances\ traveled\ in\ yards}{number\ of\ half - hourly\ distances\ measured}\right) \times 2 \times 60}{2000}$$

Review Problems: Compute the average speed traveled on each watch in nautical miles per hour.

Time	**Noon – 4 pm**	**4 – 8 pm**
1 bell	70 yards	98 yards
2 bells	72.2 yards	93.5 yards
3 bells	74 yards	93.5 yards
4 bells	76.3 yards	89.2 yards

5 bells	81 yards	91 yards
6 bells	89.2 yards	88.4 yards
7 bells	96 yards	85 yards
8 bells	95.3 yards	85.2 yards
Average Speed		

4. **Learned that every 15 degrees of longitude equals one hour of time and every degree of longitude equals 4 minutes of time.**

$$360° \div 24\ hours = 15°\ per\ hour = \frac{15°}{hour}.$$

$$1\ hour \div 15° = 60\ minutes \div 15° = 4\ minutes\ per\ degree = \frac{4'}{1°}.$$

5. **Learned equations for computing longitude at sea.**

For east longitude:

(1) $12{:}00 - Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) = h{:}mm$

(2) $h \times \frac{15°}{hour} = degrees\ east\ of\ 0°\ longitude$

(3) $mm \div \frac{4'}{1°} = additional\ degrees\ east\ of 0°\ longitude$

(4) $add\ the\ two\ degrees\ east\ together\ to\ get\ ship's\ longitude\ east$

For west longitude:

(1) $Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) - 12{:}00 = h{:}mm$

(2) $$h \times \frac{15°}{hour} = degrees\ west\ of\ 0°\ longitude$$

(3) $$mm \div \frac{4'}{1°} = additional\ degrees\ west\ of 0°\ longitude$$

(4) $add\ the\ two\ degrees\ west\ together\ to\ get\ ship's\ longitude\ west$

Review Problems: Compute your ship's longitude for the dates below.

Date	GMT of LAN	Longitude
December 8th	20:08	
December 9th	20:13	
December 10th	20:15	
December 11th	20:19	

6. **Learned how to construct a precise scale drawing of a Panama Canal lock and transiting ship using a scale factor equation to convert actual measures to drawing measures:**

$$centimeters = (scale\ factor) * meters$$

$$meteres = \frac{centimeters}{scale\ factor}$$

Review Problems: Make the following conversions using a scale factor of 0.26.

45 meters to centimeters	
67 meters to centimeters	
108 centimeters to meters	
244 centimeters to meters	
12.8 meters to centimeters	
0.67 centimeters to meters	
145 meters to centimeters	
0.005 meters to centimeters	

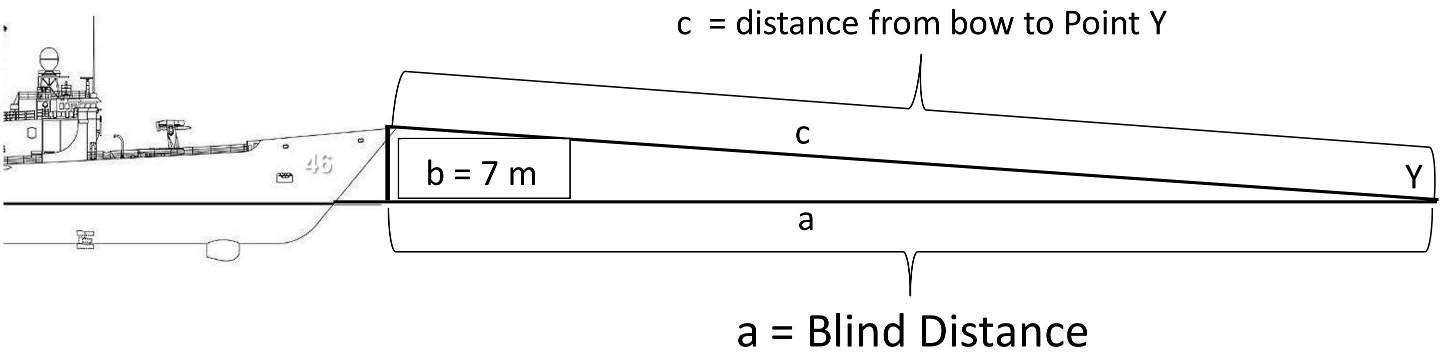
7. **Learned how to compute blind distances for your ship using the Pythagorean Theorem.**

Given any right triangle with side lengths a and b and hypotenuse length c the following relationship holds:

$$a^2 + b^2 = c^2$$

Practice Problem: Find the blind distance for the ship in the following diagram:

Blind distance in meters is: ____



8. **Learned how compute:**

   a. Voyage Time and Voyage Distance:

$$voyage\ time\ in\ hours = \frac{distance\ in\ nautical\ miles}{speed\ in\ nautical\ miles\ per\ hour}$$

$$voyage\ distance = speed * transit\ time$$

   Review Problem: Your ship is making a voyage of 3569 nautical miles at an average speed of 16 knots. How long in days will it take the ship to reach her destination?

   b. Amount of ship's fuel used on a voyage:

$$Amt.of\ fuel\ used = (fuel\ used\ per\ nautical\ mile\ at\ ship's\ speed) * transit\ distance$$

   c. Percentage of total fuel used on a voyage:

$$\%\ of\ total\ fuel\ used = \frac{amount\ of\ fuel\ used}{total\ fuel\ ship\ can\ carry}$$

Review Problem: Your ship consumes 112 gallons per nautical mile at 12 knots. Altogether she carries 306,000 gallons of fuel. How many gallons of fuel will she consume on a voyage of 2098 nautical miles and what percentage of her total fuel will she consume?

Name: ________________________

Date:_________________________

Advisor:_________________________

Task IV – Finding the Ship's Heading with a Compass

1. **Background.** We've learned how to find our ship's position at noon each day. Now we will look at how to keep our ship on a heading (a direction) that will take us to our destination. This heading is normally expressed in degrees from True North (the direction to the North Pole) on a compass and is displayed on the bridge of your ship on its compass.

Figure 7: Ship's Compass

Our ship has two types of compass. One, the gyrocompass, makes use of a gyroscope and the Earth's rotation to find True North. Because it finds True North, it is the compass most often used to find the ship's heading. The gyrocompass requires a continuous input of stable electrical power to work properly. If power is lost, the compass fails and is useless until it can be restarted and reset – normally not done at sea.

Because the gyrocompass can fail, our ship also carries a magnetic compass which does not require electrical power to operate. Magnetic compasses have a much longer history than gyrocompasses and, before the introduction of

electrical power on ships in the late nineteenth century, were the only compasses carried.

2. **How a Magnetic Compass Works.** A magnetic compass doesn't actually point to the magnetic North Pole. Instead, the compass points in the directions of the horizontal component of the Earth's magnetic field where the compass is located, and not to any single point. Knowing the magnetic declination (the angle between true north and the horizontal trace of the magnetic field) for your location allows you to correct your compass for the magnetic field in your area. A mile or two away the magnetic declination may be considerably different, requiring a different correction.[9]

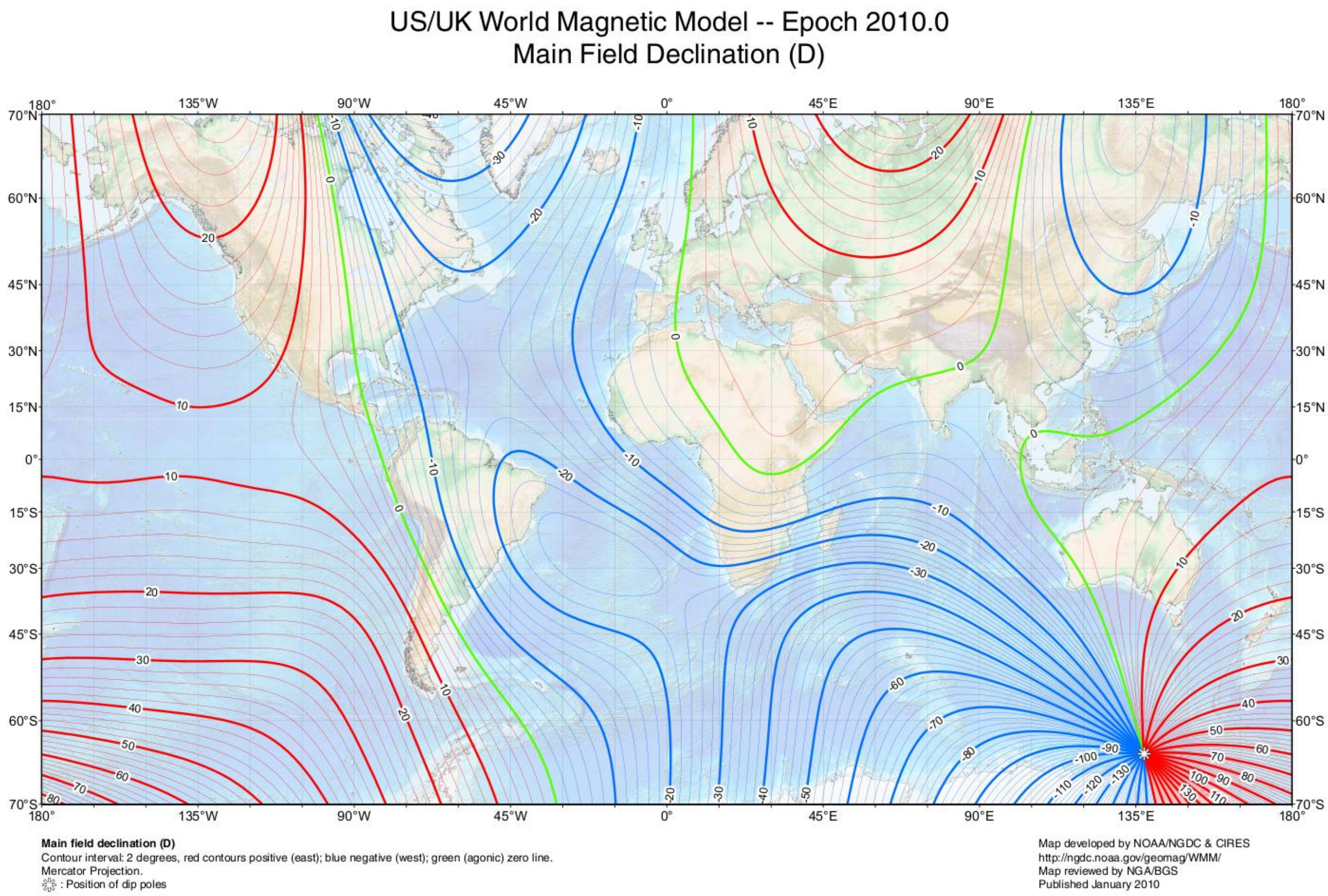
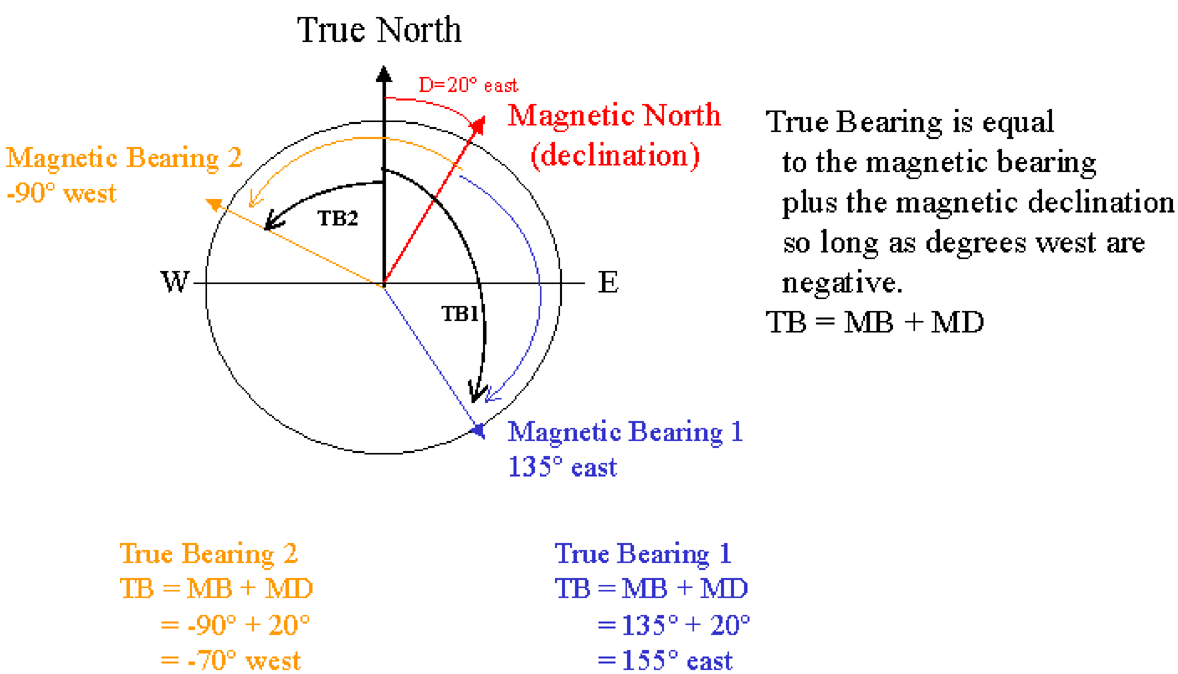


Figure 8: Magnetic Declination Worldwide

[9] National Geophysical Data Center, "Geomagnetism Frequently Asked Questions". Accessed online 09 Dec 11 at: http://www.ngdc.noaa.gov/geomag/faqgeom.shtml.

You can compute the true bearing (TB) from a magnetic bearing (MB) by adding the magnetic declination (MD) to the magnetic bearing. This works so long as you follow the convention of degrees west are negative (i.e. a magnetic declination of 10-degrees west is -10 and bearing of 45-degrees west is -45).[10] If your true bearing comes out negative, add 360°.

$$TB = MB + MD\ (+360°\ if\ result\ is\ negative)$$



Examples:

a. Magnetic bearing is 90° **West**. Magnetic declination is 20° **East**. Our true bearing is:

$$TB = -90° + 20° = -70°$$

$$-70° + 360° = 290°\,T$$

[10] Ibid.

b. Magnetic bearing is 80° **West**. Magnetic declination is 20° **West**. Our true bearing is:

$$TB = -80° - 20° = -100° + 360° = 260°\,T$$

c. Magnetic bearing is 100° **East**. Magnetic declination is 40° **West**. Our true bearing is:

$$TB = 100° - 40° = 60°\,T$$

d. Magnetic bearing is 45° **East**. Magnetic declination is 60° **East**. Our true bearing is:

$$TB = 45° + 60° = 105°\,T$$

3. **Task.** Solve the following problems. **Show all work on separate sheets of paper.**

Magnetic Bearing	**Declination**	**True Bearing**
120° W	40° E	280° T
155° E	10° E	165° T
32° W	44° W	284° T
176° E	21° E	197° T
57° W	78° W	225° T
165° W	47° E	242° T
130° E	68° W	062° T
75° W	21° E	306° T
125° W	75° W	160° T

Navigation Exercise – Voyage Plan

Waypoint	Latitude	Longitude	Bearing	Range	Time in hours at 20 knots
1 (San Diego)	32°38.2931'N	117°13.3649'W			
2	27°34.0033'N	116°8.8477'W	169°T	310 nm	15.50
3	22°18.5656'N	111°2.9883'W	138°T	421 nm	21.05
4	17°36.1283'N	105°33.3984'W	131°T	420 nm	21.00
5	14°56.6871'N	98°47.3438'W	111°T	422 nm	21.10
6	12°43.5651'N	92°1.2891'W	108°T	417 nm	20.85
7	9°26.9437'N	86°44.8828'W	122°T	368 nm	18.40
8	8°51.5000'N	85°40.0000'W	120°T	73 nm	3.65
9	8°00.0000'N	83°12.0000'W	110°T	155 nm	7.75
10	7°00.8201'N	81°46.9336'W	125°T	103 nm	5.15
11	7°09.0000'N	80°30.0000'W	085°T	77 nm	3.85
12	7°20.0000'N	79°15.8000'W	082°T	75 nm	3.75
13 (Panama Canal)	8°50.8247'N	79°29.1650'W	353°T	92 nm	4.60

Name: ________________________

Date: _________________________

Advisor: ______________________

Task V – Transit, Day 1
Show all work on separate sheets of paper attached to this one.

1. **Step 1:** Now that all planning is complete, your ship is ready to get underway for the Panama Canal. She departs Pier 4, U. S. Naval Station San Diego on the morning of November 16th, 2012 and begins her transit of San Diego Harbor to the open sea.

   As you transit the harbor, the navigator observes the following bearings on her magnetic compass and also notes the following declinations. Find the true bearings the ship is traveling on when the navigator makes each observation.

Magnetic Bearing	Declination	True Bearing
48° W	12° E	324° T
100° W	10° E	270° T
140° W	11° E	231° T

2. **Step 2.** Now that you have left San Diego harbor you will need to go alongside U.S.S. *Cimarron* (AO – 22) to refuel.

   a. Your ship can carry _______________ gallons of fuel (paragraph 5.a. from your "Panama Canal Ship Data" form. She currently has 75%. How many gallons is she carrying?

$$Full\ load\ in\ gallons \times 0.75 = Fuel\ onboard\ in\ gallons$$

   b. How much will she need to take from *Cimarron* to be full.

$$Full\ load\ in\ gallons - Fuel\ onboard\ in\ gallons = Amount\ needed$$

c. If your ship travels at **15 knots,** how far in nautical miles (nm) will she go before she burns 20% of her fuel?

$$\frac{Full\ load\ in\ gallons \times 0.20}{Gallons\ burned\ per\ nautical\ mile\ at\ 15\ kts.} = nm\ traveled$$

d. How long will it take her to burn that fuel?

$$\frac{nm\ traveled}{\frac{15\ nm}{hour}} = time\ to\ burn\ fuel$$

e. If you finish refueling by 12:00, how much fuel will you have onboard at midnight? What percentage of the total you can carry will you have onboard at midnight?

Amount onboard:

$$Amount\ burned = \left(12\ hours \times \left(15\frac{nm}{hour}\right)\right) \times gallons\ per\ nautical\ mile\ at\ 15\ knots$$

$$Amount\ onboard\ at\ close\ of\ Day\ 1 = Full\ load\ in\ gallons - Amount\ burned\ in\ gallons$$

Percentage of total onboard:

$$\frac{Amount\ onboard\ in\ gallons\ at\ close\ of\ Day\ 1}{Full\ load\ in\ gallons} = Percentage\ onboard\ at\ close\ of\ Day\ 1$$

3. **Step 3:** It is now almost 12:00 noon and time to determine the ship's position. From the following information, find the ship's latitude and longitude.

Date	**GMT of LAN**	**Sun's Altitude**	**Sun's Declination**	**Latitude**	**Longitude**
November 14th		34° 19'	23° 02' S	32° 39' N	

4. **Step 4:** By midnight, you need to file a report with Commander, Task Force 5 containing all the information you calculated above. Fill out the message template below with that information. **Your group's classroom grade for today will be based in part on how accurately you complete this message.**

O 150001Z NOV 13
FM USS ________________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT MIDNIGHT:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

A. UNDERWAY 0900T FM NAVSTA SDGO ENR PANAMA CANAL.
B. COMPLETED UNREP WITH USS CIMARRON. RECEIVED ________________________ GALLONS OF FUEL.
C. ENR PANAMA CANAL AT 15 KTS.

BT

Name: ______________________

Date: ______________________

Advisor: ______________________

Task V – Transit, Day 1
Show all work on separate sheets of paper attached to this one.

1. **Step 1:** Now that all planning is complete, your ship is ready to get underway for the Panama Canal. She departs Pier 4, U. S. Naval Station San Diego on the morning of November 16th, 2012 and begins her transit of San Diego Harbor to the open sea.

   As you transit the harbor, the navigator observes the following bearings on her magnetic compass and also notes the following declinations. Find the true bearings the ship is traveling on when the navigator makes each observation.

Magnetic Bearing	Declination	True Bearing
48° W	12° E	
100° W	10° E	
140° W	11° E	

2. **Step 2.** Now that you have left San Diego harbor you will need to go alongside U.S.S. *Cimarron* (AO – 22) to refuel.

   a. Your ship can carry ______________ gallons of fuel (paragraph 5.a. from your "Panama Canal Ship Data" form. She currently has 75%. How many gallons is she carrying?

   b. How much will she need to take from *Cimarron* to be full.

c. If your ship travels at **15 knots,** how far in nautical miles (nm) will she go before she burns 20% of her fuel?

d. How long will it take her to burn that fuel?

e. If you finish refueling by 12:00, how much fuel will you have onboard at midnight? What percentage of the total you can carry will you have onboard at midnight?

   Amount onboard:

   Percentage of total onboard:

3. **Step 3:** It is now almost 12:00 noon and time to determine the ship's position. From the following information, find the ship's latitude and longitude.

Date	**GMT of LAN**	**Sun's Altitude**	**Sun's Declination**	**Latitude**	**Longitude**
November 14th		34° 19'	23° 02' S		

4. **Step 4:** By midnight, you need to file a report with Commander, Task Force 5 containing all the information you calculated above. Fill out the message template below with that information. **Your group's classroom grade for today will be based in part on how accurately you complete this message.**

O 150001Z NOV 13
FM USS ________________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______ //
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT MIDNIGHT:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

D. UNDERWAY 0900T FM NAVSTA SDGO ENR PANAMA CANAL.
E. COMPLETED UNREP WITH USS CIMARRON. RECEIVED ________________________ GALLONS OF FUEL.
F. ENR PANAMA CANAL AT 15 KTS.

BT

Name: ________________________

Date:_________________________

Advisor:_________________________

Task VI – Transit, Day 2
Show all work on separate sheets of paper attached to this one.

Today is the second day of your transit to the Panama Canal. You have traveled **384** nautical miles (nm) South-Southeast of San Diego from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		40° 23.15'	23° 08' S		

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you burn from 12:00 yesterday until 12:00 today?

$$nm\ traveled\ in\ 24\ hours \times gallons\ per\ nm\ at\ 15\ knots = fuel\ burned\ in\ 24\ hours$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

Percentage burned:

$$\frac{Fuel\ burned\ in\ 24\ hours\ at\ 15\ knots}{(Full\ load\ in\ gallons)} = Percentage\ burned$$

Percentage left onboard:

$$100\% - Percentage\ burned$$

c. At 12 noon your ship receives an order from Commander, Third Fleet (COMTHRIDFLT) to proceed to 23° 43' N / 111° 23' W – a position just offshore of the Mexican resort of Cabo San Lucas to pick up two important passengers at the fishing port for transport to Panama City. Your ship must be in position by 12:00 noon tomorrow (24 hours from now). The position is 402 nm from your ship's current position.

   Is 15 knots sufficient speed to travel to this new position in 24 hours? Why not?

$$15\ knots * 24\ hours = 360\ nm < 402\ nm$$

d. Is 20 knots sufficient speed?

$$20\ knots \times 24\ hours = 480\ nm > 402\ nm$$

e. What percentage of fuel will she consume at 20 knots? What percentage will remain?

$$\frac{(Gallons\ burned\ per\ nm\ at\ 20\ knots) \times 402\ nm}{Full\ load\ in\ gallons} = Percentage\ of\ fuel\ burned\ at\ 20\ knots$$

$$Percentage\ onboard\ at\ close\ of\ Day\ 1 - Percentage\ of\ fuel\ burned\ at\ 20\ knots$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 2$$

3. **Step 3: Speed Computation.** During the afternoon watch (12 noon – 4 PM), you measure the distance the ship travels in 30 seconds every half hour. The following are your results:

Time	**Noon – 4 pm**
1 bell	280 yards
2 bells	285 yards
3 bells	278 yards
4 bells	289 yards
5 bells	288 yards
6 bells	274 yards
7 bells	270 yards
8 bells	285 yards
Average Speed	16.87 knots

a. What is your average speed in nautical miles per hour (knots)?

   16.87 knots

b. Is this speed sufficient to get to your destination on time?

   No.

4. **Task 4:** By midnight, you need to file a report with Commander, Task Force 5 containing all the information you calculated above. Fill out the message template below with that information.

O 160001Z NOV 13
FM USS ___________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS________________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:
A. ENR 23° 43' N / 111° 23' W IAW COMTHRIDFLT 152000Z TO PICK UP PAX. ETA 141200 LOCAL.
B. PAC FIRE
C. CONDITION I DRILLS.

BT

Name: ________________________

Date:_________________________

Advisor:_________________________

Task VI – Transit, Day 2
Show all work on separate sheets of paper attached to this one.

Today is the second day of your transit to the Panama Canal. You have traveled 384 nautical miles (nm) South-Southeast of San Diego from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		40° 23.15'	23° 08' S		

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you burn from 12:00 yesterday until 12:00 today?

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

Percentage burned:

Percentage left onboard:

c. At 12 noon your ship receives an order from Commander, Third Fleet (COMTHRIDFLT) to proceed to 23° 43’ N / 111° 23’ W – a position just offshore of the Mexican resort of Cabo San Lucas to pick up two important passengers at the fishing port for transport to Panama City. Your ship must be in position by 12:00 noon tomorrow (24 hours from now). The position is 480 nm from your ship’s current position.

   Is 15 knots sufficient speed to travel to this new position in 24 hours? Why not?

d. Is 20 knots sufficient speed?

e. What percentage of fuel will she consume at 20 knots? What percentage will remain?

3. **Step 3: Speed Computation.** During the afternoon watch (12 noon – 4 PM), you measure the distance the ship travels in 30 seconds every half hour. The following are your results:

Time	Noon – 4 pm
1 bell	280 yards
2 bells	285 yards
3 bells	278 yards
4 bells	289 yards
5 bells	288 yards
6 bells	274 yards
7 bells	270 yards
8 bells	285 yards
Average Speed	16.87 knots

c. What is your average speed in nautical miles per hour (knots)?

d. Is this speed sufficient to get to your destination on time?

4. **Step 4:** By midnight, you need to file a report with Commander, Task Force 5 containing all the information you calculated above. Fill out the message template below with that information.

O 160001Z NOV 13
FM USS ___________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS________________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________
2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:
A. ENR 23° 43' N / 111° 23' W IAW COMTHRIDFLT 152000Z TO PICK UP PAX. ETA 141200 LOCAL.
B. PAC FIRE
C. CONDITION I DRILLS.

BT

Name: ________________________

Date:__________________________

Advisor:__________________________

Task VII – Transit, Day 3
Show all work on separate sheets of paper attached to this one.

Today is the third day of your transit to the Panama Canal. You have traveled **360** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		43° 06'	23°11' S		

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 20 knots, how much fuel did you consume from 12:00 noon yesterday until 12:00 noon today?

$$480\ nm * Gallons\ burned\ per\ nm\ at\ 20\ knots = Gallons\ burned\ last\ 24\ hours$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Gallons\ burned\ last\ 24\ hours}{Full\ load\ in\ gallons} = Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3$$

$$Percentage\ onboard\ at\ close\ of\ Day\ 2 - Percentage\ of\ fuel\ burned\ at\ 20\ knots$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3$$

c. The distance between San Diego and the Pacific Ocean entrance to the Panama Canal is 2844 nautical miles. How much of that distance remains after two days?

$$2844\ nm - 360\ nm - 480\ nm = 2004\ nm$$

d. If your ship travels at 15 knots for the remainder of the voyage how much fuel in gallons will remain and what percentage will remain?

$$2004\ nm * (Gallons\ burned\ per\ nm\ at\ 15\ knots) = Gallons\ burned\ for\ remainder\ of\ voyage$$

$$\frac{Gallons\ burned\ for\ remainder\ of\ voyage}{Full\ load\ in\ gallons} = Percentage\ burned\ for\ remainder\ of\ voyage$$

$$(Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3) - Percentage\ burned\ for\ remainder\ of\ voyage$$
$$= Percentage\ remaining\ onboard\ at\ end\ of\ voyage$$

e. Commander, Third Fleet requires that no ship drop below 50% fuel while operating in the Third Fleet area. Therefore, you will have to stop at a port to refuel. You have two options:

Puerto Quetzel, Guatemala	**1163 nm distant**
Golfito, Costa Rica	**1703 nm distant**

What percentage of fuel will you have left when you reach each port?

Puerto Quetzel:

$$1163\ nm * (Gallons\ burned\ per\ nm\ at\ 15\ knots) = Gallons\ burned\ enroute\ Puerto\ Quetzel$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3 - \frac{Gallons\ burned\ enroute\ Puerto\ Quetzel}{Full\ load\ in\ gallons}$$
$$= Percentage\ remaining\ at\ Puerto\ Quetzel$$

Golfito:

$$1703\ nm * (Gallons\ burned\ per\ nm\ at\ 15\ knots) = Gallons\ burned\ enroute\ Golfito$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3 - \frac{Gallons\ burned\ enroute\ Golfito}{Full\ load\ in\ gallons}$$
$$= Percentage\ remaining\ at\ Golfito$$

f. Which port do you choose and why?

3. **Step 3:** By midnight, you need to file a report with Commander, Task Force V containing all the information you calculated above. Fill out the message template below with that information.

O 170001Z NOV 13
FM USS ______________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS ________________//
1. SHIP'S 1200 POSITION: LATITUDE __________________________

LONGITUDE _________________________
2. FUEL ONBOARD AT 1200:

GALLONS ____________________________

PERCENT ____________________________

3. EVENTS COMPLETED:
A. COMPLETED PAX TRANSFER AT 23° 43' N / 111° 23' W IAW COMTHRIDFLT 152000Z.
B. ENR __________________________________ TO REFUEL. ETA AND LOGREQ TO FOLLOW.

BT

Name: ________________________

Date: ________________________

Advisor: ________________________

Task VII – Transit, Day 3
Show all work on separate sheets of paper attached to this one.

Today is the third day of your transit to the Panama Canal. You have traveled **360** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		43° 06'	23°11' S		

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 20 knots, how much fuel did you consume from 12:00 noon yesterday until 12:00 noon today?

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

c. The distance between San Diego and the Pacific Ocean entrance to the Panama Canal is 2844 nautical miles. How much of that distance remains after two days?

d. If your ship travels at 15 knots for the remainder of the voyage how much fuel in gallons will remain and what percentage will remain?

e. Commander, Third Fleet requires that no ship drop below 50% fuel while operating in the Third Fleet area. Therefore, you will have to stop at a port to refuel. You have two options:

Puerto Quetzel, Guatemala	**1163 nm distant**
Golfito, Costa Rica	**1703 nm distant**

What percentage of fuel will you have left when you reach each port?

Puerto Quetzel:

Golfito:

f. Which port do you choose and why?

3. **Step 3:** By midnight, you need to file a report with Commander, Task Force V containing all the information you calculated above. Fill out the message template below with that information.

O 170001Z NOV 13
FM USS ______________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS ________________//
1. SHIP'S 1200 POSITION: LATITUDE ___________________________

LONGITUDE __________________________
2. FUEL ONBOARD AT 1200:

GALLONS _____________________________

PERCENT _____________________________

3. EVENTS COMPLETED:
A. COMPLETED PAX TRANSFER AT 23° 43' N / 111° 23' W IAW COMTHRIDFLT 152000Z.
B. ENR __________________________________ TO REFUEL. ETA AND LOGREQ TO FOLLOW.

BT

Name: ____________________

Date: ____________________

Advisor: ____________________

Task VII – Transit, Day 4
Show all work on separate sheets of paper attached to this one.

Today is the fourth day of your transit to the Panama Canal. You have traveled **345** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 19th		47° 28'	23° 14' S	19° 18' N	

2. **Step 2 – Logistics Requirements Message (LOGREQ).** Complete the LOGREQ message at the end of this document by filling in the blanks. All of the information you need has already been provided or determined by you.

3. **Step 3 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you burn from 12:00 noon yesterday until 12:00 noon today?

$$345\ nm \times Gallons\ burned\ per\ nm\ at\ 15\ knots = Fuel\ burned\ in\ gallons$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Fuel\ burned\ in\ gallons}{Full\ load\ in\ gallons} = Percentage\ burned$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3 - Percentage\ burned$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 4$$

3. **Step 4: Weather Forecast.** Your course will take you past the Gulf of Tehuantepec on the western coast of Guatemala. The northern boundary of the gulf is 750 nautical miles from your current position.

   a. How long before you arrive at the northern boundary (in days and hours)?

$$\frac{750\ nm}{15\ knots} = 50\ hours = 2\ days, 2\ hours$$

   b. Find two online resources that describe what makes the Gulf of Tehuantepec dangerous to ships and boats passing through it. **In your own words on a separate sheet of paper**, **type** a one paragraph (3 – 4 sentences) description of the danger. **Ensure you cite both sources. Attach your paragraph to this paper.**

   c. Search online for the High Seas Forecast for the Gulf of Tehuantepec (provided by the National Hurricane Center in Miami, Florida). Find the following forecast information:

   Forecast Winds (direction and speed): ______________________________

   Forecast Seas (height in feet): ____________________________________

   d. Your ship will ride comfortably in seas up to 10 feet in height. Will your ship and her crew have a comfortable passage across the Gulf of Tehuantepec?

4. **Step 5: Daily Operations Summary.** Complete and submit the Daily OPSUM on the next page along with your LOGREQ message.

Daily Operations Summary

O 200001Z NOV 13
FM USS ________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________
2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

A. ENR _________________ TO REFUEL. ETA AND LOGREQ TO FOLLOW.

BT

Logistics Requirement Message

P 200001Z NOV 13
FM USS ____________________
TO USNAO GUATEMALA CITY GU
INFO CTF FIVE
COMTHIRDFLT
BT

C O N F I D E N T I A L //N04000//

MSGID/LOGREQ/_________________________/001/NOV//
REF/NWP 1-03.1//

ALFA: ESTIMATED TIME OF ARRIVAL __________________: ______________________

BRAVO:

(1) PILOT, ONE TUG.

(2) SHIP'S LENGTH: ____

(3) SHIP'S BEAM: ____

(4) SHIP'S DRAUGHT: ____

FOXTROT: REQUEST ___________________________ GALLONS FUEL UPON ARRIVAL.

HOTEL: DRY AND REFRIGERATED FOOD STORES REQUIREMENTS PROVIDED TO HUSBANDING AGENT SEPCOR.

KILO: SEWAGE AND OILY WASTE DISPOSAL. TRASH AND GARBAGE DISPOSAL.

PAPA: REQUEST FREE PRATIQUE.

UNIFORM: __________________________________/COMMANDER/USN//

BT

Name: ____________________________

Date: ____________________________

Advisor: ____________________________

Task VII – Transit, Day 4
Show all work on separate sheets of paper attached to this one.

Today is the fourth day of your transit to the Panama Canal. You have traveled **390** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 19th		47° 28'	23° 14' S	19° 18' N	

2. **Step 2 – Logistics Requirements Message (LOGREQ).** Complete the LOGREQ message at the end of this document by filling in the blanks. All of the information you need has already been provided or determined by you.

3. **Step 3 – Ship's Fuel Consumption.**

   a. At 15 knots on one engine, how much fuel did you burn from 12:00 noon yesterday until 12:00 noon today?

$$390\ nm \times Gallons\ burned\ per\ nm\ at\ 15\ knots = Fuel\ burned\ in\ gallons$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Fuel\ burned\ in\ gallons}{Full\ load\ in\ gallons} = Percentage\ burned$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3 - Percentage\ burned$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 4$$

4. **Step 4: Weather Forecast.** Your course will take you past the Gulf of Tehuantepec on the western coast of Guatemala. The northern boundary of the gulf is 750 nautical miles from your current position.

   a. How long before you arrive at the northern boundary (in days and hours)?

$$\frac{750\ nm}{15\ knots} = 50\ hours = 2\ days, 2\ hours$$

   b. Find two online resources that describe what makes the Gulf of Tehuantepec dangerous to ships and boats passing through it. **In your own words on a separate sheet of paper**, **type** a one paragraph (3 – 4 sentences) description of the danger. **Ensure you cite both sources. Attach your paragraph to this paper.**

   c. Search online for the High Seas Forecast for the Gulf of Tehuantepec (provided by the National Hurricane Center in Miami, Florida). Find the following forecast information:

   Forecast Winds (direction and speed): ______________________________

   Forecast Seas (height in feet): ____________________________________

   d. Your ship will ride comfortably in seas up to 10 feet in height. Will your ship and her crew have a comfortable passage across the Gulf of Tehuantepec?

5. **Step 5: Daily Operations Summary.** Complete and submit the Daily OPSUM on the next page along with your LOGREQ message.

Daily Operations Summary

O 200001Z NOV 13
FM USS ________________________
TO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

B. ENR _________________ TO REFUEL. ETA AND LOGREQ TO FOLLOW.

BT

Logistics Requirement Message

P 200001Z NOV 13
FM USS ____________________
TO USNAO GUATEMALA CITY GU
INFO CTF FIVE
COMTHIRDFLT
BT

C O N F I D E N T I A L //N04000//

MSGID/LOGREQ/_________________________/001/NOV//
REF/NWP 1-03.1//

ALFA: ESTIMATED TIME OF ARRIVAL __________________: ______________________

BRAVO:

(1) PILOT, ONE TUG.

(2) SHIP'S LENGTH: ____

(3) SHIP'S BEAM: ____

(4) SHIP'S DRAUGHT: ____

FOXTROT: REQUEST ___________________________ GALLONS FUEL UPON ARRIVAL.

HOTEL: DRY AND REFRIGERATED FOOD STORES REQUIREMENTS PROVIDED TO HUSBANDING AGENT SEPCOR.

KILO: SEWAGE AND OILY WASTE DISPOSAL. TRASH AND GARBAGE DISPOSAL.

PAPA: REQUEST FREE PRATIQUE.

UNIFORM: __________________________________/COMMANDER/USN//

BT

Name: ___________________________

Date: ___________________________

Advisor: ___________________________

Task VIII – Transit, Day 5
Show all work on separate sheets of paper attached to this one.

Today is the fifth day of your transit to the Panama Canal. You have traveled **370** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 20th					

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you consume from 12:00 noon yesterday until 12:00 noon today?

$$370\ nm \times Gallons\ burned\ per\ nm\ at\ 15\ knots = Fuel\ burned\ in\ gallons$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Fuel\ burned\ in\ gallons}{Full\ load\ in\ gallons} = Percentage\ burned$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 4 - Percentage\ burned$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 5$$

3. At 1200 you receive an intelligence report that a major transfer of pure cocaine (several tons) will be conducted between a large fishing vessel and several "go-fast" high speed boats. The transfer will take place in 3 hours approximately 75 nautical miles south from your 1200 position.

   a. Plot your 1200 position on the chart and lay – out a DR track, 180° T, for three hours. Locate the estimated position of the drug transfer.

   b. At what speed will you need to travel to arrive at this position on time?

$$\frac{75\ nm}{3\ hours} = 25\ knots$$

   c. How much fuel (gallons and percent) will you burn traveling at this speed in the next three hours?

$$75\ nm \times Gallons\ burned\ per\ nm\ at\ 25\ knots = Gallons\ burned$$

$$\frac{Gallons\ burned}{Full\ load\ in\ gallons} = Percentage\ burned$$

4. At 1230 you receive the following encoded message from the Commandant of the Fifteenth Naval District headquartered at U.S. Naval Station Panama Canal.

Z 201900Z NOV 12
FM COMFIFTEEN
TO USS ____________________________
INFO CTF FIVE
COMTHIRDFLT
BT
S E C R E T (WHEN DECODED)
400 324 1 196 361 36 25 324 - 225 256 25 324 1
400 81 225 196

400 225 - 400 1 121 25 - 256 144 1 9
25 - **180** - 400

- **65** - 196 169 - 36 324 225 169 - 625 225
441 324 - **1200**
256 225 361 81 400 81 225 196
BT

a. Decode this message.

TRANSFER OPERATION TO TAKE PLACE 180T 65 NM FROM YOUR POSITION AT 1200.

b. How does this message change (if it does change) your answers to questions 3a. and 3b.?

$$65\ nm \times Gallons\ burned\ per\ nm\ at\ 25\ knots = Gallons\ burned$$

$$\frac{Gallons\ burned}{Full\ load\ in\ gallons} = Percentage\ burned$$

Daily Operations Summary

O 210001Z NOV 13
FM USS ____________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS ________________//

1. SHIP'S 1200 POSITION: LATITUDE ___________________________

LONGITUDE ___________________________

2. FUEL ONBOARD AT 1200:

GALLONS ______________________________

PERCENT ______________________________

3. EXPENDED _______________________________ GALLONS OF FUEL IN SUPPORT OF COMFIFTEEN 201900Z NOV 13 OPERATIONS.

4. EVENTS COMPLETED:

A. CONDUCTING OPERATIONS IAW COMFIFTEEN 201900Z NOV 13.

BT

Name: ______________________

Date: ______________________

Advisor: ______________________

Task VIII – Transit, Day 5

Show all work on separate sheets of paper attached to this one.

Today is the fifth day of your transit to the Panama Canal. You have traveled **370** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 20th					

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you consume from 12:00 noon yesterday until 12:00 noon today?

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

5. At 1200 you receive an intelligence report that a major transfer of pure cocaine (several tons) will be conducted between a large fishing vessel and several "go-fast" high speed boats. The transfer will take place in 3 hours approximately 75 nautical miles south from your 1200 position.

    a. Plot your 1200 position on the chart and lay – out a DR track, 180° T, for three hours. Locate the estimated position of the drug transfer.

    b. At what speed will you need to travel to arrive at this position on time?

    c. How much fuel (gallons and percent) will you burn traveling at this speed in the next three hours?

6. At 1230 you receive the following encoded message from the Commandant of the Fifteenth Naval District headquartered at U.S. Naval Station Panama Canal.

Z 201900Z NOV 13
FM COMFIFTEEN
TO USS ____________________________
INFO CTF FIVE
COMTHIRDFLT
BT
S E C R E T (WHEN DECODED)

400	324	1	196	361	36	25	324	- 225	256	25	324	1
400	81	225	196									
400	225	-	400	1	121	25	-	256	144	1	9	
25 -	**180**	-	400									
-	**65**	-	196	169	-	36	324	225	169	-	625	225
	441	324	-	**1200**								
256	225	361	81	400	81	225	196					

BT

a. Decode this message.

TRANSFER OPERATION TO TAKE PLACE 180T 65 NM FROM YOUR POSITION AT 1200.

b. How does this message change (if it does change) your answers to questions 3a. and 3b.?

Daily Operations Summary

O 210001Z NOV 13
FM USS ____________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS ________________//

1. SHIP'S 1200 POSITION: LATITUDE ___________________________

LONGITUDE ___________________________

2. FUEL ONBOARD AT 1200:

GALLONS ______________________________

PERCENT ______________________________

3. EXPENDED ________________________________ GALLONS OF FUEL IN SUPPORT OF COMFIFTEEN 201900Z NOV 13 OPERATIONS.

4. EVENTS COMPLETED:

A. CONDUCTING OPERATIONS IAW COMFIFTEEN 201900Z NOV 13.

BT

Name: ________________________

Date:_________________________

Advisor:_________________________

Task XI – Transit, Day 7
Show all work on separate sheets of paper attached to this one.

Today is the seventh day of your transit to the Panama Canal. You left your refueling port two days ago and have traveled **895** nautical miles (nm) South-Southeast from 12:00 local time two days ago until 12:00 local time today.

1. **Step 1 – Ship's Position.**

   a. Find the declination of the Sun for today from and online resource.

      Sun's Declination is: ____

      The resource I used is: ____

   b. Find a government online resource for **Complete Sun and Moon for One Day** for the Panama Canal. Use the following information:

      Latitude: 9° N
      Longitude: 80° W
      Time Zone: 8 hours west of Greenwich

      Find the following information (assume the time is local apparent noon):

      Sunrise: ____

      Sunset: ____

      GMT of LAN: ____

      Sun's Altitude: ____

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 20 knots, how much fuel did you consume in the last 48 hours?

$$895\ nm \times Gallons\ burned\ per\ nm\ at\ 15\ knots = Fuel\ burned\ in\ gallons$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Fuel\ burned\ in\ gallons}{Full\ load\ in\ gallons} = Percentage\ burned$$

$$\begin{aligned} &Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 5 - Percentage\ burned \\ &\quad = Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 7 \end{aligned}$$

3. **Step 3: Speed Computation.** During mid-watch (12 midnight – 4 AM), you measure the distance the ship travels in 30 seconds every half hour. The following are your results:

Time	**Midnight – 4AM**
1 bell	290 yards
2 bells	295 yards
3 bells	288 yards
4 bells	299 yards
5 bells	298 yards
6 bells	284 yards
7 bells	280 yards
8 bells	295 yards
Average Speed	17.46 knots

O 270001Z NOV 13
FM USS ____________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________//

1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

a. ENROUTE PANAMA CANAL.

Name: ________________________

Date: ________________________

Advisor: ________________________

Assessment Review – Navigation Unit

1. **Task.** In each section below, use the variables I give you to set up the formula to solve each type of problem.

   a. Latitude and Longitude:

      i. Latitude:

Variables:

Latitude (La)
Observed Altitude of Sun (O)
Sun's Declination (D)

Formula:

$La = 90° - O + D$
(Sun in Northern Hemisphere)

$La = 90° - O - D$
(Sun in Southern Hemisphere)

      ii. Longitude:

Variables:

Longitude: (Lo)
GMT of LAN (GMT)
Result of Subtraction (S)
Hours from Subtraction (H)
Minutes from Subtraction (M)

Formulas:

1. $S = 12:00 - GMT\ (12:00 > GMT)$
   $S = GMT - 12:00\ (12:00 < GMT)$

2. $Lo = H * 15° + \frac{M}{(4' per\ °)}$

   b. True Bearing:

Variables:

True Bearing (TB °T)
Magnetic Bearing (M °E or °W)
Declination (D °E or °W)

Formulas:

TB °T = $\pm$M $\pm$ D + 360° (if initial result is negative.

c. Distance Traveled:

Variables:

Distance Traveled (D nm)
Speed in Knots (S nm/hr)
Time of Journey (T hrs.)

Formulas:

$$D\ nm = \left(\frac{S\ nm}{hr}\right) * T\ hrs.$$

d. Fuel Burned:

Variables:

Fuel Burned on Journey (F gal.)
Gallons Burned per NM (G gal.)
Distance Traveled (D nm)

Formulas:

$$F\ gal. = \left(\frac{G\ gal}{nm}\right) * D\ nm$$

e. Percentage of Fuel Burned:

Variables:

Percentage of Fuel Burned (B)
Fuel Burned on Journey (F gal.)
Total Fuel Carried (T gal.)

Formulas:

$$B = \frac{F\ gal.}{T\ gal.}$$

f. Percentage of Fuel Remaining:

Variables:

Percentage of Fuel Remaining (P)
% Fuel Onboard fm Previous Day (PD)
Percentage of Fuel Burned Today (B)

Formulas:

$$P = PD - B$$

2. **Additional Information:**

Your Ship Fuel Consumption

Ship's Speed (Knots)	Gallons per Nautical Mile (GPNM) - Single Engine	Gallons per Nautical Mile (GPNM) - Duel Engine
5	95	134
6	80	112.5
7	70	97
8	61	86
9	56	78
10	52	71.5
11	48.5	67
12	46.5	63.5
13	45	60
14	44.5	58.5
15	44.5	58
16	44.9	57.5
17	45.1	58
18	47	59
19	49	60
20	50.5	62
21	54	63.5
22	56	65.5
23	60	69
24	63.5	72
25		75
26		79.5
27		84
28		88
29		94.5
30		100

Name: ________________________

Date:_________________________

Advisor:_________________________

Assessment Review – Navigation Unit

1. **Task.** In each section below, use the variables I give you to set up the formula to solve each type of problem.

   a. Latitude and Longitude:

      i. Latitude:

      Variables:

      Latitude (La)
      Observed Altitude of Sun (O)
      Sun's Declination (D)

      Formula:

      ii. Longitude:

      Variables:

      Longitude: (Lo)
      GMT of LAN (GMT)
      Result of Subtraction (S)
      Hours from Subtraction (H)
      Minutes from Subtraction (M)

      Formulas:

   b. True Bearing:

      Variables:

      True Bearing (TB °T)
      Magnetic Bearing (M °E or °W)
      Declination (D °E or °W)

      Formulas:

c. Distance Traveled:

Variables:

Distance Traveled (D nm)
Speed in Knots (S nm/hr)
Time of Journey (T hrs.)

Formulas:

d. Fuel Burned:

Variables:

Fuel Burned on Journey (F gal.)
Gallons Burned per NM (G gal.)
Distance Traveled (D nm)

Formulas:

e. Percentage of Fuel Burned:

Variables:

Percentage of Fuel Burned (B)
Fuel Burned on Journey (F gal.)
Total Fuel Carried (T gal.)

Formulas:

f. Percentage of Fuel Remaining:

Variables:

Percentage of Fuel Remaining (P)
% Fuel Onboard fm Previous Day (PD)
Percentage of Fuel Burned Today (B)

Formulas:

2. **Additional Information:**

Your Ship Fuel Consumption

Ship's Speed (Knots)	Gallons per Nautical Mile (GPNM) - Single Engine	Gallons per Nautical Mile (GPNM) - Duel Engine
5	95	134
6	80	112.5
7	70	97
8	61	86
9	56	78
10	52	71.5
11	48.5	67
12	46.5	63.5
13	45	60
14	44.5	58.5
15	44.5	58
16	44.9	57.5
17	45.1	58
18	47	59
19	49	60
20	50.5	62
21	54	63.5
22	56	65.5
23	60	69
24	63.5	72
25		75
26		79.5
27		84
28		88
29		94.5
30		100

Standard: ________________ Name: ______________________

HL: ________________ Date: ______________________

Advisor: ______________________

Navigation Unit Assessment

Show all work on separate sheets of paper stapled to this one. Put answers on this sheet

Today is the last day of your transit to the Panama Canal. You have traveled 895 nautical miles (nm) since you left Puerto Quetzel, Guatemala. You have traveled at an average speed of 15 knots since you left port.

1. **Task 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 27th	17:18	60° 59'	21° 01'	8° 30' N	079° 30' W

2. **Task 2 – Ship's Fuel Consumption.**

   a. How long in hours has it taken you to reach the Panama Canal after leaving Puerto Quetzel, Guatemala? (Extra credit if you provide the time in days, hours, minutes, seconds)

$$\frac{895\,nm}{15\,\frac{nm}{hr}} = 59.67\ hours = 2\ days, 11.67\ hours$$

$$0.67\ hours * 60\left(\frac{min.}{hr}\right) = 40.2\ minutes$$

$$0.2\ minutes * 60\,\frac{sec}{min} = 12\ seconds$$

$$2\ days, 9\ hours, 40\ minutes, 12\ seconds$$

   b. On one engine, how much fuel have you burned?

$$44.5 \frac{gal.}{nm} * 895\ nm = 39827.5\ gallons$$

c. What percentage of fuel have you burned?

$$\frac{39827.5\ gallons}{205{,}000\ gallons} * 100 = 19.43\%$$

d. What percentage do you have left onboard? (*your ship* can carry a total of 205,000 gallons of fuel). ***Be careful on this one.***

$$100\% - 19.43\% = 80.57\%$$

e. If you had travelled on two engines, how much fuel **in gallons and percent** would you have burned?

$$58 \frac{gal.}{nm} * 895\ nm = 51910\ gallons$$

$$\frac{51910\ gallons}{205000\ gallons} = 25.32\%$$

3. As *your ship* approaches the canal, the navigator observes the following bearings on her magnetic compass and also notes the following declinations. Find the true bearings the ship is traveling on when the navigator makes each observation.

Magnetic Bearing	**Declination**	**True Bearing**
90° W	45° E	315° T
60° W	10° E	310° T
59° W	11° E	312° T

Standard: ________________ Name: ______________________

HL: ________________ Date: ________________________

Advisor: ________________________

Navigation Unit Assessment

Show all work on separate sheets of paper stapled to this one. Put answers on this sheet

Today is the last day of your transit to the Panama Canal. You have traveled 895 nautical miles (nm) since you left Puerto Quetzel, Guatemala. You have traveled at an average speed of 15 knots since you left port.

1. **Task 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 27th	17:18	60° 59'	21° 01'		

2. **Task 2 – Ship's Fuel Consumption.**

   a. How long in days and hours has it taken you to reach the Panama Canal after leaving Puerto Quetzel, Guatemala? (Extra credit if you provide the time in days, hours, minutes, seconds)

   b. On one engine, how much fuel have you burned?

c. What percentage of fuel have you burned?

d. What percentage do you have left onboard? (*your ship* can carry a total of 205,000 gallons of fuel). ***Be careful on this one.***

e. If you had travelled on two engines, how much fuel **in gallons and percent** would you have burned?

3. As *your ship* approaches the canal, the navigator observes the following bearings on her magnetic compass and also notes the following declinations. Find the true bearings the ship is traveling on when the navigator makes each observation.

Magnetic Bearing	**Declination**	**True Bearing**
90° W	45° E	
60° W	10° E	
59° W	11° E	

PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth

“Geometry of the Ship”

Surface Area and Volume of 3-Dimensional Shapes

Thomas R. Beall
Captain, U. S. Navy (Ret.)

P C H S

CURRICULUM MAPPING: Middle and High School Geometry – Geometry of the Ship

CENTRAL QUESTION(S)/THEMES

1. What is the practical application of accurately measuring segments and angles?
2. When will I ever need to know how to find the surface area and volume of a shape?

TEXT(S) AND RESOURCES

1. Clark, I. C. *Ship Dynamics for Mariners.*
2. Hooyer, H. H. *Behavior and Handling of Ships.*
3. Lewis, *Principles of Naval Architecture.*
4. NCTM *Illuminations.* Popcorn Prisms Anyone?
5. NCTM *Illuminations.* Scaling Away.

STUDENTS WILL KNOW AND BE ABLE TO . . .

Content, Skills, & Standards to be assessed by rubric(s) in this unit (Common Core, GSE, HOL and PCHS Expectations)

1. **M(G&M)–10–2:** Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem).

2. **M(G&M)-10-5:** Applies the concepts of similarity by solving problems within mathematics or across disciplines or contexts.

3. **M(G&M)-10-6:** Solves problems involving perimeter, circumference, area, surface area, and volume.

PROJECT/PRODUCT & PUBLIC DEMONSTRATION

1. Container Scale Model Project.

2. Loading Container Project.

3. Ship Model Construction Project.

4. Ship Stability Project.

OTHER EVIDENCE OF STUDENT LEARNING

1. Diagnostic Pre-assessment.

2. Pencil and Paper Post-assessments.

SCAFFOLDED TEACHING AND LEARNING ACTIVITIES

1. **"Layout of a Ship" Activity.** Students identify and define parts of a ship on a general layout diagram and relate those parts to geometric shapes, defining those as well.

2. **"Where Did That Product Come From" Activity.** Builds on student knowledge, learned in middle school, of how to find the surface area and volume of a rectangular prism to illustrate application in shipping cargo around the world.

3. **"Building a Scale Model of a Container" Project.** Building on review of rectangular prisms and how to find their dimensions, students will:

   A. Accurately design a scale model of a standard 20 ft. CEU shipping container.
   B. Build a scale model to design dimensions.
   C. Determine the surface area and volume of the scale model and discover the relationship between the model's dimensions and the actual container's dimensions.

4. **"Stowing and Loading of a Ship" Project.** Using the scale model of the container they have built and various shapes, students will determine the optimum loading of the container to maximize the shippers profit.

5. **Model Shipbuilding Project.** Using a simple naval architect's drawing of the contours of a ship's hull, students will:

   A. Create a geometric reflection of the forward and aft contours.
   B. Build a model of the ship using athwart ship frames traced from the contour drawing.

6. **Ship Stability Project.** Students will measure weights and dimensions of various shapes and the angle of heel of a floating platform when those weights are moved various distances from the centerline.

7. **Coastal Navigation Exercise.** Students will apply the Pythagorean Theorem and distance measures to develop piloting plans in coastal waters.

Unit Outline and Table of Contents

This unit comprises eight lessons. Given that students learn at varying rates, instruction will likely take place over 3 – 5 weeks. The outline of the plan is as follows:

Unit Goals

Students will:

1. Place learning in a real-world context by exploring the connections between basic geometry and the design and operation of merchant ships at sea.

2. Design and build accurate models of three dimensional shapes.

3. Derive equations for surface area and volume of rectangular and triangular prisms and cylinders through inductive reasoning.

4. Accurately evaluate changes in surface area and volume of three dimensional shapes when one or more of the dimensions is changed.

5. Convert volume measurements into liquid measurements in both the English and metric systems.

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

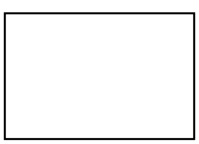
Name:________________________________
Advisor:______________________________
Date:_________________________________

Geometry Unit Pre-Assessment

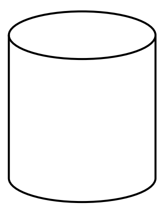
I. Definitions. Match the term by placing its letter next to its definition on the right:

Term	***Definition***
A. Radius of a Circle:	___C__ The sum of the lengths of the sides of a polygon.
B. Rectangle:	___A__ Any line segment from its center to its perimeter.
C. Perimeter:	___G__ The amount of space a solid body occupies.
D. Chord:	___E__ The size of the region enclosed by a figure.
E. Area:	___F__ The ratio of the circumference of a circle to its diameter.
F. Pi:	___D__ Line segment whose endpoints both lie on the circle.
G. Volume:	___B__ A quadrilateral with opposite sides of equal lengths and with four right angles.

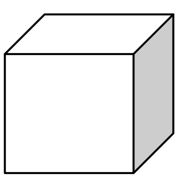
II. Identify each of the following shapes:



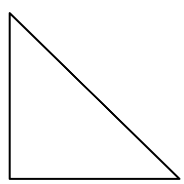
Rectangle



Cylinder

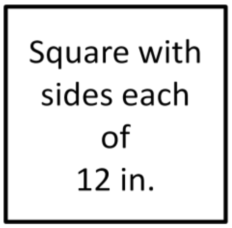


Rectangular Prism/ Cube



Triangle / Right Triangle

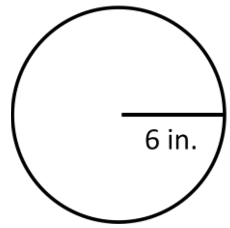
III. What is the **area** and the **perimeter** of each of the following shapes?



Area**: 144 sq. in.**

$Length = width = 12\ in.$
$Area = 12\ in. \times 12\ in. =$
$144\ sq.in.$

$Length = width = 12\ in.$
$Perimeter = 12 + 12 + 12 + 12 = 48\ in.$

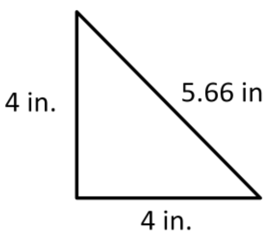


Area: **113.094 sq. in.**

$A = \pi \times r^2 = 3.1415 \times 36\ in.^2$
$= 113.094\ in.^2$

Perimeter: **37.698 in.**

$P = 2 \times \pi * r = 2 \times 3.1415 \times 6\ in.$
$= 37.698\ in.$

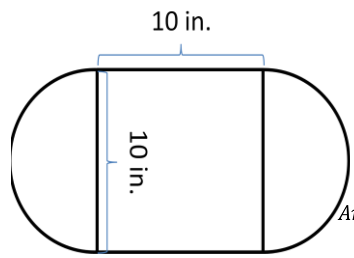


Area**: 8 sq. in.**

$A = \frac{1}{2} \times (l \times w) = \frac{1}{2} \times 16\ in.^2$
$= 8\ in.^2$

Perimeter: **13.66 in.**

$P = 4\ in. + 4\ in. + 5.66\ in. = 13.66\ in.$



Area**: 178.54 sq. in.**

$Area\ of\ rectangle = (l \times w) = 100\ in.^2$

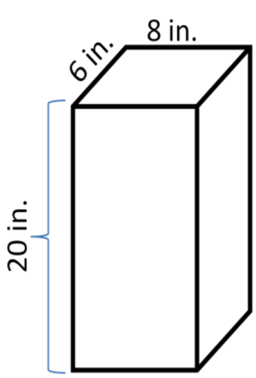
$Area\ of\ half\ circles = 2 \times \left(\frac{1}{2} \times (\pi \times 5^2)\right) = 2 \times$
$\left(\frac{1}{2} \times (3.1415 \times 25)\right) = 78.54\ in.^2$

$Area = 100\ in.^2 + 78.54\ in.^2 = 178.54\ in.^2$

Perimeter**: 51.415 in.**

$P = 2 \times (\frac{1}{2} \times (2 \times 3.1414 \times 5\ in.) +$
$(2 \times 10\ in.) = 51.415\ in.$

IV. What is the **surface area** and **volume** of each of the following shapes?



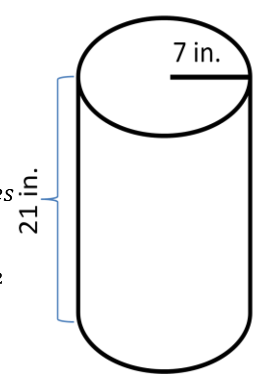
Surface Area**: 656 sq. in.**

$Surface\ Area$
$= the\ sum\ of\ the\ areas\ of\ the\ 6\ sides$

$= 2 \times (6\ in. \times 8\ in.) + 2 \times (6\ in. \times$
$20\ in.) + 2 \times (8\ in. \times 20\ in.) = 656\ in^2$

Volume**: 960 cu. in.**

$Volume = l \times w \times h = 8\ in. \times 6\ in. \times$
$20\ in. = 960\ in^3$



Surface Area**: 1231.47 sq. in.**

$Surface\ Area = 2 \times (3.1415 \times r^2) +$
$(2 \times \pi \times r) \times height$

$= 2 \times (3.1415 \times (7in.)^2) + (2 \times 3.1415 \times$
$7in.) \times 21 = 1231.47\ in.^2$

Volume**: 3232.6 cu. in.**

$Volume = Area\ of\ the\ cirle \times height$

$= \pi \times r^2 \times h = 3.1415 \times 49 \times 21 = 3232.6in^2$

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

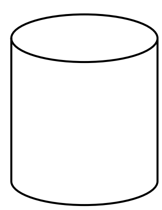
Name:___________________________________
Advisor:_________________________________
Date:___________________________________

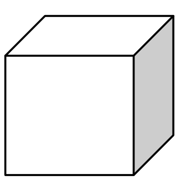
Geometry Unit Pre-Assessment

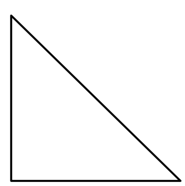
I. Definitions. Match the term by placing its letter next to its definition on the right:

Term	***Definition***
A. Radius of a Circle:	______ The sum of the lengths of the sides of a polygon.
B. Rectangle:	______ Any line segment from its center to its perimeter.
C. Perimeter:	______ The amount of space a solid body occupies.
D. Chord:	______ The size of the region enclosed by a figure.
E. Area:	______ The ratio of the circumference of a circle to its diameter.
F. Pi:	______ Line segment whose endpoints both lie on the circle.
G. Volume:	______ A quadrilateral with opposite sides of equal lengths and with four right angles.

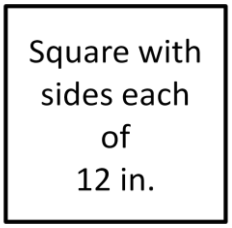
II. Identify each of the following shapes:





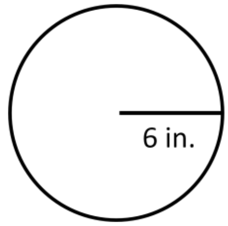


III. What is the **area** and the **perimeter** of each of the following shapes?



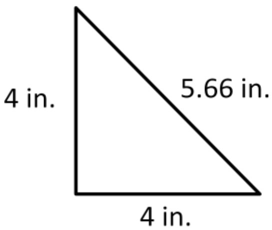
Area:

Perimeter:



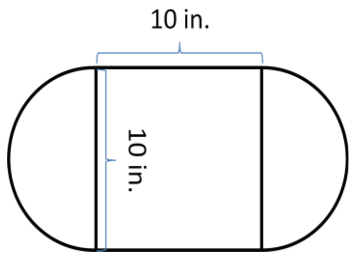
Area:

Perimeter:



Area:

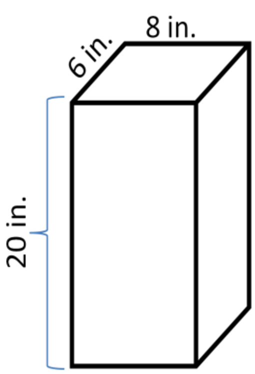
Perimeter:



Area:

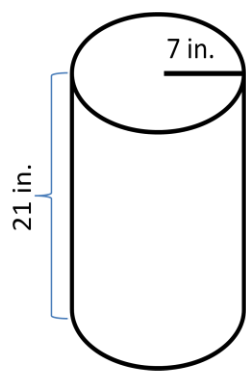
Perimeter:

IV. What is the **surface area** and **volume** of each of the following shapes?



Surface Area:

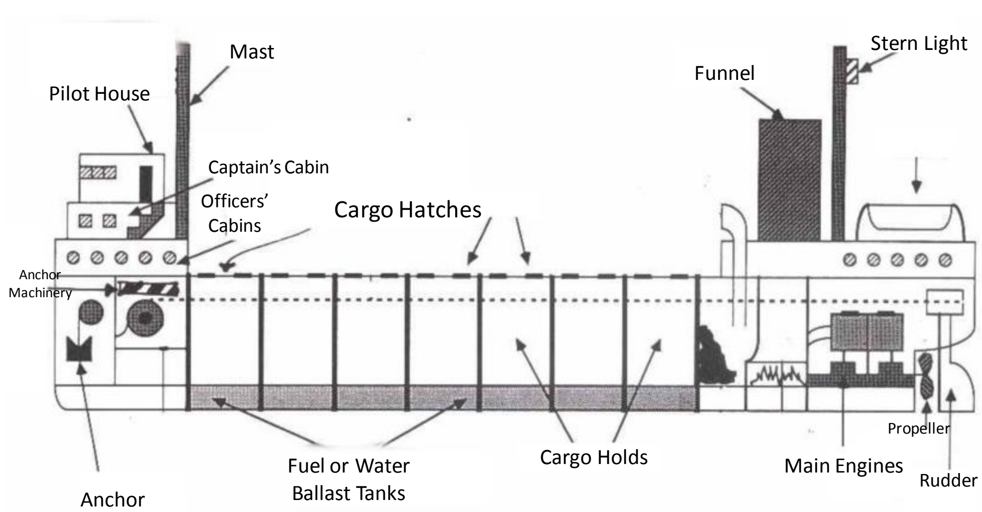
Volume:



Surface Area:

Volume:

Grade / Content Area	# 8^{th} / 9^{th} Grade: Geometry
Lesson Title	**Understanding the General Layout of a Ship and its Relationship to Geometry**
Guiding Question	***How do common components of a ship relate to geometric shapes?***
Content Standards	*NCTM Standards*: I. Instructional programs from prekindergarten through grade 12 should enable all students to— 1. Recognize reasoning and proof as fundamental aspects of mathematics; 2. Make and investigate mathematical conjectures; 3. Develop and evaluate mathematical arguments and proofs; **4.** Select and use various types of reasoning and methods of proof.
	Common Core Standards: High school students will: I. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
Student Learning Objectives	Students will be able to: I. Identify parts of a ship such as deck gear, maneuvering equipment, propulsion equipment, compartmentation. II. Relate these parts to geometric shapes.
Preparation	I. *Classroom Organization*: Classroom Organization: Students will work in pairs. Desks will be rearranged to permit two students to work together with a common writing surface. II. *Differentiation*. A. Students will be required to identify on a simple diagram of a ship the components of that ship using proper terminology and then relate those components with geometric shapes; defining those as well. B. To accommodate learners at different levels and with different language abilities, I will: 1. Provide pictures of geometric shapes, allowing students to match those

	with the components. 2. Provide a list of definitions to selected students. III. *Materials:* A. For each group of students: 1. “General Layout of a Ship” Diagram. 2. List of components of the ship to be labeled. 3. Definitions and pictures of geometric shapes. B. Additional materials: 1. Large classroom version of “General Layout of a Ship” diagram.
Instruction and Engagement	I. *Warm-up* (**10 minutes**). II. *Launch* (**5 minutes**). A. Introduce the idea that Geometry has practical application in the world by showing projected pictures of ships, especially dry-cargo, RO / RO and container ships. B. Ask the question, “What Geometry knowledge might be helpful to the crew of a cargo ship?” III. *Engagement* (**40 minutes**). A. Students working in pairs will label the diagram “General Layout of a Ship. When complete, it should look like this: 

B. Students will then relate these terms to geometric shapes and their definitions:

Part of Ship	Geometric Term	Definition
Captain's Cabin Windows	Square	A 4-sided regular polygon with all sides equal and all internal angles 90°.
Officers' Cabin Windows	Circle	A line forming a closed loop, every point on which is a fixed distance from a center point.
Cargo Holds	Rectangular Prism	A solid (3-dimensional) object which has six faces that are rectangles.
The intersection of two corners of a cargo hold.	Angle	A shape, formed by two lines or rays diverging from a common point (the vertex).
Funnel	Cylinder	A three-dimensional object that has two parallel ends (called bases) that are either circles or ellipses, connected by straight, parallel sides.
Rudder (the flat surface of the rudder)	Plane	Part of a flat surface that is infinitely large and with zero thickness.

C. Students will then answer the question, "How would an understanding of how to determine the area, surface area, and volume of these shapes help those who:

1. Design and build ships?
2. Move cargo in ships?"

IV. *Closing* (**10 minutes**). We will review how to determine the surface area and volume of rectangular prisms as a precursor to the following lesson. Homework will involve practice.

Assessment	In-class worksheets and homework.

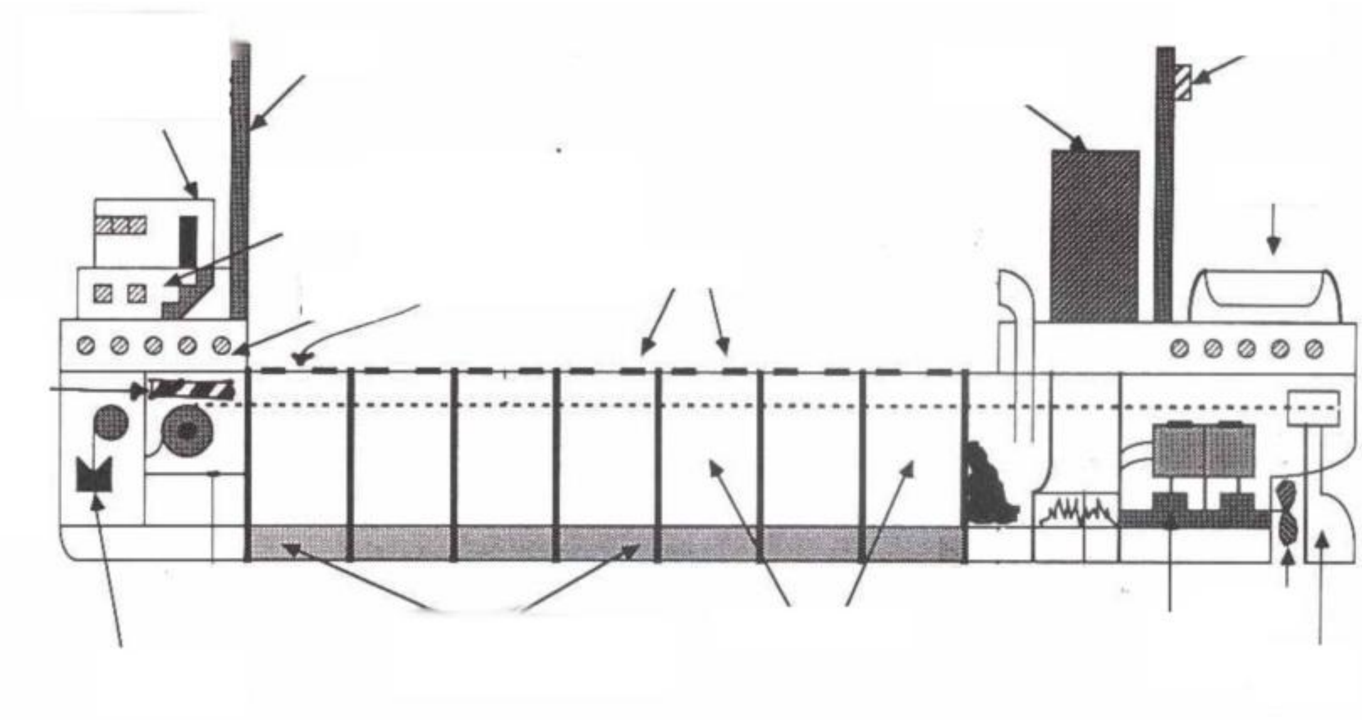
Part 1.

Directions:

I. Write the correct label next to the arrow pointing to that object on the ship diagram.

II. Underneath the label, write the geometric shape that most closely resembles the object you labeled.

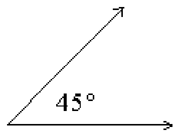
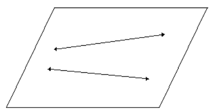
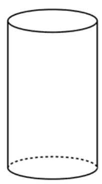
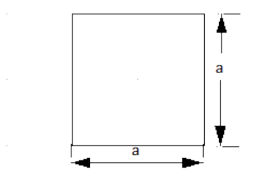
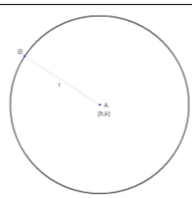
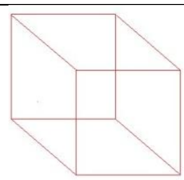
Labels
Captain's Cabin Windows
Officers' Cabin Windows
Funnel
Cargo Holds
Water Ballast Tanks
Main Engines
Anchor Machinery
Propeller
Rudder



Geometric Shapes
Angle
Rectangular Prism
Cylinder
Circle
Plane
Square

Part 2.

Directions: Draw a line to connect the term with its picture and its definition.

Picture	Geometric Term	Definition
	Square	A line forming a closed loop, every point on which is a fixed distance from a center point.
	Circle	A 4-sided regular polygon with all sides equal and all internal angles 90°.
	Rectangular Prism	A flat surface that is infinitely large and with zero thickness.
	Angle	A three-dimensional object that has two parallel ends (called bases) that are either circles or ellipses, connected by straight, parallel sides.
	Cylinder	A shape, formed by two lines or rays diverging from a common point (the vertex).
	Plane	A solid (3-dimensional) object which has six faces that are rectangles.

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name: ______________________________

Date: ______________________________

Advisor: ____________________________

Geometry of the Ship: Practice 1: Perimeter and Area of 2 – Dimensional Shapes

Formulas: Use the following formulas to complete this practice exercise:

I. Square of side length *s*:

A. **Perimeter:** $\boldsymbol{P = s + s + s + s = 4s}$
B. **Area:** $\boldsymbol{A = s \times s = s^2}$

II. Rectangle of length *l* and width *w*:

A. **Perimeter:** $\boldsymbol{P = l + w + l + w = 2l + 2w}$
B. **Area:** $\boldsymbol{A = l \times w}$

III. Circle of radius *r*:

A. **Perimeter:** $\boldsymbol{P = 2 \times \pi \times r = 2\pi r}$
B. **Area:** $\boldsymbol{A = \pi \times r \times r = \pi r^2}$

IV. Triangle of base length *b* and height *h*

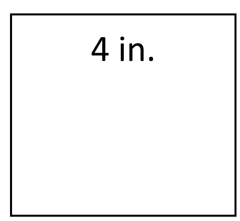
A. **Area:** $\boldsymbol{A = \left(\frac{1}{2}\right) bh}$

Solve the following problems. Show all work and answers on separate sheets of paper attached to this one.

1. Use the following example as a guide to solving the problems on the next page.

   Example: A square of side length $s = 4\ in.$

   $P = 4s$ | $A = s^2$
   $P = 4 \times 4\ in.$ | $A = (4\ in.)^2$
   $P = 16\ in.$ | $A = 16\ in.^2$



2. Find the perimeter and area of the following 2 – dimensional shapes. Draw each shape, using ruler and compass, labeling the dimensions. The drawings do not have to match the exact dimensions.

   a. A square of side length $s = 7\ cm$.

   b. A square of side length $s = 12\ mm$.

   c. A square of side length $s = 22\ ft$.

   d. A rectangle with length $l = 6\ in.$ and width $w = 4\ in.$

   e. A rectangle with length $l = 13\ in.$ and width $w = 6\ in.$

   f. A rectangle with length $l = 15\ m.$ and width $w = 7.5\ m.$

   g. A rectangle with length $l = 22\ m.$ and width $w = 110\ cm.$

   h. A rectangle with length $l = 18\ ft.$ and width $w = 108\ in.$

   i. A circle of radius $r = 2\ in.$

   j. A circle of radius $r = 17\ in.$

   k. A circle of radius $r = 19\ ft.$

   l. A circle of diameter $d = 12\ in.$

   m. A circle of diameter $d = 42\ cm.$

   n. A circle of diameter $d = 1\ in.$

   o. A triangle of base length $b = 2\ cm.$ and height $h = 3\ cm.$ (Area only)

   p. A triangle of base length $b = 12\ m.$ and height $h = 10\ m.$ (Area only)

   q. A triangle of base length $b = 15\ ft.$ and height $h = 144\ in.$ (Area only)

Grade / Content Area	**8th / 9th Grade: Geometry**
Lesson Title	**Where Did That Product Come From? (1 day)**
Guiding Question	***What factors do merchants consider when shipping products across great distances?***
Content Standards	*NCTM Standards:* I. Instructional programs from prekindergarten through grade 12 should enable all students to— A. Recognize reasoning and proof as fundamental aspects of mathematics; B. Make and investigate mathematical conjectures; C. Develop and evaluate mathematical arguments and proofs; D. Select and use various types of reasoning and methods of proof. *Common Core Standards:* None for this lesson.
Student Learning Objectives	I. Gain insight into to the various factors involved in shipping products overseas. II. Make the connection between cargo shipment and Geometry.
Preparation	I. *Classroom Organization*: Classroom Organization: Students will work in groups of three or four. Desks will be rearranged to permit four students to work together with a common writing surface. II. *Differentiation*. III. *Materials:* A. For each group of students: 1. Worksheet, camera box. B. Additional materials: None.
Instruction and Engagement	I. *Warm-up* (**10 minutes**). II. *Launch* (**5 minutes**).

A. I will start with, *"In middle school, you learned about the surface area and volume of a rectangular prism. Who can tell me the formulas for each?"*

$$V = length \times width \times height = Bh$$

$$SA = 2((l \times w) + (l \times h) + (w \times h) = 2B + Ph$$

I will do an example on the board, introducing how $B = l \times w$ and $P = 2l + 2w$.

B. *"Who can give me an example of why it is important to understand this?"* Introduce the concept of shipping in a box using the digital camera box.

C. **Group Work:** Since this will be the first time students work in groups, we will discuss what makes for good group work:

1. Work quietly – do not distract other groups.
2. Every member has a role: leader, timekeeper, recorder, moderator.
3. Every member works.
4. Conduct discussions respectfully – one member should not dominate. Listen more than talk.

III. *Engagement* (**50 minutes**).

A. **First Activity** (**10 minutes**): Students, working in groups, will begin to work on the attached worksheet, asking themselves several questions including:

1. *When you buy an electronic device, what is most important to you: what it looks like, what it can do, how much it costs, or how big it is?* Give them a couple of minutes to answer the question.

   Each group should report group consensus. Answers will vary. What do students value most?

2. *As the shipping company, if you are paid a fixed amount of money for every camera shipped, what is your goal?* Give them a couple of minutes to answer the question and come to consensus.

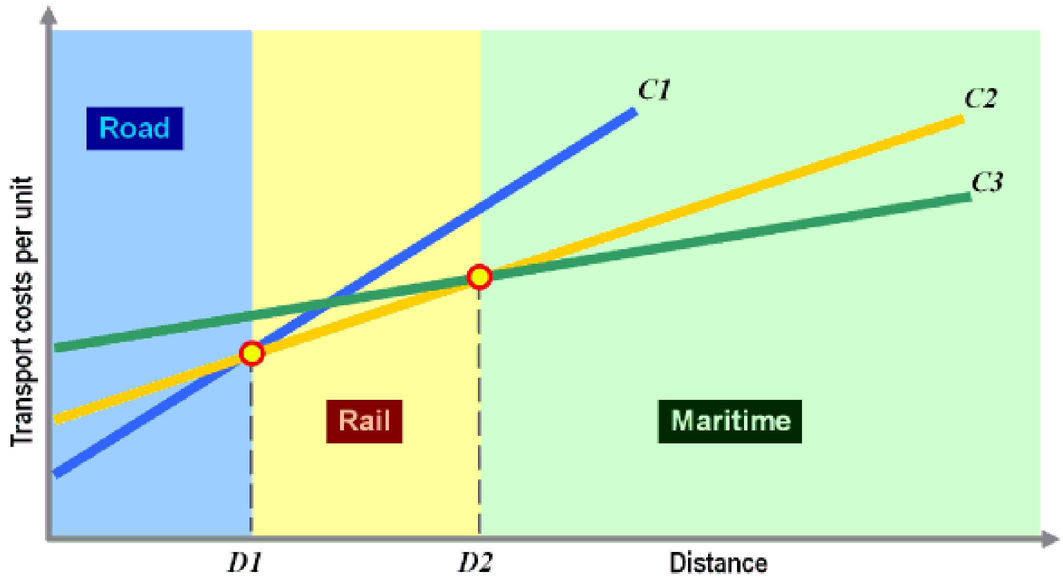
   Each group should report group consensus. Again, answers will vary. What do students consider the most important factors?

3. *How do you think your digital cameras should be shipped at each stage of their journey? Consider cost, speed, and how much each*

mode (plane, truck, train, ship) can carry.

Student answers most likely will be based on understanding of how much each mode can carry, how fast it is, how safe, and how expensive.

B. **Second Activity** (**10 minutes**): Students, in groups will be asked to evaluate the meaning of the following graph and information:



1. C1 is transport costs per unit for transport by road – i.e. truck.
2. C2 is transport costs per unit by rail – i.e. train.
3. C3 is transport costs per unit by maritime – i.e. ship.

4. D1 is a distance of 300 – 450 miles.
5. D2 is a distance of 1000 miles.

I may have to give some support such as indicating what the *x*-axis and *y*-axis mean and what the distances mean. The following more detailed example might be used:

Distance (100's miles)	Transportation Cost by Ship	Transportation Cost by Train	Transportation Cost by Truck
0	$500.00	$250.00	$200.00
1	$520.00	$287.50	$250.00
2	$540.00	$325.00	$300.00
3	$560.00	$362.50	$350.00
4	$580.00	$400.00	$400.00
5	$600.00	$437.50	$450.00
6	$620.00	$475.00	$500.00
7	$640.00	$512.50	$550.00
8	$660.00	$550.00	$600.00
9	$680.00	$587.50	$650.00
10	$700.00	$625.00	$700.00
11	$720.00	$662.50	$750.00
12	$740.00	$700.00	$800.00
13	$760.00	$737.50	$850.00
14	$780.00	$775.00	$900.00
15	$800.00	$812.50	$950.00
16	$820.00	$850.00	$1,000.00
17	$840.00	$887.50	$1,050.00
18	$860.00	$925.00	$1,100.00
19	$880.00	$962.50	$1,150.00
20	$900.00	$1,000.00	$1,200.00

C. **Third Activity** (**10 minutes**): Students in groups will be asked to reevaluate their answer to the question:
"How do you think your digital cameras should be shipped at each stage of their journey? Consider cost, speed, and how much each mode (plane, truck, train, ship) can carry," given the chart and the following

	information: 1. Factory to Ocean: 700 miles. 2. Across the Ocean: 6000 miles. 3. Ocean to Store: 3000 miles. Students will be asked to come to group consensus and report their consensus. D. **Fourth Activity** (**10 minutes**): I will state, *"Let's keep in mind our choices and consider that, regardless of how we ship, we will need to ship in a large container. As we said before, the standard container measures 20 ft. by 8 ft. by 8 ft. Another type of container measures 40 ft. by 8 ft. by 8 ft. In your groups, consider the questions on the worksheet."* Students will answer the questions and report their findings. I will then ask, *"What equation did you use to solve the problems concerning the amount of cameras that could be fit into the container box.?"* Answer: $V = lwh = Bh$ *"How can we prove that equation is always true?"* E. **Fifth Activity** (**10 minutes**): Finally, I will ask students to consider and reflect on the question: *"What does all this have to do with Geometry?"* ...asking them to list some specific Geometric concepts that might apply to the movement of cargo. Students will be asked to share their reflections. IV. *Closing* (**5 minutes**). I will state, *"We've considered the real world considerations of cargo movement, we've considered how they relate to Geometry – specifically how cargo is moved in geometric shapes such as rectangular prisms. Next we will consider how to prove the simple formulae we use to calculate the volume of these shapes"*
Assessment	1. Worksheets and class participation. 2. Homework Worksheet: Surface Area and Volume of Rectangular Prisms

Name: ________________________

Where Did That Product Come From?

When you buy a cell phone or other electronic device, list, in order from 1 (most important) to 4 (least important) the things you consider when making your choice:

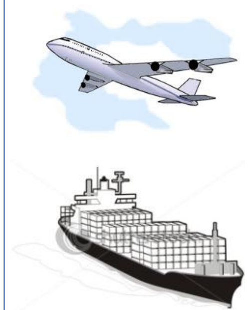
What it can do.	1.	________________________________
How big it is.	2.	________________________________
How much it costs.	3.	________________________________
What it looks like.	4.	________________________________

You are CEO a shipping company whose job it is to transport products made in China for sale in the United States. You've just received an order to ship a large order of digital cameras that come packed in small boxes like the one I will show you.

Consider the following questions:

1. As the shipping company, if you are paid a fixed amount of money for every camera shipped, what is your goal?

2. How do you think your digital cameras should be shipped at each stage of their journey? Consider cost, speed, and how much each mode (plane, truck, train, ship) can carry. Circle your choice on the picture below and tell why you chose it in the space below:

	From factory to Ocean	**Across the Ocean**	**From Ocean to Store**	
				
Why did you choose ..... Consider: 1. Cost 2. Speed 3. How much it can carry				

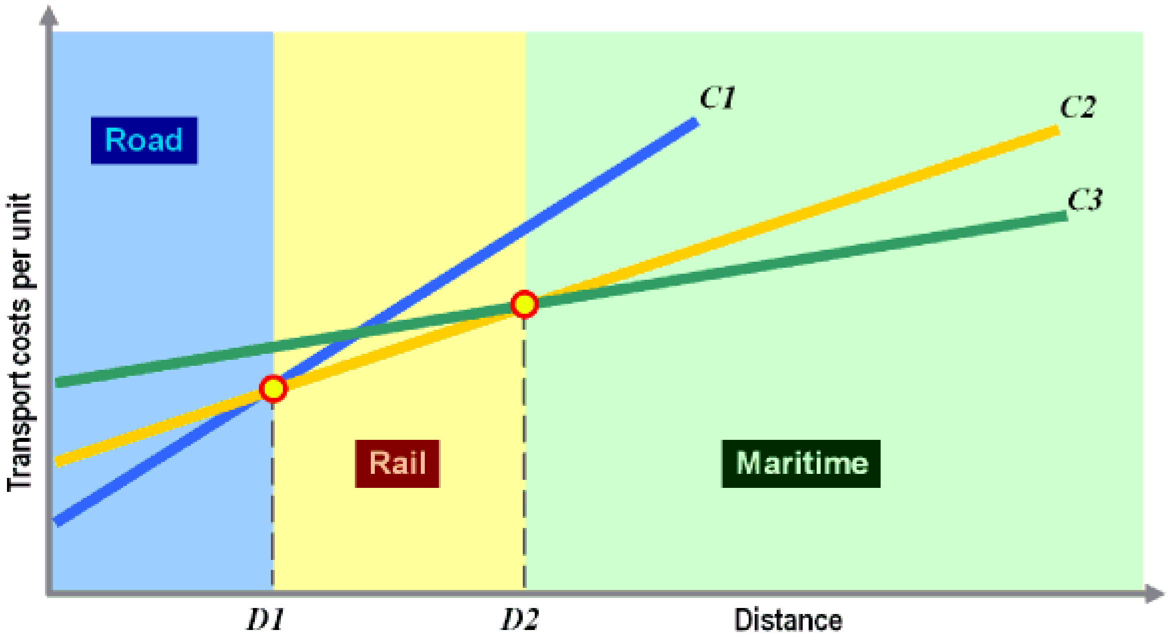
3. Explain which mode of transport at each stage is best and why it is best.

   a. From Factory to Ocean: ______________________________

   b. Across the Ocean: ______________________________

   c. Ocean to Store: ______________________________

4. Now look at the following graph. Decide what this graph means and be prepared to tell the class what it means.



a. C1 is transport costs per unit for transport by road – i.e. truck.
b. C2 is transport costs per unit by rail – i.e. train.
c. C3 is transport costs per unit by maritime – i.e. ship.

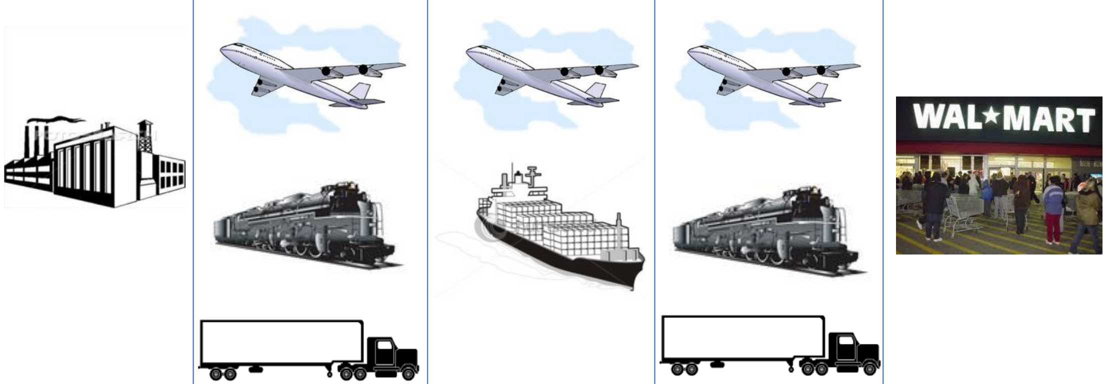
d. D1 is a distance of 300 – 450 miles.
e. D2 is a distance of 1000 miles.

5. The following are distances from factory to ocean, across the ocean, and from the ocean to the store.

   a. Factory to Ocean: 700 miles.
   b. Across the Ocean: 6000 miles.
   c. Ocean to Store: 3000 miles.

6. Now, which would you pick:

	From factory to Ocean	Across the Ocean	From Ocean to Store



Why did you choose .....

Consider:
1. Cost
2. Speed
3. How much it can carry

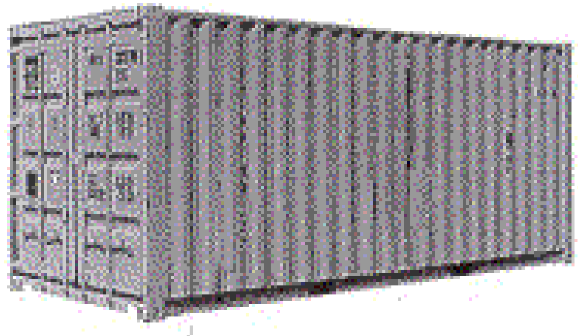
7. Decide which mode of transport at each stage is best and why it is best.

   a. From Factory to Ocean: ____

   b. Across the Ocean: ____

   c. Ocean to Store: ____

8. Commercial cargo is normally shipped in containers like the one pictured below.



9. There are two standard sizes:

   $B = 20\ ft. \times 8\ ft.$ and $h = 8\ ft.$

   $B = 40\ ft. \times 8\ ft.$ and $h = 8\ ft.$

   a. If our digital camera box is $12\ in.\ x\ 6\ in.\ x\ 6\ in.$, how many can fit into each type of container?

      $B = 20\ ft. \times 8\ ft.$ and $h = 8\ ft.$ ____________________

      $B = 40\ ft. \times 8\ ft.$ and $h = 8\ ft.$ ____________________

   b. What if the cameras came in cylindrical containers with radius 4 in. and height __________ in.?

      Would you be able to ship more or fewer?

      How many more or fewer?

10. Reflect on what we have discussed and answer the question (in your own words) **"What does all this have to do with Geometry?"**

    List some specific Geometric concepts that might apply to the movement of cargo.

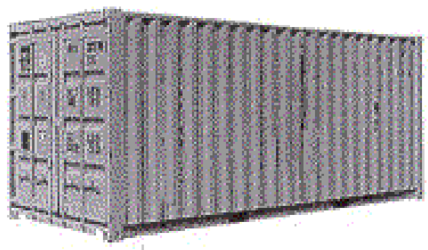
1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________
Date:_____________________
Advisor: ___________________

Homework Worksheet: Surface Area and Volume of Rectangular Prisms

(WRITE ALL FORMULAS AND SHOW ALL WORK ON SEPARATE SHEETS OF PAPER)

1. Standard cargo shipping containers are of two types. Answer the following questions about each.



A. The most common container is 20 ft. long, 8 ft. wide, and 8 ft. high. Find the following:

Surface Area: **768 sq. ft.**

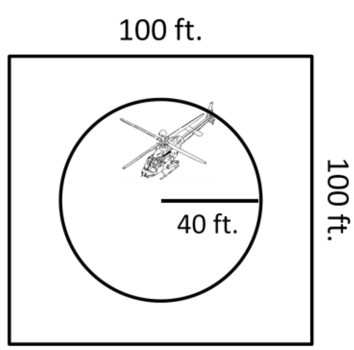
Volume: **1280 cu. ft.**

B. The other standard container is also 8 ft. wide and 8 ft. wide and has a total volume of 2560 $ft.^3$ Find the following:

Length of the container: **40 ft.**

Surface Area: **1408 sq. ft.**

2. The U. S. Navy Hospital Ship *Comfort* (which is providing relief in Haiti) has a helicopter flight deck (on top of the ship) with dimensions of 100 feet long and 100 feet wide. The helicopters land on a circular landing pad with radius of 40 feet.



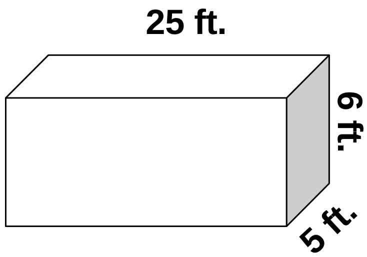
A. What is the area of the circular landing pad?

$$\pi \times 40 \times 40 = 5027\ sq.ft.$$

B. What is the area of the ***remainder*** of the flight deck?

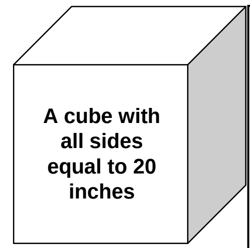
$$10000 - 5027 = 4973\ sq.ft.$$

Find the surface area and volume of the following shapes:



$SA = 2B + Ph$
$SA = 2(25 \times 5) + 2(25 + 5) \times 6$
$SA = 610\ ft.^2$

$Volume = B \times h$
$Volume = 25\ ft. x\ 5\ ft. x\ 6\ ft.$
$= 750\ ft.^3$

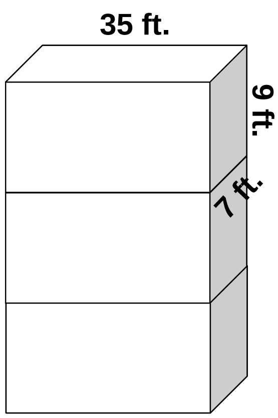


$SA = 2B + Ph$
$SA = 2(20^2) + (4 \times 20)20$
$SA = 2400\ ft.^2$

$Volume = Bh$
$Volume = 20x20x20$
$= 8000\ in.^3$

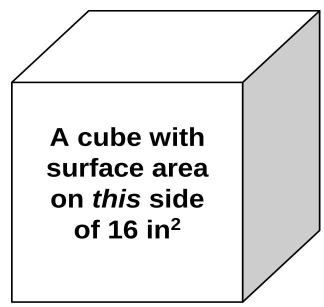
3.a Surface Area: **610 sq. ft.**
3.b. Volume: **750 cu. ft.**

4.a Surface Area: **2400 sq. ft.**
4.b Volume: **8000 cu. ft.**



$SA = 2B + Ph$
$SA = 2(35 \times 7) + 2(35 + 7) \times 9$
$SA = 610\ ft.^2$

$Volume = B \times h$
$Volume = 35\ ft. \times\ 7\ ft. \times\ 9\ ft.$
$= 6615\ ft.^3$



$SA = 2B + Ph$
$SA = 2(4^2) + (4 \times 4)4$
$SA = 2400\ ft.^2$

$Volume = Bh$
$Volume = 4^3 = 64\ in.^3$

Each box is 35 x 7 x 9 ft.

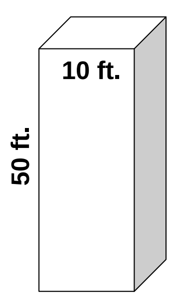
5.a Surface Area: **2506 sq. ft.**

5.b. Volume: **6615 cu. ft.**

6.a Surface Area: **96 sq. in.**

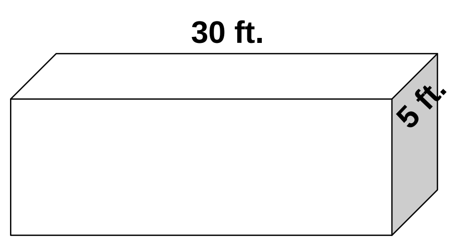
6.b Volume: **64 cu. in.**

A. Given the volume of the shape and two of the dimensions, find the third dimension.



Volume = 4000 ft³

$4000/(50x10) = 8\ ft.$



$1500/(30x5) = 10\ ft.$

Volume = 1500 ft³

6.a. **8 ft.**

6.b. **10 ft.**

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________

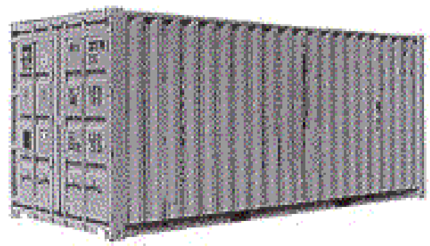
Date:____________________

Advisor: ____________________

Homework Worksheet: Surface Area and Volume of Rectangular Prisms

(WRITE ALL FORMULAS AND SHOW ALL WORK ON SEPARATE SHEETS OF PAPER)

1. Standard cargo shipping containers are of two types. Answer the following questions about each.



A. The most common container is 20 ft. long, 8 ft. wide, and 8 ft. high. Find the following:

Surface Area:

Volume:

B. The other standard container is also 8 ft. wide and 8 ft. wide and has a total volume of 2560 $ft.^3$ Find the following:

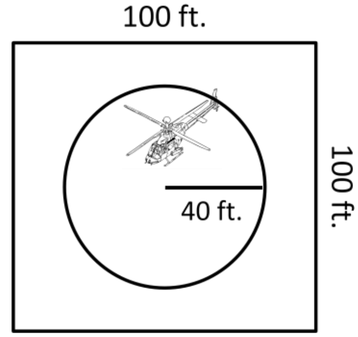
Length of the container:

Surface Area:

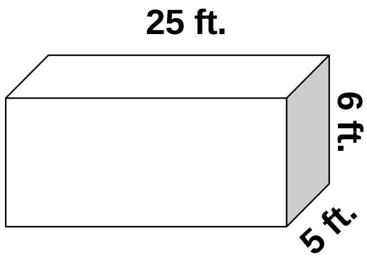
2. The U. S. Navy Hospital Ship *Comfort* (which is providing relief in Haiti) has a helicopter flight deck (on top of the ship) with dimensions of 100 feet long and 100 feet wide. The helicopters land on a circular landing pad with radius of 40 feet.

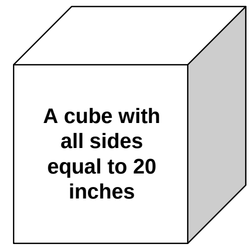
A. What is the area of the circular landing pad?

B. What is the area of the ***remainder*** of the flight deck?



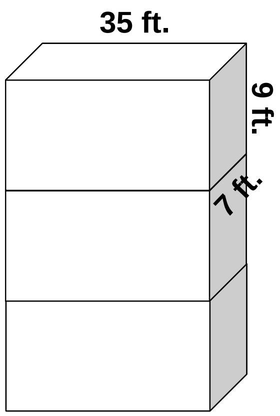
Find the surface area and volume of the following shapes:



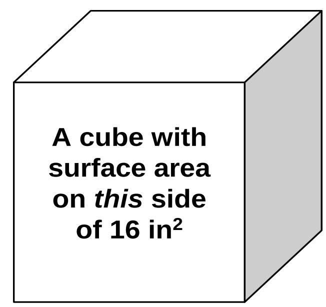


3.a Surface Area:
3.b. Volume:

4.a Surface Area:
4.b Volume:



Each box is 35 x 7 x 9 ft.



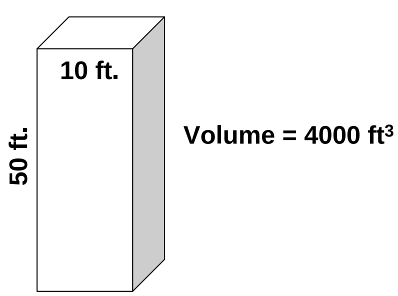
5.a Surface Area:

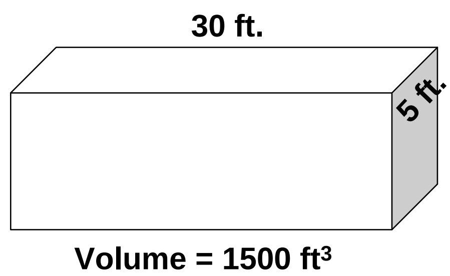
5.b. Volume:

6.a Surface Area:

6.b Volume:

B. Given the volume of the shape and two of the dimensions, find the third dimension.





6.a.

6.b.

Grade / Content Area	8th / 9th Grade Geometry
Lesson Title	**Building and Filling a Scale Model of a Container (6 days)**
Guiding Question	***What is a scale model and how to I build one?*** ***Why is it built to these dimensions?***
Content Standards	*State Content Standards:* I. **M(G&M)–8–5:** Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local) II. **M(G&M)-10-6:** Solves problems involving perimeter, circumference, area, surface area, and volume. III. **M(G&M)-10-7:** Uses units of measure appropriately and consistently when solving problems across content strands; makes conversions within or across systems and makes decisions concerning an appropriate degree of accuracy in problem situations involving measurement in other GSE's.
	NCTM Standards: Students should: I. **Analyze characteristics:** precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship. II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: High school students will: I. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
Student Learning Objectives	*Goals:* Students will be able to: I. Accurately design a scale model of a standard 20 ft. CEU shipping

<table>
<tr>
<td></td>
<td>container.

II. Build a scale model to design dimensions.

III. Determine the surface area and volume of the scale model and discover the relationship between the model's dimensions and the actual container's dimensions.</td>
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<tr>
<td>Preparation</td>
<td>I. Classroom Organization: Students will work in groups of three or four the first day and groups of two the second day. Desks will be rearranged to permit students to work together with a common work surface.

II. Differentiation. There are many things in this lesson that will appeal to the multiple intelligences of the students: new terminology, worksheets, and lots of hands-on exercises. Further working in groups promotes inclusion provided students are organized such that they can leverage off each others' strengths and minimize their individual weaknesses. I will be particularly mindful when grouping students of where I place my English language learners, those with reading comprehension challenges, and those who are already having difficulty with the course material. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing.

III. Materials (for each group of students):

A. Day 1 – 5:

1. Posterboard.
2. Material to strengthen sides of box (plastic or heavy paper).
3. Glue / glue guns.
4. Tape.
5. Washable paint (many basic colors).
6. Brushes.
7. Cleaning buckets (two).
8. Cubic inch blocks to fill the box.
9. Rulers.
10. Calculators.

B. Day 6 – 7:

1. NCTM Illuminations handout "Popcorn Prisms Anyone?" (http://illuminations.nctm.org/Lessons/Popcorn/Popcorn-AS-Prisms.pdf)
2. Two sheets of 8 ½ x 11 inch construction paper per student.
3. Tape.</td>
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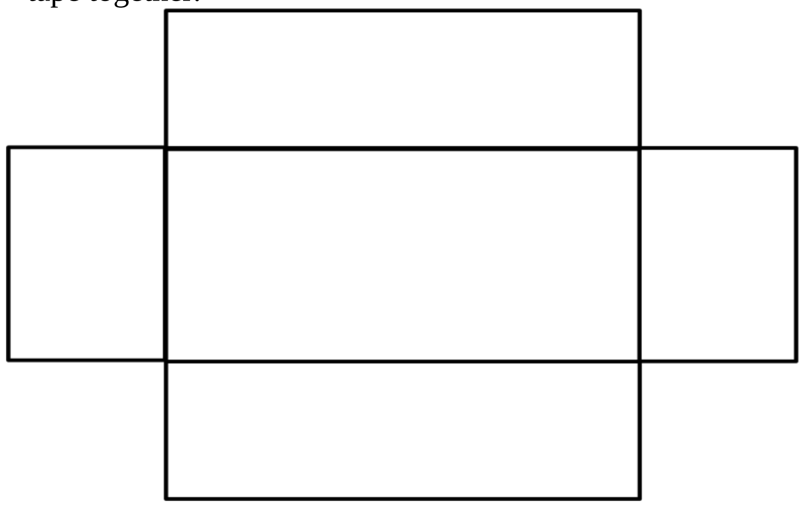
</table>

	4. Lots of popcorn.
Instruction and Engagement	I. *Launch.* **Day 1** (**15 minutes**): A. We will begin with a review of concepts learned previously, specifically the following definitions (these will be pre-posted so students can copy them in their vocabulary notes. 1. *Surface area:* The sum of all the areas of all the shapes that cover the surface of the object. 2. *Volume:* The amount of "space" a three-dimensional shape occupies. “It is he room in the box” (we will generalize this definition in this lesson). 3. *Scale Model:* A representation or copy of an object that is usually smaller than the actual size of the object, which seeks to maintain the relative proportions (the scale factor) of the physical size of the original object. Very often the scale model is used as a guide to making the object in full size. II. *Engagement.* **Day 1 – 5:** A. Students working in groups will design and construct a durable scale model of a container box. 1. The dimensions of the real container are: $20' \times 8' \times 8'$ $12(20)'' \times 12(8)'' \times 12(8)''$ $240'' \times 96'' \times 96''$ 2. The model should be precisely scaled. 3. The model should have a removable top. 4. The model should have hinged doors on one side. 5. The model should be large enough to place some objects inside. 6. The model should be painted, illustrating as closely as possible an actual shipping container (show some pictures to students and suggest they look online). B. Construction involves several steps for which students may need prompting or help: 1. Choose the length of one dimension of the model in inches – for example: 20". 2. Discover the lengths of the other dimensions using the proportional

relationship:

$$\frac{20"}{240"} = \frac{x}{96"} \Rightarrow \frac{20" \times 96"}{240"} = 8"$$

3. Draw an outline of the box on a piece of poster board. Cut out and tape together.



C. Decoration: Students can research online the appearance of container boxes. I will provide some examples:

III. **Launch (Day 6):**

A. We will begin with the question, *“Why is a container box built to the dimensions of* $20 \times 8 \times 8$*?”*

B. Answer is that the dimensions are an ideal balance between the need to fit as much into a box as possible (reducing cost – recall last lesson) and need to stack them efficiently on a ship.

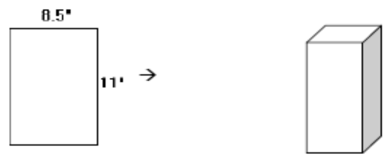
IV. **Engagement (Day 6):** We will explore the relationship between dimension and volume, arriving at some rather surprising conclusions. This exercise was developed by Jamie Chaikin and can be found on the NCTM *Illuminations* website at http://illuminations.nctm.org/Lessons/Popcorn/Popcorn-AS-Prisms.pdf.

Answer Key – Popcorn Prisms Anyone?

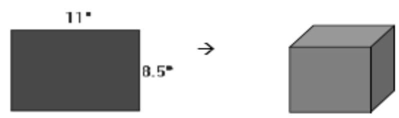
For this activity you will be comparing the volume of 2 prisms created using the same sheet of paper. You will be determining which can hold more popcorn. To do this, you will have to find a pattern for the dimensions for containers.

Materials:
- 8.5 inch by 11 inch white paper
- 8.5 inch by 11 inch colored paper
- Tape
- Popcorn
- Plate
- Cup
- Ruler

Take the white paper and fold it in half the long way. Do this a 2nd time. You are forming a baseless rectangular prism that is tall and narrow. Do not overlap the sides. Tape along the edge. Measure the length, width, and height of each dimension with a ruler. Record your data below and on the rectangular prism. Label it Prism A.



Take the colored paper and fold it in half the wide way. Do this a 2nd time. You are forming a baseless rectangular prism that is short and stout. Do not overlap the sides. Tape along the edge. Measure the length, width, and height of each dimension with a ruler. Record your data below and on the rectangular prism. Label it Prism B.



1.

Dimension	Prism A	Prism B
Length (in.)	[2.125 in]	[2.75 in]
Width (in.)	[2.125 in]	[2.75 in]
Height (in.)	[11 in]	[8.5 in]

2. Do you think the two prisms will hold the same amount? Do you think one will hold more than the other? Which one? Why?

Answers will vary.

3. Place Prism B on the paper plate with Prism A inside it. Use your cup to pour popcorn into Prism A until it is full. Carefully, lift Prism A so that the popcorn falls into Prism B. Describe what happened. Is Prism B full, not full, or overflowing?

Prism B is not full. There is still room in the prism for more popcorn.

As you share your popcorn snack, answer the questions below.

4. a) Was your prediction correct? How do you know?

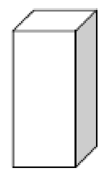
Answers will vary.

b) If your prediction was incorrect, describe what actually happened.

Prism B has a greater volume than Prism A.

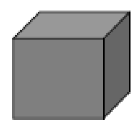
5. a) State the formula for finding the volume of a prism.

b) Calculate the volume of Prism A? Label the dimensions in the figure.



$V = lwh = (2.125)(2.125)(11) \approx 49.7 \text{ in}^3$

c) Calculate the volume of Prism B? Label the dimensions in the figure.



$V = lwh = (2.75)(2.75)(8.5) \approx 64.3 \text{ in}^3$

d) Explain why the prisms do or do not hold the same amount. Use the formula for the volume of a prism to guide your explanation.

The prisms have different dimensions, so the volumes are different.

6. a) What do you notice about the length and the width?

They are the same.

b) Rewrite the formula with only two variables to reflect this observation.

$V = l^2 h$ or $V = w^2 h$

7. By how much would you have to decrease the height of Prism B to make the volumes of the two prisms equal?

$$V_A \approx 49.7 \text{ in}^3$$
$$V_B \approx 49.7 \text{ in}^3 = (2.75)(2.75)(h)$$
$$h \approx 6.6 \text{ in}$$
The height would need to be decreased by about $8.5 - 6.6 \approx 1.9$ in.

A. Together we will construct the rectangular prisms. I will then let the students begin work on the questions as I walk around the room helping as necessary.

B. After students answer Question 2, I will pass out cooked popcorn (ideally prepared in the classroom or, at least, just moments before) to be used for the rest of the exercise. There should be enough to enjoy a

snack as well as to conduct the exercise. Classroom management will be a bit of a challenge. Ideally, I should not do this on a Friday or just before a holiday.

C. Some students are likely to be surprised that the two prisms do not hold the same amount, given they were constructed from identical sheets of construction paper. As they complete the exercise and consider the volume formula as it applies to either prism:

length *l* * width *w** height *h* = Volume *V*

So, the taller prism is:

$$\left(\frac{8.5\ in.}{4}\right)^2 * 11\ in. = 49.6719\ cu.in.$$

The, shorter, wider prism is:

$$\left(\frac{11\ in.}{4}\right)^2 * 8.5\ in. = 64.28143\ cu.in.$$

WHY DOES THIS HAPPEN?

D. Algebraically, it can be seen that the value $\frac{11}{4}$ is greater than $\frac{8.5}{4}$. When they are both squared, the $\left(\frac{11}{4}\right)^2$ is 1.67 times greater than $\left(\frac{8.54}{4}\right)^2$ whereas 11 is only 1.29 times greater than 8.5. Therefore, the magnitude of the length and width together has greater impact than the magnitude of the height.

V. *Closing.* **Day 6:**

A. Upon completion of the exercise and in closing, I will ask the question, *"So what kind of box would you want your furniture to go into?"* Probably one more like the shorter, broader prism because it can carry more and probably fit bigger things.

B. I will then ask them to think about our shipping container because it is into that which we will need to fit our shipping crates. I will leave them with the following question, *"Remember, you want to get as much into that container as possible. What are our limitations on the dimensions of our shipping crates?"*

Assessment	I. The container building project will be graded using the following rubric:

	4	3	2	1
Accuracy of Scale		Model is scaled such that dimensions are proportional to those of a $20' \times 8' \times 8'$ container box.		Model is not accurately scaled.
Working Parts		Model has durable hinged doors and removable top.	Model has durable hinged doors or removable top.	Model has no working parts.
Decoration	Model is decorated in great detail – realistically and thoroughly depicting an actual container box.	Model is decorated in sufficient detail to make it an acceptable depiction of an accurate container box but is lacking in some details.	Model is decorated neatly but not in sufficient detail to visually represent an accurate container box.	Model is not decorated.

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:________________________________
Date:_________________________________
Advisor: ______________________________

Container Scale Model Project

Project:

You will construct a model of a shipping container. These are pictures of standard shipping containers.


4 Locking Bars
Waterproof Gasket Seals Around Doors
5 Vents Per Side
Double Doors For Easy Storage Access
CBCU 200061 5
22G1
M.G.W.
TARE
NET
Lockbox For Lock Protection
Corten Steel Panels
CU.CAP.
Waist High Locking Handles
Fork Lift Pockets

Directions:

1. The dimensions of the real container are:

$$20' \times 8' \times 8'$$

2. The model should be precisely scaled.
3. The model should have a removable top.
4. The model should have hinged doors on one of the smaller sides.
5. The model should be large enough to place some objects inside.
6. The model should be painted, illustrating as closely as possible an actual shipping container (you can find pictures online).

Grading: Your model will be graded using the following rubric:

	4	**3**	**2**	**1**
Accuracy of Scale		Model is scaled such that dimensions are proportional to those of a $20' \times 8' \times 8'$ container box.		Model is not accurately scaled.
Working Parts		Model has durable hinged doors and removable top.	Model has durable hinged doors or removable top.	Model has no working parts.
Decoration	Model is decorated in great detail – realistically and thoroughly depicting an actual container box.	Model is decorated in sufficient detail to make it an acceptable depiction of an accurate container box but is lacking in some details.	Model is decorated neatly but not in sufficient detail to visually represent an accurate container box.	Model is not decorated.

Project Proposal

1. My container model will have the following dimensions (show your work below the table):

	Actual Container Dimensions in Feet	Actual Container Dimensions in Inches	Scale Factor	Model Container Dimensions in Inches
Length	20 feet			
Width	8 feet			
Height	8 feet			

2. Attached are pictures of my model on separate sheets of paper. The lengths, widths, and heights of the drawings should match the dimensions I have selected for my model. These pictures include

   a. A picture of the top of the container.
   b. A picture of one of the longer sides of the container.
   c. A picture of one of the shorter sides of the container.
   d. Each drawing is illustrated as I will decorate my model.

Submitted:

____

Student Signature

Project Proposal Approved:

____

Teacher Signature

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________________________
Date:_____________________________________
Advisor: __________________________________

Directions:

1. Find the dimensions and volume of each 3-D shape:
2. Using a **drawing** of your container box, determine how many of each shape you will place in the container box to **fill it completely at maximum profit to you**.
3. Fill your container box with the 3-D shapes to determine if your fill plan is correct.

Green Cube: Length: _____ Width: _____ Height: _____

Volume: ____________________

Price: $0.25

Yellow Rectangle: Length: _____ Width: _____ Height: _____

Volume: ____________________

Price: $0.80

Yellow Square: Length: _____ Width: _____ Height: _____

Volume: ____________________

Price: $3.25

Orange Rectangle: Length: _____ Width: _____ Height: _____

Volume: ____________________

Price: $0.85

Orange Square: Length: _____ Width: _____ Height: _____

Volume: ____________________

Price: $3.40

Light Orange Rectangle:	Length: _____	Width: _____	Height: _____
	Volume: __________________		
	Price: $3.50		
Yellow Prism:	Length: _____	Width: _____	Height: _____
	Volume: __________________		
	Price: $13.50		
Orange Prism:	Length: _____	Width: _____	Height: _____
	Volume: __________________		
	Price: $14.00		
Light Orange Prism	Length: _____	Width: _____	Height: _____
	Volume: __________________		
	Price: $14.50		
Large Wood Prism	Length: _____	Width: _____	Height: _____
	Volume: __________________		
	Price: $16.00		

Shape	**Total Number**	**Total Volume**	**Total Price**
Your Container Box			
Green Cube			
Yellow Rectangle			
Yellow Square			
Orange Rectangle			
Orange Square			
Light Orange Rectangle			
Yellow Prism			
Orange Prism			
Light Orange Prism			
Large Wood Prism			
Total			

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

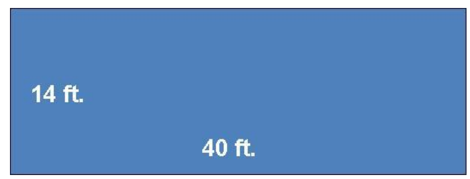
Name:______________________________
Date:______________________________
Advisor: ____________________________

Problem Solving Strategy: *Draw a Picture*[11]

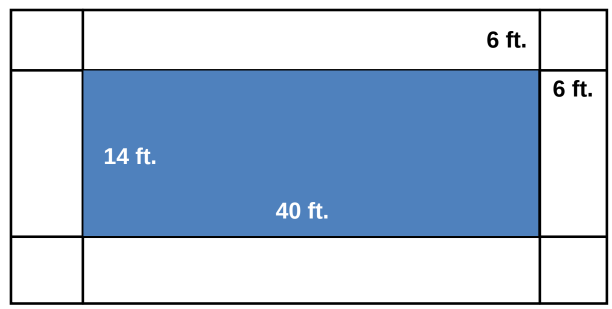
Curly dug his own swimming pool by hand with a shovel. He figured he needed a pool because digging it was hard work, and he could use it to cool off after working on it all day. He also planned a rectangular concrete deck around the pool that would be six feet wide at all points. The pool is rectangular and measures 14 feet by 40 feet. What is the area of the deck?

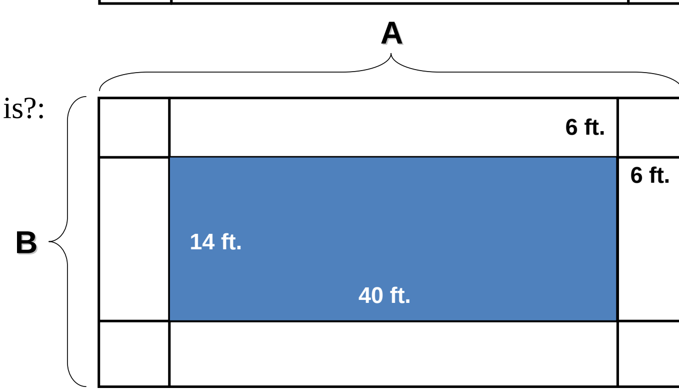
1. Draw a picture. What do we know?

   a. The pool is 14 feet by 40 feet:



   b. The deck is 6 feet wide around the edges:





2. So what do you think the length of *A* is? *B* is?:

   A = 40 + 6+ 6 = 52 feet

   B = 14 + 6 + 6 = 26 feet

3. What is the area of the pool? *Area of pool = 40 * 14 = 560 sq. ft.*

4. What is the area of the pool and the deck? *Area = 52 * 26 = 1352 sq. ft.*

5. What is the area of the deck? *Area of deck = Area of pool & deck – Area of pool.*
1352 – 560 = 792 sq. ft.

[11] From Herr and Johnson, *Problem Solving Strategies: Crossing the River With Dogs*, pp. 16 – 18.

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:___________________________________
Date:___________________________________
Advisor: ________________________________

Problem Solving Strategy: *Draw a Picture*[12]

Curly dug his own swimming pool by hand with a shovel. He figured he needed a pool because digging it was hard work, and he could use it to cool off after working on it all day. He also planned a rectangular concrete deck around the pool that would be six feet wide at all points. The pool is rectangular and measures 14 feet by 40 feet. What is the area of the deck?

1. Draw a picture. What do we know?

   a. The pool is 14 feet by 40 feet:

   b. The deck is 6 feet wide around the edges:

2. So what do you think the length of *A* is? *B* is?:

3. What is the area of the pool?

4. What is the area of the pool and the deck?

5. What is the area of the deck?

---

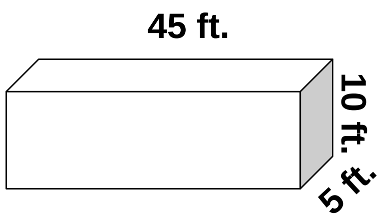
[12] From Herr and Johnson, *Problem Solving Strategies: Crossing the River With Dogs*, pp. 16 – 18.

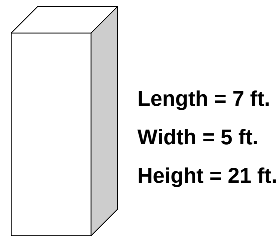
1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

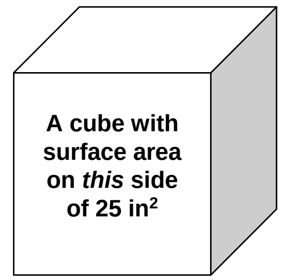
Name:____________________
Date:______________________
Advisor: __________________

Assessment Review: Surface Area and Volume of Rectangular Prisms

1. Find the surface area and volume (if asked for) of the following shapes:

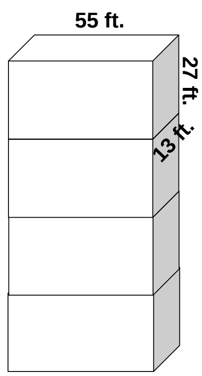


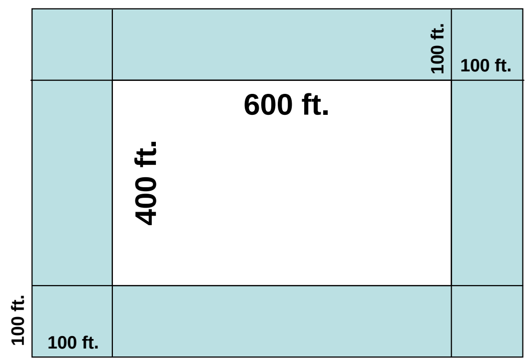


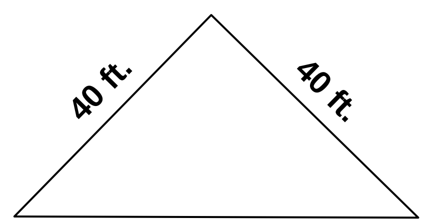


1.a Surface Area: _______ 1.c Surface Area: _______ 1.e Surface Area: _______

1.b Volume: _______ 1.d Volume: _______ 1.f Volume: _______







1.g Surface Area: _______ 1.i Entire Surface Area _______ 1.k Surface Area: _______

1.h Volume: _______ 1.j Surface Area of Shaded Area _______

2. John wants to frame a photograph of his shipmates on the cargo ship SS *Cape Mohican*. The photograph measures 10 inches long by 8 inches wide. He wants to put a frame around the picture that extends 2 inches from each side of the photograph.

   a. Draw a picture of the photograph and the frame showing all the measurements you have been given.

   b. How much wall space will the picture and the frame take up (what is the total surface area of the picture and frame)?

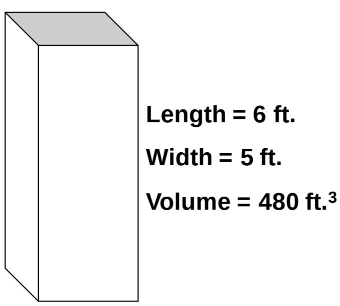
   ____

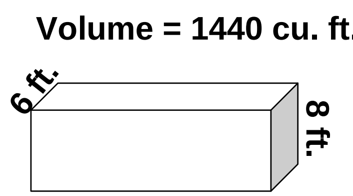
   c. What is the surface area of the frame alone? ___________________________

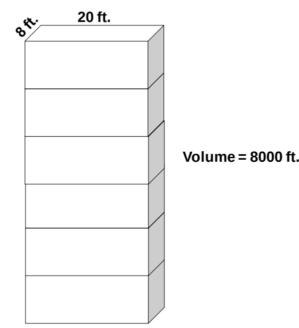
3. **Definitions**. Match the term by placing its letter next to its definition on the right:

a. Perimeter:	________	The amount of space, inside and out, a solid body occupies.
b. Surface Area:	________	A quadrilateral with opposite sides of equal lengths and with four right angles.
c. Rectangle:	________	A polygon with four sides and four corners.
d. Volume:	________	The sum of the areas of the surfaces of a three-dimensional object.
e. Square:	________	The sum of the lengths of the sides of a polygon.
f. Quadrilateral:	________	A quadrilateral with four equal sides and four right

4. Given two dimensions and the volume of the figure, find the third dimension.







4.a Height: _______ 4.b Length: _______ 4.c Height: _______

5. The cargo bay of the Space Shuttle *Discovery* is a rectangular prism that measures 60 ft. x 15 ft. x 30 ft. *Discovery's* mission is to carry as many cargo containers for the International Space Station in its cargo bay as possible on each flight. Each cargo container measures 30 ft. x 20 ft. x 10 ft.

   a. What is the surface area of *Discovery's* cargo bay? ____________________

   b. What is the volume of *Discovery's* cargo bay? ____________________

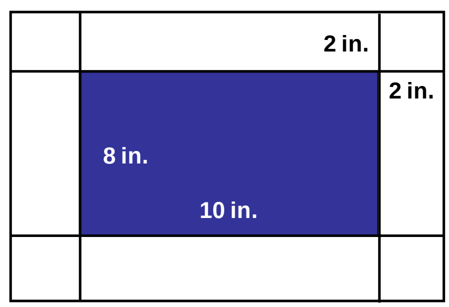
   c. What is the surface area of one cargo container? ____________________

   d. What is the volume of one cargo container? ____________________

   e. ***EXTRA CREDIT*** How many cargo containers can *Discovery* carry? ____________________

Review: Answer Key

1.a.	1450 ft.^{2}	2.a.	
1.b.	2250 ft.^{3}	2.b.	168 in^2
1.c.	574 ft.^{2}	2.c.	88 in^2
1.d.	735 ft.^{3}	3.a.	The sum of the lengths of the sides of a polygon.
1.e.	150 ft.^{2}	3.b.	The sum of the areas of the surfaces of a three-dimensional object.
1.f.	125 ft.^{2}	3.c.	A quadrilateral with opposite sides of equal lengths and four right angles.
1.g.	16,118 ft.^{2}	3.d.	The amount of space, inside and out, a solid body occupies.
1.h.	77,220 ft.^{3}	3.e.	A quadrilateral with four equal sides and four right angles.
1.i.	480,000 ft.^{2}	3.f.	A polygon with four sides and four corners.
1.j.	240,000 ft.^{2}	4.a.	16 ft.
1.k.	800 ft.^{2}	4.b.	30 ft.
		4.c.	50 ft.



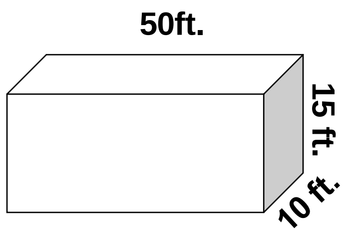
5.a.	6300 ft.^{2}
5.b.	27,000 ft.^{3}
5.c.	2200 ft.^{2}
5.d.	6000 ft.^{3}
5.e.	3

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________
Date:______________________
Advisor: ___________________

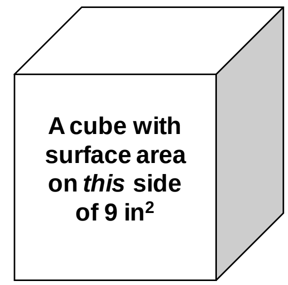
Assessment: Surface Area and Volume of Rectangular Prisms
SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. Find the surface area and volume (if asked for) of the following shapes:



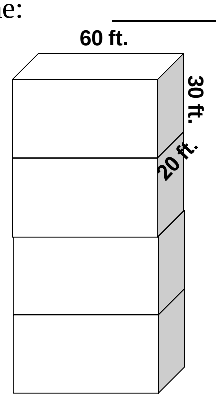
1.a Surface Area: _______

1.b Volume: _______



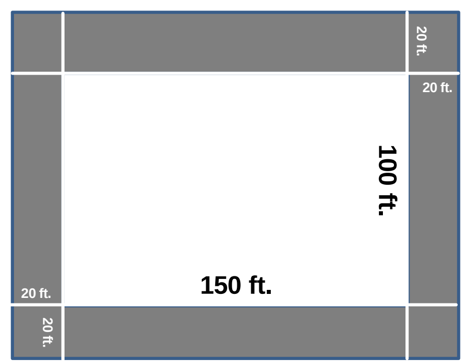
1.c Surface Area: _______

1.d Volume: _______



1.e Surface Area: _______

1.f Volume: _______



1.g Entire Surface Area _______

1.h Surface Area of Shaded Area _______

2. **Definitions**. Match the term by placing its letter next to its definition on the right:

a. Quadrilateral: ________ A quadrilateral with four equal sides and four right angles.

b. Rectangle: ________ A polygon with four sides and four corners.

c. Square: ________ The sum of the lengths of the sides of a polygon.

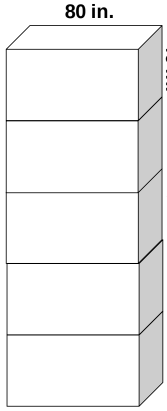
d. Perimeter: ________ The amount of space, inside and out, a solid body occupies.

e. Surface Area: ________ A quadrilateral with opposite sides of equal lengths and with four right angles.

f. Volume: ________ The sum of the areas of the surfaces of a three-dimensional object.

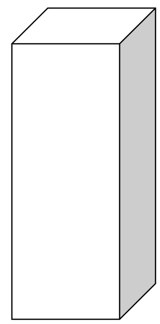
3. Given two dimensions and the volume of the figure, find the third dimension.

80 in.



40 in.

Volume = 96,000 in^3



Length = 15 in.

Width = 12 in.

Volume = 10,800 in^3

3.a Width: _______ 3.b Height: _______

4. One of the world's largest cargo aircraft is the U. S. Air Force C-5 Galaxy. Its cargo bay is a rectangular prism that measures 120 ft. x 19 ft. x 13 ft. The C-5's mission is to lift people and cargo by air into war zones and into areas that need disaster relief. Among other things, the C-5 can carry cargo containers measuring 20 ft. x 8 ft. x 8 ft.

a. What is the surface area of C-5 Galaxy's cargo bay? ___________________

b. What is the volume of C-5's Galaxy's cargo bay? ___________________

c. What is the surface area of one cargo container? ___________________

d. What is the volume of one cargo container? ___________________

How many cargo containers can the C-5 Galaxy carry? ___________________

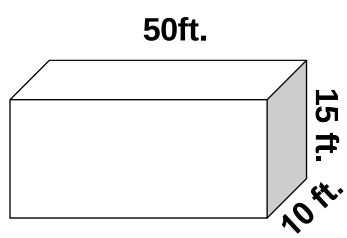
1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

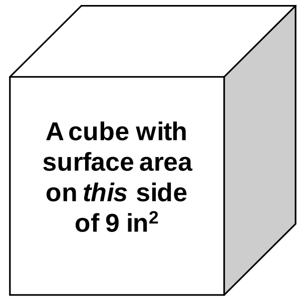
Differentiated Exam

Name:____________________
Date:_____________________
Advisor: __________________

Assessment: Surface Area and Volume of Rectangular Prisms
SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. Find the surface area and volume (if asked for) of the following shapes **(5 points apiece)**:



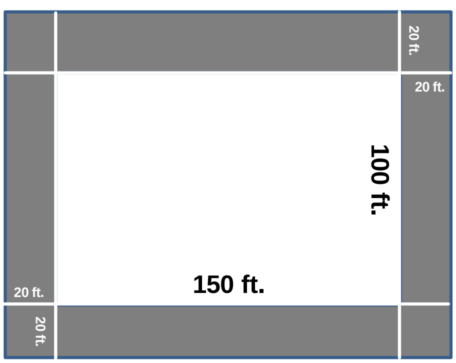


1.a Surface Area: ________

1.b Volume: ________

1.c Surface Area: ________

1.d Volume: ________



1.e Entire Surface Area ________

1.f Surface Area of Shaded Area ________

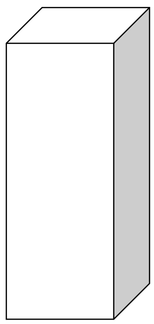
2. **Definitions**. Match the term by placing its letter next to its definition on the right:

a. Quadrilateral:	________	A quadrilateral with four equal sides and four right angles.
b. Rectangle:	________	A polygon with four sides and four corners.
c. Square:	________	The sum of the lengths of the sides of a polygon.
d. Perimeter:	________	The amount of space, inside and out, a solid body occupies.

e. Surface Area: ________ A quadrilateral with opposite sides of equal lengths and with four right angles.

f. Volume: ________ The sum of the areas of the surfaces of a three-dimensional object.

3. Given two dimensions and the volume of the figure, find the third dimension.



Length = 15 in.

Width = 12 in.

Volume = 10,800 in^3

3.a Height: ________

4. One of the world's largest cargo aircraft is the U. S. Air Force C-5 Galaxy. Its cargo bay is a rectangular prism that measures 120 ft. x 19 ft. x 13 ft. The C-5's mission is to lift people and cargo by air into war zones and into areas that need disaster relief. Among other things, the C-5 can carry cargo containers measuring 20 ft. x 8 ft. x 8 ft.

a. What is the surface area of C-5 Galaxy's cargo bay? ____________________

b. What is the volume of C-5's Galaxy's cargo bay? ____________________

c. What is the surface area of one cargo container? ____________________

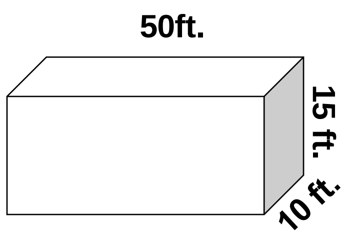
d. What is the volume of one cargo container? ____________________

e. ***EXTRA CREDIT*** How many cargo containers can The C-5 Galaxy carry? ____________________

Answer Key

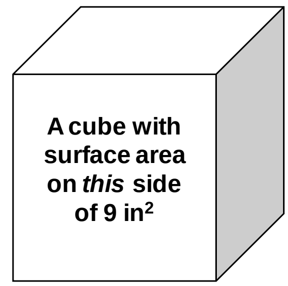
Assessment: Surface Area and Volume of Rectangular Prisms
SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. Find the surface area and volume (if asked for) of the following shapes:



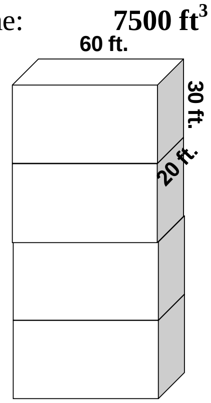
1.a Surface Area: **2800 ft^2**

1.b Volume: **7500 ft^3**



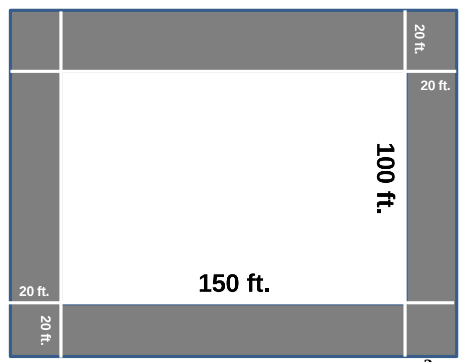
1.c Surface Area: **54 in^2**

1.d Volume: **27 in^3**



1.e Surface Area: **21,600 ft^2**

1.f Volume: **144,000 ft^3**



1.g Entire Surface Area **26,600 ft^2**

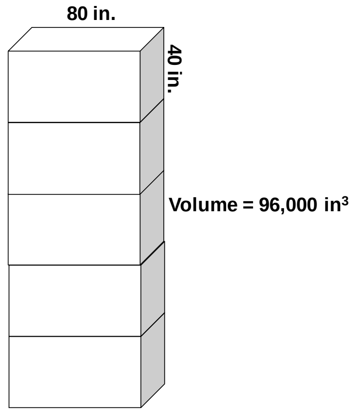
1.h Surface Area of Shaded Area **11,600 ft^2**

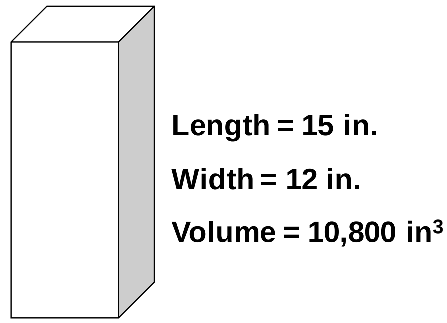
2. **Definitions**. Match the term by placing its letter next to its definition on the right:

a. Quadrilateral:	**c**	A quadrilateral with four equal sides and four right angles.
b. Rectangle:	**a**	A polygon with four sides and four corners.
c. Square:	**d**	The sum of the lengths of the sides of a polygon.
d. Perimeter:	**f**	The amount of space, inside and out, a solid body occupies.
e. Surface Area:	**b**	A quadrilateral with opposite sides of equal lengths and with four right angles.
f. Volume:	**e**	The sum of the areas of the surfaces of a three-dimensional object.

Answer Key

3. Given two dimensions and the volume of the figure, find the third dimension.





3.a Width: **6 in.**

3.b Height: **60 in.**

4. One of the world's largest cargo aircraft is the U. S. Air Force C-5 Galaxy. Its cargo bay is a rectangular prism that measures 120 ft. x 19 ft. x 13 ft. The C-5's mission is to lift people and cargo by air into war zones and into areas that need disaster relief. Among other things, the C-5 can carry cargo containers measuring 20 ft. x 8 ft. x 8 ft.

   a. What is the surface area of C-5 Galaxy's cargo bay? **8174 ft^2**

   b. What is the volume of C-5's Galaxy's cargo bay? **29,640 ft^3**

   c. What is the surface area of one cargo container? **768 ft^2**

   d. What is the volume of one cargo container? **1280 ft^3**

   e. ***EXTRA CREDIT*** How many cargo containers can The C-5 Galaxy carry? **12**

Grade / Content Area	8th / 9th Grade Geometry
Lesson Title	**Scaling (2 days)**
Guiding Question	*If we change one, two, or all three dimensions of an object, how does its surface area and volume change?*
Content Standards	*State Content Standards:* I. **M(G&M)–8–5:** Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local) II. **M(G&M)–8–6:** Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, cylinders, pyramids, or cones. Expresses all measures using appropriate units. (Local)
	NCTM Standards: Middle and high school students should: I. **Analyze characteristics:** precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship. II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: I. Verify experimentally the properties of dilations given by a center and a scale factor:

	A. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. B. The dilation of a line segment is longer or shorter in the ratio given by the scale factor."
Student Learning Objectives	I. Students will identify both empirically and algebraically the impact of changing the dimensions of rectangular prisms on surface area and volume
Preparation	I. *Classroom Organization.* Students will work together in groups of two or four. Desks will be rearranged to permit two or four students to work together with a common writing surface. II. *Differentiation.* Again, this lesson has much to appeal to multiple intelligences including the use of manipulatives as an aid to deriving and understanding mathematical results. As always, I will be mindful of students having difficulty with comprehension. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials* (for each group of students): A. **Day 1:** 1. Container models built in class or 16 – 20 “Kleenex” boxes. 2. Worksheet for warm-up and at home exercise. 3. Ruler, pencil, paper. B. **Day 2:** 1. Worksheet for warm-up exercise.
Instruction	I. *Warm-up* (**10 minutes**): II. *Launch* **Day 1:** (**10 minutes**). I will open with a warm-up exercise which will ask students to determine the surface area and volume of a rectangular prism and then ask them to determine the dimensions of the shape if we double its volume. I expect that some of the students will speculate that all three dimensions are doubled if the volume is doubled. Others will determine that only one dimension is doubled. This will set the stage for the lesson.

III. *Engagement* **Day 2:**

A. (10 minutes) Students will be divided into groups of two to four and asked to compare their answers in the warm-up and make a group prediction or hypothesis – writing their findings and sharing with the class. I will compile these hypotheses for the entire class.

B. (10 minutes) Students will then be given two "Kleenex" boxes and then asked to prove their prediction or explain why they now believe their prediction is wrong.

C. (10 minutes) If / when a group demonstrates that doubling only one dimension is necessary, they will be asked to demonstrate the impact of doubling two and then three dimensions. Groups will be given additional "Kleenex" boxes to experiment and evaluate.

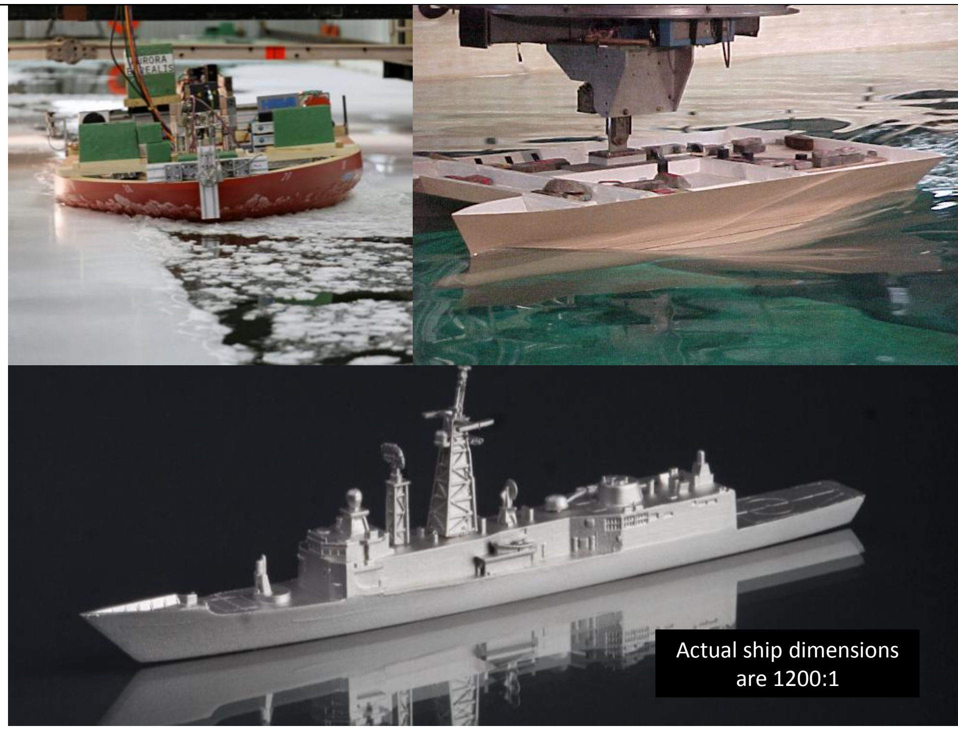
IV. *Closing* **Day 1** (**10 minutes**)**:** Now I will ask the question, *"OK, we have determined that for a rectangular prism, we need to double one dimension to double the volume, double, double two dimensions to increase the volume by* 2^2*, and double all three to increase the volume by* 2^3*.*

Double:	Volume increases by:	Or by:
One dimension	Factor of two	2^1
Two dimensions	Factor of four	2^2
Three dimensions	Factor of eight	2^3

Note how the exponent equals the number of dimensions we have doubled. Now what happens if we triple the dimensions.

V. *Warm-up* **Day 2** (**10 minutes**):

VI. *Launch* **Day 2** (**10 minutes**)**:** I will start by asking students to consider the ships we discussed earlier. "*How do you think they are tested to ensure they can operate safely at sea – in storms or icy waters, etc.?"* I will then show some pictures of scale models of ships:



"To build the actual ship from the model, one multiplies all the dimensions by the scale factor. For my ship, U.S.S. Rentz, *the builders would take this model and multiply all of the dimensions by 1200.*

VII. *Engagement* **Day 2** (**40 minutes**): We will now consider scaling and particularly its relationship to the volume and surface area of scaled objects. We will use a worksheet from the NCTM Illuminations lesson "Scaling Away" by Rhonda Naylor. I will ask the following questions:

A. Question: *What happens to the volume when an object is enlarged by a given scale factor? What happens to the surface area?*

   Answer: *Both the surface area and volume increase when an object is enlarged.*

B. Question: *What is the ratio of the surface area of the original object to the surface area of the model? What is the ratio of the volumes? How does this compare to the ratio of the side lengths?*

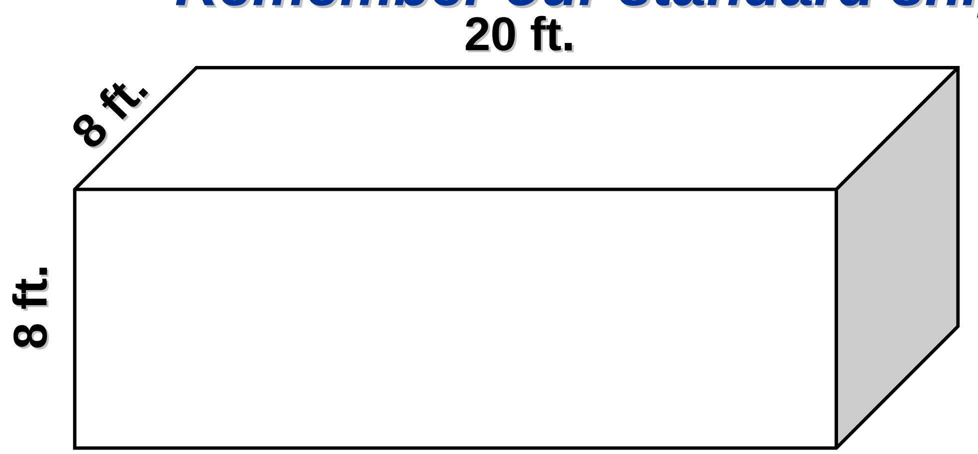
   Answer: *When an object is enlarged (or, for that matter, shrunk) by a scale factor n, the resulting surface area is n2 times the original surface area, and the resulting volume is n3 times the original volume. Consequently, the ratio of side lengths is 1:n, the ratio of surface areas*

	is 1:n^2, and the ratio of volume is 1:n^3. VIII. *Closing* **Day 2** (**10 minutes**): Algebraically: If the original object was a prism, it had dimensions l, w, and h. The dimensions of the model are equal to the dimensions of the object multiplied by the scale factor n, which are nl, nw, and nh. **<u>Original surface area:</u>** $SA = 2lw + 2lh + 2wh = 2B + Ph$ **<u>New surface area:</u>** $2(nl)(nw) + 2(nl)(nh) + 2(nw)(nh) = 2n^2lw + 2n^2lh + 2n^2wh$ $= \boldsymbol{n^2}\,(2lw + 2lh + 2wh)$ **<u>Original volume:</u>** lwh **<u>New volume:</u>** $(nl)(nw)(nh) = \boldsymbol{n^3}(lwh)$ Try it.
Assessment	I. In this lesson, I will assess: A. Readiness to learn material covered in each day's lesson through the warm-up exercise. B. Understanding of the concepts of this lesson through observation of group work, report out, and completion of the worksheets. C. Final comprehension through a graded assessment.

Warm-up

March 2nd, 2010

Remember our standard shipping container…



Volume: ____________

What are the new dimensions when you double the volume?

Doubled Volume: ____________

Length: ____________

Width: ____________

Height: ____________

Hint: There is more than one correct answer!

Prove your work using formulas here and a calculator.

Why do you think these are the *new* dimensions?

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:_____________________
Date:______________________
Advisor: ________________ __

Scaling of Rectangular Prisms I

1. Refer to your proposal for a model of a container box. Record the dimensions of your container box model.

Length	Width	Height	Surface Area	Volume

2. Now, consider two identical container boxes with the dimensions of your model. Draw a picture of the two boxes with one placed atop the other. If one were placed atop the other, determine the dimensions, surface area and volume.

Length	Width	Height	Surface Area	Volume

   a. Does the volume double when you put the two boxes together? **YES** **NO**

   b. How does each of the dimensions change when you put the two boxes together?

	Length	Width	Height	Volume
One container box				
Two container boxes				

   c. So to double the volume, how many dimensions change and by what percent? Explain.

3. What do you think will happen to the volume when all three dimensions are doubled? Explain your answer.

Warm-up

March 3rd, 2010

"overstock.com" sells 36-box cartons of "Kleenex" tissue. If each "Kleenex" box measures

4.75 in. x 4 in. x 9 in.

1. **What is the volume of each "Kleenex" box?**

   ____

2. **What is the total volume of the carton?**

   ____

3. **What might be the dimensions of the carton?**

   ____

4. **If "overstock.com" wants to sell 72-box cartons, what might be the dimensions of the new carton.**

   ____

Scaling Away

NAME ______________________

Select an object that is a rectangular prism or a cylinder. Record all of your work below.

Object: ______________________________

Shape: ______________________________

1. What do you think will happen to the volume when you enlarge a common object by your given scale factor? What do you think will happen to the surface area? Write your **hypothesis**.

If I multiply each dimension by the scale factor, then I think...

the surface area will ______________________________

and the volume will ______________________________

because ______________________________.

2. Measure and Record the dimensions of your object. Include the correct units.

Length ____________ Width ____________ Height ____________

– or –

Diameter ____________ Height ____________

3. Compute the surface area of your object. Include the correct units.

4. Compute the volume of your object. Include the correct units.

Imagine you are going to enlarge your object by building a scale model. To do this, you will multiply each dimension by a number known as the **scale factor**.

Choose a scale factor (from 3 to 8): ____________

5. Multiply each dimension by the scale factor, and record the new dimensions below.

   Length ______________ Width ________________ Height______________

   – or –

   Diameter ____________ Height _______________

6. Compute the surface area of the model. Include the correct units.

7. Compute the volume of model. Include the correct units.

8. Determine the ratio of the surface area of the original object to the surface area of the model.

9. Determine the ratio of the volume of the original object to the volume of the model.

10. Was your hypothesis correct? Why or why not? Explain what you have discovered about multiplying a side length by a scale factor. What happens to the surface area? What happens to the volume?

11. If you had used a scale factor of 8, by what factor would the surface area have increased? By what factor would the volume have increased?

Warm-up

March 4th, 2010

	Dimensions (inches)	Volume (in^3)	Number of doubled from dimensions of Box 1	Number of times volume increased over Box 1
Box 1	5 x 3 x 2	30 in^3		
Box 2	5 x 6 x 2			
Box 3	5 x 6 x 4			
Box 4	10 x 6 x 4			

If you double one dimension, the volume increases by a factor of _____________ or 2—.

If you double two dimensions, the volume increases by a factor of _____________ or 2—.

If you double three dimensions, the volume increases by a factor of _____________ or 2—.

Warm-up

March 3rd, 2010

"overstock.com" sells 36-box cartons of "Kleenex" tissue. If each "Kleenex" box measures

4.75 in. x 4 in. x 9 in.

1. **What is the volume of each "Kleenex" box?**

   ____

2. **What is the total volume of the carton?**

   ____

3. **What might be the dimensions of the carton?**

   ____

4. **If "overstock.com" wants to sell 72-box cartons, what might be the dimensions of the new carton.**

   ____

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name: ____________________
Date: ____________________
Advisor:____________________

Scaling of Rectangular Prisms II

1. For a rectangular prism, if you:

   double one dimension, the volume increases by a factor of ____________ or $2^{—}$,

   double two dimensions, the volume increases by a factor of ____________ or $2^{—}$.

   double three dimensions, the volume increases by a factor of ____________ or $2^{—}$.

2. Standard shipping containers come in various sizes. In the table below, fill in the missing information for the three types of containers:

Type	Length	Width	Height	Volume
Type 1	20 ft.	________	8 ft.	1280 ft.^{3}
Type 2	________	8 ft.	________	2560 ft.^{3}
Type 3	45 ft.	8 ft.	________	3456 ft.^{3}

3. Complete the following table:

	Dimensions (inches)	Volume (in^3)	Number of doubled dimensions from Box 1	Number of times volume increased over Box 1
Box 1	10 x 7 x 3	______		
Box 2	10 x 14 x 3	420 in^3	________	2
Box 3	________	______	2	________
Box 4	20 x 14 x 6	______	3	________

Be sure to check your answers!

4. When you scale all three dimensions of an object such as a box or a cylinder by a scaling factor, n, the volume of the object increases by a factor of:

____

5. “Kleenex” boxes are sold in packages of 10. If each “Kleenex” box has the following dimensions:

9 in. x 4.5 in. x 4 in.

a. What is the volume of each “Kleenex” box” ____________

b. What is the total volume of a package of 10 “Kleenex” boxes? ____________

c. List two possible sets of dimensions for the package? ____________

____

d. If the seller wants to sell 20-box packages, what is volume of the new 20-box package? ____________

e. If the seller wants to sell 20-box packages, list two possible sets of dimensions for the new 20-box package. ____________

____

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name: ____________________
Date: ____________________
Advisor: ____________________

Scaling of Rectangular Prisms II

1. For a rectangular prism, if you:

   double one dimension, the volume increases by a factor of 2 or 2^1.

   double two dimensions, the volume increases by a factor of 4 or 2^2.

   double three dimensions, the volume increases by a factor of 8 or 2^3

2. Standard shipping containers come in various sizes. In the table below, fill in the missing information for the three types of containers:

Type	Length	Width	Height	Volume
Type 1	20 ft.	8 ft.	8 ft.	1280 $ft.^3$
Type 2	40 ft.	8 ft.	8 ft.	2560 $ft.^3$
Type 3	45 ft.	8 ft.	9.6 ft.	3456 $ft.^3$

3. Complete the following table:

	Dimensions (inches)	Volume (in^3)	Number of doubled dimensions from Box 1	Number of times volume increased over Box 1
Box 1	10 x 7 x 3	210 in^3		
Box 2	10 x 14 x 3	420 in^3	1	2 or 2^1
Box 3	10 x 14 x 6 20 x 14 x 3	840 in^3	2	4 or 2^2
Box 4	20 x 14 x 6	1680 in^3	3	8 or 2^3

Be sure to check your answers!

4. When you scale all three dimensions of an object such as a box or a cylinder by a scaling factor, n, the volume of the object increases by a factor of:

$$\mathbf{n^3}$$

5. "Kleenex" boxes are sold in packages of 10. If each "Kleenex" box has the following dimensions:

9 in. x 4.5 in. x 4 in.

a. What is the volume of each "Kleenex" box" **162 in.^{3}**

b. What is the total volume of a package of 10 "Kleenex" boxes? **1620 in.^{3}**

c. List two possible sets of dimensions for the package? **9 in. x 8 in. x 22.5 in.**
9 in. x 4 in. x 45 in.
18 in. x 4 in. x 22.5 in.

d. If the seller wants to sell 20-box packages, what is volume of the new 20-box package? **3240 in.^{3}**

e. If the seller wants to sell 20-box packages, list two possible sets of dimensions for the new 20-box package. **18 in. x 8 in. x 22.5 in.**
9 in. x 8 in. x 45 in.
18 in. x 4 in. x 45 in.

Grade / Content Area	**8th / 9th Grade Geometry**
Lesson Title	**Fueling / Loading the Ship (1 day)**
Guiding Question	*How much liquid is needed to fill a three-dimensional shape?*
Content Standards	*State Content Standards:* I. **M(G&M)–8–5:** Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local) II. **M(G&M)–8–6:** Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, cylinders, pyramids, or cones. Expresses all measures using appropriate units. (Local)
	NCTM Standards: Middle and high school students should: I. **Analyze characteristics:** precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship. II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
Planning	I. *Classroom Organization.* Students will work together in groups of three or four. Desks will be rearranged to permit four students to work together with a common writing surface.

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II. Differentiation. For my students with special needs, and guided by my collaborating special needs teacher (if I have one) I will prepare a worksheet which incorporates hints and some answers filled in to provide these students with a starting point helpful to their learning. As always, I will be mindful of those students who have difficulty with comprehension. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing.

III. Materials (for each student):

A. Liquid load worksheet.

B. Calculator.
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<tr><td>Student Learning Objectives</td><td>
I. Students will determine the volume of fuel necessary to fill a tank.

II. Students will demonstrate ability to calculate the volume of a triangular prism.

III. Students will demonstrate ability to use metric measures including conversion from measures of volume to liquid measures.
</td></tr>
<tr><td>Instruction and Engagemen t</td><td>
I. Warm-up (10 minutes):

II. Launch (10 minutes): I will open with the following, “We’ve been talking about filling cargo containers with solid materials but much of what is loaded onto our ship is liquid; primarily fuel. The interior of a container ship looks like this picture:”


“Fuel is normally loaded into tanks just below the cargo hold (point to where the fuel tanks are on the “General Layout of a Ship” diagram). How can we
</td></tr>
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calculate how much fuel we need to fill the tanks?" At this point I will let students speculate on the answer. If necessary, I will prompt them with, *"Well, we can calculate the volume of the tanks, can't we? Does this give us our answer?"* If the answer is yes, I will prompt further thinking with the following question and answer discussion:

A. Question: *So, do you fill your car with cubic feet or cubic inches of gasoline?"*

   Answer: *No, we fill it with gallons of gasoline.*

B. Question: *So, is there a way to convert the volume of a space like a container or a fuel tank into liquid measures?* After they ponder this for a while, I will ask the final question.

C. Question: *What do you notice about the shape of some of these tanks?* Answer is that some are not prisms. In today's exploration, we will find how to calculate the surface area and volume of these shapes and how to convert the those measures into the amount of liquid we can put into the tanks.

D. **Vocabulary:** I will introduce the following:

   1. **Triangular Prism:** a prism whose bases are triangles.

      Surface area of a regular triangular prism:

$$SA = 2\left(\frac{1}{2}bh\right) + PH$$

      Volume of a regular rectangular prism:

$$V = (\frac{1}{2}bh) \times H$$

      - *b* is the length of one side of the base triangle.
      - *h* is the height of the base triangle.
      - *P* is the perimeter of the base.
      - *H* is the height of the prism

   2. **Liquid Conversions:**

$$1\ foot^3 = 7.48051948\ gallons$$

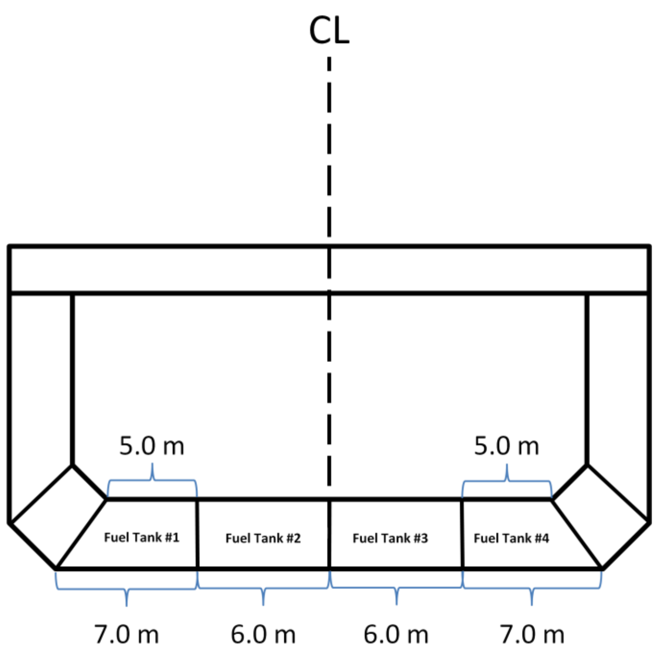
$$1\ meter^3 = 1000\ liters$$

	III. *Engagement* (**30 minutes**). Students will fill in the worksheet (see below). I will explain the diagram to them, relating it to the photograph of a container ship cross-section I showed them before. Then I will let them begin work. At first, the exercise may appear to be straightforward but it might prove more challenging when they try to find the volume of fuel tanks #1 and #4 because they are not rectangular prisms. If students are struggling, I will offer the following prompts, as necessary, group-by-group: A. Question: *Why don't you try drawing the side of the tank and separating it into two smaller shapes, what would they be?* Answer: *We can divide it into a triangle and a rectangle.* B. Question: *Can you find the area of these two shapes?* Answer: *We can find the area of the rectangle, it is 20 square meters.* C. Question: *What about the triangle? Does it look like something or half of something?* Answer: *Sure! It looks like one half of a rectangle. So if I multiply the lengths of the sides and divide by 2 I will get the area?* D. Question: *That's right, so do you know enough now to compute the volume of the tank?* Answer: *I can add the areas of the rectangle and triangle and multiply by the height.* IV. *Closing* (**10 minutes**): We will share our findings and share how we were able to find the volume of the tanks that our not rectangular prisms. I will then ask the following question, *"Which tanks should be loaded first and why?"* I would like them to visualize the problem of stability – how a weight added to the ship will heel it over more as the weight is moved from the center line of the ship. There is a geometrical relationship to be considered – beyond the scope of this lesson but worthy of some discussion, especially for the more inquiring minds.
Assessment	Completed worksheets will serve as informal assessments of student ability to compute volume, convert volume to liquid measure, and determine the volume of a non-rectangular prism.

MAERSK LINE

Container Ship Liquid Load Planning Sheet

Tank Number	Tank Length	Tank Width	Tank Height	Tank Volume	Fuel Necessary to Fill Tank
1					
2					
3					
4					



Height of all tanks is 4.0 m
Length of all tanks is 20.0 m
1 cu. m. = 1000 liters

MAERSK LINE

Container Ship Liquid Load Planning Sheet

Version 2

Tank Number	Tank Length	Tank Width	Tank Height	Tank Volume	Fuel Necessary to Fill Tank
1	20.0 m	5.0 m / 7.0 m	4.0 m		
2	20.0 m	6.0 m	4.0 m		
3	20.0 m	6.0 m	4.0 m		
4	20.0 m	5.0 m / 7.0 m	4.0 m		

Surface Area and Volume Formulas

1. Surface area of a rectangular prism:

$$SA = 2((l \times w) + (l \times h) + (w \times h))$$

2. Volume of a rectangular prism:

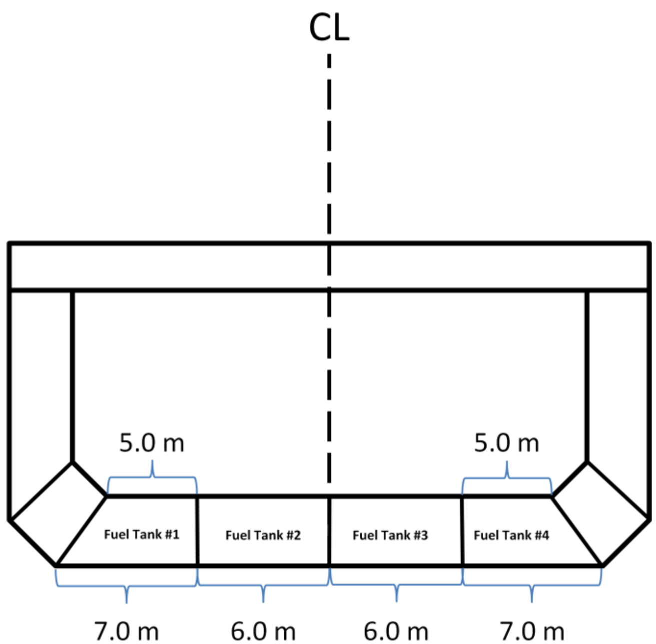
$$V = l \times w \times h$$

3. Surface area of a triangular prism:

$$SA = 2\left(\frac{1}{2}bh\right) + PH$$

4. Volume of a triangular prism:

$$V = \left(\frac{1}{2}bh\right) \times H$$

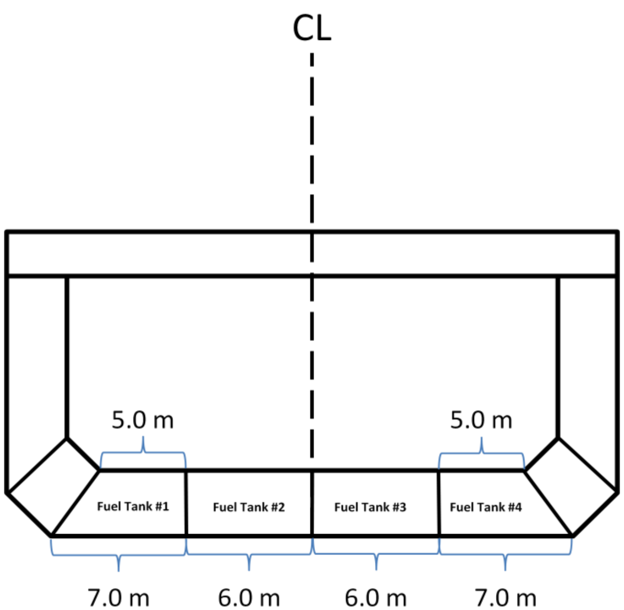


Height of all tanks is 4.0 m
Length of all tanks is 20.0 m
1 cu. m. = 1000 liters

Container Ship Liquid Load Planning Sheet

(Answer Key)

Tank Number	Tank Length	Tank Width	Tank Height	Tank Volume	Fuel Necessary to Fill Tank
1	20.0 m	5.0 m / 7.0 m	4.0 m	$(20 * 5 * 4) + (0.5 * 20 * 2 * 4) = 480\ cu.m.$	$Fuel\ needed = 480 * 1000 = 480{,}000\ litres$
2	20.0 m	6.0 m	4.0 m	$(20 * 7 * 4) = 560\ cu.m.$	$Fuel\ needed = 560 * 1000 = 560{,}000\ litres$
3	20.0 m	6.0 m	4.0 m	$(20 * 7 * 4) = 560\ cu.m.$	$Fuel\ needed = 560 * 1000 = 560{,}000\ litres$
4	20.0 m	5.0 m / 7.0 m	4.0 m	$(20 * 5 * 4) + (0.5 * 20 * 2 * 4) = 480\ cu.in.$	$Fuel\ needed = 480 * 1000 = 480{,}000\ litres$



Height of all tanks is 4.0 m

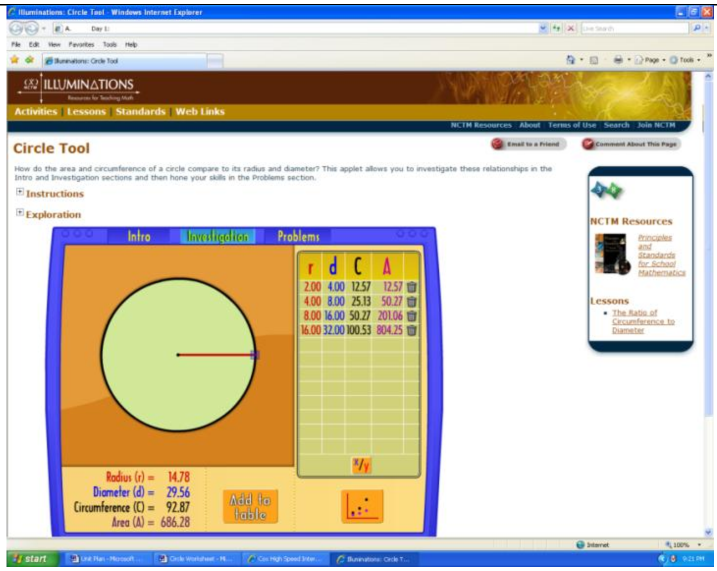
Length of all tanks is 20.0 m

1 cu. m. = 1000 liters

Grade / Content Area	# 8^{th} / 9^{th} Grade Geometry
Lesson Title	**Cylinders and the General Formula for Prisms (3 – 4 days)**
National and State Content Standards	*State Content Standards:* I. **M(G&M)–8–5:** Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local) II. **M(G&M)–8–6:** Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, cylinders, pyramids, or cones. Expresses all measures using appropriate units. (Local)
	NCTM Standards: In grades 6 – 8 all students should: I. **Analyze characteristics:** precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship. II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
Goals, Context of the Lesson, and Rationale	I. *Goals:* My goal in this lesson is to help students make the connection between physically filling a cylinder and the mathematical equation for the volume of a cylinder. II. *Context and Rationale:* This is the third lesson in this unit on Geometry, expanding our focus on surface area and volume to cylindrical objects. In the

	last lesson, we derived the equation for the volume of a rectangular prism. In a similar fashion and with similar objects, we will derive the equations for the volume of a cylindrical object. We will then explore how varying the dimensions of cylindrical objects produces some surprising results – just as we discovered with prisms.
Opportunities to Learn	I. *Classroom Organization*: Students will work in groups of two at computer workstations for the first activity and then in groups of three or four. Desks will be rearranged to permit students to work together with a common work surface. II. *Differentiation*. There are many things in this lesson that will appeal to the multiple intelligences of the students: new terminology, worksheets, and lots of hands-on exercises. Further working in groups promotes inclusion provided students are organized such that they can leverage off each others' strengths and minimize their individual weaknesses. I will be particularly mindful when grouping students of where I place my English language learners, those with reading comprehension challenges, and those who are already having difficulty with the course material. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials* (for each group of students)*:* A. Day 1: 1. A soup can. 2. One computer workstation for each group of two students. 3. *Illuminations* Circle Tool Applet: (http://illuminations.nctm.org/ActivityDetail.aspx?ID=116). 4. Circle worksheet. B. Day 2: 1. Cylindrical objects (each student will be asked to bring one). 2. Construction paper. 3. Rulers and pencils. C. Day 3: 1. 8.5 x 11 in. white paper 2. 8.5 x 11 in. colored paper.

	3. Tape. 4. Popcorn. 5. Plate. 6. Cup. 7. Ruler.
Student Learning Objectives	I. Students derive the formula for surface area of a circle and, from that, derive the formula for the volume of a cylindrical shape. II. Students compare the volumes of cylindrical shapes by filling them with liquids / solids as well as algebraically to understand the relationship between a cylinder's dimensions and its volume.
Timing	I. Circle Tool Applet Lesson: **45 minutes**. II. In-class group practice on worksheet: **30 minutes**. III. Measuring cylinders: **45 minutes.** IV. Popcorn cylinders: **45 minutes.** **Total: 165 minutes.**
Instructional Procedures	I. *Opening.* **Day 1:** I will begin by pointing out that not all of the shapes that will go into our shipping container will be rectangular. Another common shape for shipping goods, particularly liquids, is a cylinder. In our previous lesson, we derived the equation to calculate the volume of a rectangular box. Now we will do the same for a cylinder. A. I will hold up a soup can and ask the question, *"Can anyone tell me the volume of this can?"* B. Given that students should have learned the equation for the surface area of a circle in the seventh grade, it is possible that someone will make the connection with the previous lesson and state that the volume equals the surface area of the base circle times the height. C. Regardless of whether someone does or not, we will conduct an activity in groups of two at computer workstations to derive the equation. II. *Engagement.* **Day 1:** In groups of two, students will work at computer stations with internet access, using the *Illuminations* Circle Tool Applet:



A. Using the worksheet below, students will use the applet to determine the diameter, circumference, and area of circles of radius 2, 4, 8, and 16. Using the applet's ratio tool (x/y), students will derive the formulas for the circumference and area of a circle.

Finding the Area and Circumference of a Circle

1. Using the Circle Tool (http://illuminations.nctm.org/ActivityDetail.aspx?ID=116), set the radius at r = 2 and record the diameter, circumference, and area of the circle. Do the same for r = 4, r = 8, and r = 16.

Radius (r)	Diameter (d)	Circumference (C)	Area (A)	d/r	C/d	C/r	C
2	4	12.57	12.57	2	3.14	2*3.14	2*3.14*r
4	8	25.13	50.27	2	3.14	2*3.14	2*3.14*r
8	16	50.27	201.06	2	3.14	2*3.14	2*3.14*r
16	32	100.53	804.25	2	3.14	2*3.14	2*3.14*r

2. Calculate the relationship between the diameter and the radius.

Ans. d/r is always 2 so, solving for d,

$$d = 2r$$

3. Calculate the relationship between the circumference and the diameter.

	Ans. C/d is always 3.14 so, solving for C, $$C = 3.14d$$ ***…and, since d = 2r,*** $$C = 2 * 3.14 * r$$ *So, the circumference C of a circle equals 2*radius*3.14. Now let's try to derive a formula for the area of the circle.* 4. Calculate the relationship between Area and Circumference. ***Ans. Area divided by the circumference equals one half the radius. So:*** $$\frac{A}{C} = \frac{A}{2 * 3.14 * r} = 0.5 * r$$ ***Solving for A, we get:*** $$A = (2 * 3.14 * r) * 0.5 * r = (2 * 0.5) * 3.14 * (r * r) = 1 * 3.14 * r^2$$ 5. Do you remember what 3.14 is? ***Ans. 3.14 is an estimate for the value of π which is 3.1415926535898…so the area of a circle is*** $\boldsymbol{A = \pi * r^2}$ B. We will review the students' findings on the worksheets, ensuring everyone understands how to derive the formula. I will then conduct the following dialog: 1. Question: *Do you recall how we found the volume of a rectangular prism, like a box?* 2. Answer: *We measured the surface area of the base and multiplied it by the height of the box.* 3. Question: *Then what do you think is the volume of the cylinder? Now that we know the area of the base (the circle) what should we do?* 4. Answer: *The same thing, multiply the surface area of the base by the height of the cylinder.* 5. Question: *Now, remember we discovered yesterday that the volume of a box varies with its dimensions and that two boxes formed from the same sheet of construction paper can have different dimensions. Does this work for cylinders too? Let's find out.* At this point, I will produce two cylinders each formed from identical sheets of construction paper. I will ask two students to come forward and fill both

with popcorn, demonstrating that the same thing happens with cylinders.

III. *Closing.* **Day 1:** I will conclude with, "*So now we know that we can find the volume of a cylinder using the surface area of the base circle and the height of the cylinder. We also know that cylinders and rectangular prisms have similar properties with regard to their volume. What we don't know yet is how to calculate the surface area of a cylinder. We will explore tomorrow how we can find the surface area of the cylinder. Try to think about how to find the surface area of the cylinder tonight. Oh, and by the way, I would like each of you to bring a cylindrical object to school tomorrow – a food can would be best.*"

IV. *Opening.* **Day 2:** We will begin with a review of the previous day's lesson. I will have the *Illuminations* applet we used in that lesson and review how we derived the equation of the circle and then calculated the volume of the sphere. Then I will ask if anyone can tell me how to calculate its surface area. Chances are, someone will know or have some idea – i.e. he / she might know that it is the sum of the areas of the base, the top, and the "sides" but may not know how to calculate the area of the "sides." In our exercise today, we will derive that formula.

V. *Engagement.* **Day 2:** In groups of four and with the cylindrical objects they brought to class, students will determine the surface area of the objects by calculating the surface area of the base and the top using the equation we derived and, using the construction paper and rulers, the surface area of the side. They will enter their observations into the following table. Students will also calculate the volume of the can using our equation and then *speculate on how many such cans could fit into the shoe box whose volume we calculated previously* (this will set the stage for a follow-on lesson).

Can	Surface Area of Top	Surface Area of Base	Surface Area of Side	Surface Area of Can	Volume of Can	How Many Cans Fit Into a Shoe Box?

A. **Question**: *We know how to calculate the area of the two circles that make up the top and the base. How can we calculate the area of the side?*

Answer: *We can use our construction paper. By wrapping it around the cylinder, trimming the excess paper, unwrapping the paper, and then*

measuring the length of the sides, we can determine the area of the paper sufficient to wrap around the cylinder and, thus, the surface area of cylinder itself.

B. **Question**: *That does work but can we derive an equation from this? Wrap the paper around the cylinder again. Look at the side that wraps around the cylinder. That side has become what?*

Answer: *A circle! So, if we compute the circumference of the circle as one of the sides and multiply it by the length of the other side, we have the surface area. The equation is:*

$$Side\ Surface\ Area = (2 * \pi * r) * h$$

C. **Question**: *Are we done?*

Answer: *No. We need to add the surface areas of the top and the base. The complete equation is:*

$$Surface\ Area\ of\ Cylinder = (2 * \pi * r) * h + 2 * (\pi * r^2)$$
$$General\ Formula\ for\ Surface\ Area = 2B + PH$$

VI. *Closing.* **Day 2:** Now we will explore the final question, *"How many cans fit into the shoe box?"* Perhaps some students will simply divide the volume of their can into the volume of the shoe box to get the answer. Perhaps others will realize that the answer is more complicated. We will speculate on possible answers to this question to conclude the lesson.

VII. *Opening.* **Day 3:** Just as we did with the "Popcorn Prisms", we will explore the relationship between dimension and volume, arriving at some rather surprising conclusions. This exercise was developed by Jamie Chaikin and can be found on the NCTM Illuminations website at http://illuminations.nctm.org/LessonDetail.aspx?ID=L797.

VIII. *Engagement.* **Day 3:**

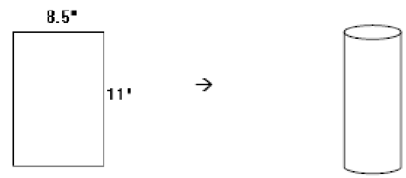
Answer Key – Popcorn Cylinders Anyone?

For this activity you will be comparing the volume of 2 cylinders created using the same sheet of paper. You will be determining which can hold more popcorn. To do this, you will have to find a pattern for the dimensions for containers.

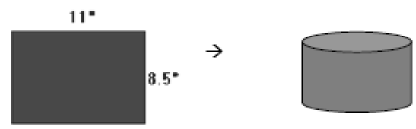
Materials:

- 8.5 inch by 11 inch white paper
- 8.5 inch by 11 inch colored paper
- Tape
- Popcorn
- Plate
- Cup
- Ruler

Take the white paper and roll it up along the longest side to form a baseless cylinder that is tall and narrow. Do not overlap the sides. Tape along the edge. Measure the dimensions with a ruler. Record your data below and on the cylinder. Label it Cylinder A.



Take the colored paper and roll it up along the shorter side to form a baseless cylinder that is short and stout. Do not overlap the sides. Tape along the edge. Measure the height and diameter with a ruler. Record your data below and on the cylinder. Label it Cylinder B.



1.

DIMENSION	CYLINDER A	CYLINDER B
HEIGHT (in.)	[11 in]	[8.5 in]
DIAMETER (in.)	[~2.7 in]	[~3.5 in]
RADIUS (in.)	[~1.4 in]	[~1.8 in]

2. Do you think the two cylinders will hold the same amount? Do you think one will hold more than the other? Which one? Why?

Answers will vary.

6. Which measurement impacts the volume more: the radius or the height? Work through the example below to help you answer the question.

a) Assume that you have a cylinder with a radius of 3 inches and a height of 10 inches. Increase the radius by 1 inch and determine the new volume. Then using the original radius, increase the height by 1 inch and determine the new volume.

CYLINDER	RADIUS	HEIGHT	VOLUME
ORIGINAL	3 in	10 in	[~282.7 in^3]
INCREASED RADIUS	[4 in]	[10 in]	[~502.7 in^3]
INCREASED HEIGHT	[3 in]	[11 in]	[~311.0 in^3]

b) Which increased dimension had a larger imact on the volume of the cylinder? Why do you think this is true?

Increasing the radius increased the volume more than increasing the height. This is because the radius is squared to find the volume, which increases its impact on the volume.

7. By how much would you have to decrease the height of Cylinder B to make the volumes of the two prisms equal?

$$V_A \approx 67.7 \text{ in}^3$$
$$V_B \approx 67.7 \text{ in}^3 = \pi(1.8)^2(h)$$
$$h \approx 6.7 \text{ in}$$

The height would need to be decreased by about $8.5 - 6.7 \approx 1.8$ in .

8. Compare and contrast your results from the prism activity and the cylinder activity. What conclusions can you make about the relationship between dimensions, area, and volume?

Answers will vary. Students may point out the similarity in the volume formulas $V = l^2h$ and $V = \pi r^2 h$ and how this effected their results.

3. Place Cylinder B on the paper plate with Cylinder A inside it. Use your cup to pour popcorn into Cylinder A until it is full. Carefully, lift Cylinder A so that the popcorn falls into Cylinder B. Describe what happened. Is Cylinder B full, not full, or overflowing?

Cylinder B is not full. There is still room in the cylinder for more popcorn.

As you share your popcorn snack, answer the questions below.

4. **a)** Was your prediction correct? How do you know?

Answers will vary.

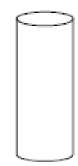
b) If your prediction was incorrect, describe what actually happened.

Cylinder B has a greater volume than Cylinder A.

5. **a)** State the formula for finding the volume of a cylinder.

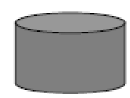
$V = \pi r^2 h$

b) Calculate the volume of Cylinder A? Label the dimensions in the figure.



$V = \pi r^2 h \approx \pi (1.4)^2 (11) \approx 67.7 \text{ in}^3$

c) Calculate the volume of Cylinder B? Label the dimensions in the figure.



$V = \pi r^2 h \approx \pi (1.8)^2 (8.5) \approx 86.5 \text{ in}^3$

d) Explain why the cylinders do or do not hold the same amount. Use the formula for the volume of a cylinder to guide your explanation.

The cylinders have different radii and heights, so the volumes are different.

ILLUMINATIONS Resources for Teaching Math

A. Together we will construct the cylindrical prisms. I will then let the students begin work on the questions as I walk around the room helping as necessary.

B. After students answer Question 2, I will pass out cooked popcorn (ideally prepared in the classroom or, at least, just moments before) to be used for the rest of the exercise. There should be enough to enjoy a snack as well as to conduct the exercise. Classroom management will be a bit of a

	challenge. Ideally, I should not do this on a Friday or just before a holiday. C. Some students are likely to be surprised that the two cylinders do not hold the same amount, given they were constructed from identical sheets of construction paper. As they complete the exercise and consider the volume formula as it applies to either prism: $$Volume = B \times h = \pi \times r^2 \times h$$ So, the taller prism is: $$V = \pi \times r^2 \times h = \pi \times (1.4)^2 \times 11 \cong 67.7 in^3$$ The, shorter, wider prism is: $$V = \pi \times r^2 \times h = \pi \times (1.8)^2 \times 8.5 \cong 86.5\ in^3$$ *WHY DOES THIS HAPPEN?* D. Algebraically, it can be seen that the radius of the second cylinder squared is approximately 1.7 times greater than that of the first cylinder. Therefore, the magnitude of the area of the base circle has greater impact than the magnitude of the height. IX. *Closing*. **Day 3.** I will ask students to think about our shipping container because it is into that which we will need to fit our cylinders. I will leave them with the following question, *"Remember, you want to get as much into that container as possible. Which of the two cylinders would you want to use to load cargo into the container?"*
Assessment	I will assess progress through a number of instruments to include in-class and at-home activities and worksheets. There will be a formative assessment at the conclusion of this lesson. 1. *Circles and Rectangles*: a review of how to determine the circumference and area of a circle. 2. *Queen Mary 2 Menu*: more review and introduction to cylinders. 3. *Circles and Cylinders*: further review of the circumference and area of a circle as well as practice in calculating the surface are and volume of a cylinder. 4. *Popcorn Cylinders*: an in-class exercise from NCTM *Illuminations*.

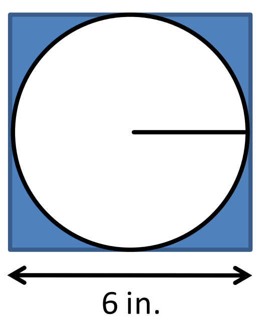
1. List all formulas.
2. Show all work on separate paper attached to this one.
3. Ensure you include units of measure (in^2, ft^3, etc.)

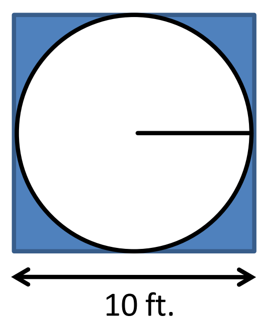
Name:____________________
Date:____________________
Advisor:____________________

Worksheet: Circles and Rectangles

1. What are the equations for the circumference of a circle? ________________
2. What is the equation for the area of a circle? ________________
3. What is the value of pi? ________________
4. For each of the following:

   A. What is the circumference of the circle inside the square?
   B. What is the area of the circle inside the square?
   C. What is the total area of the square?
   D. What is the area of the shaded region?





Circumference: ______________

Circle area: ______________

Area of square: ______________

Shaded region area: ______________

Circumference: ______________

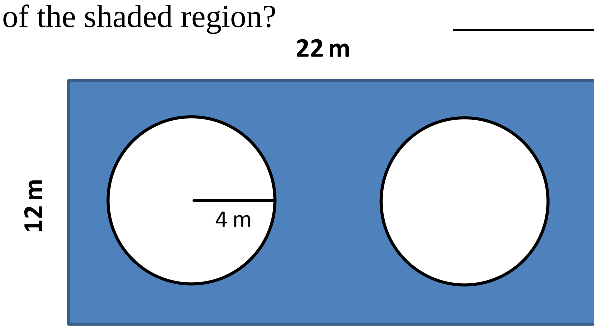
Circle area: ______________

Area of square: ______________

Shaded region area: ______________

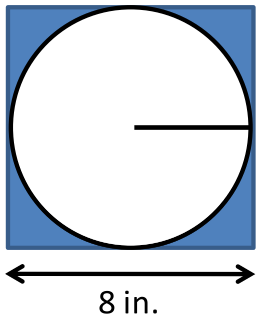
5. In the following figure:

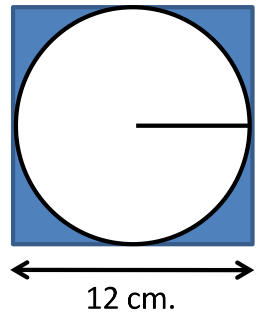
   A. What is the circumference of each small circle? ________________________
   B. What is the area of each small circle? ________________________
   C. What is the area of the shaded region? ________________________



6. For each of the following:

   A. What is the circumference of the circle inside the square?
   B. What is the area of the circle inside the square?
   C. What is the total area of the square?
   D. What is the area of the shaded region?





Circumference: ______________ Circumference: ______________

Circle area: ______________ Circle area: ______________

Area of square: ______________ Area of square: ______________

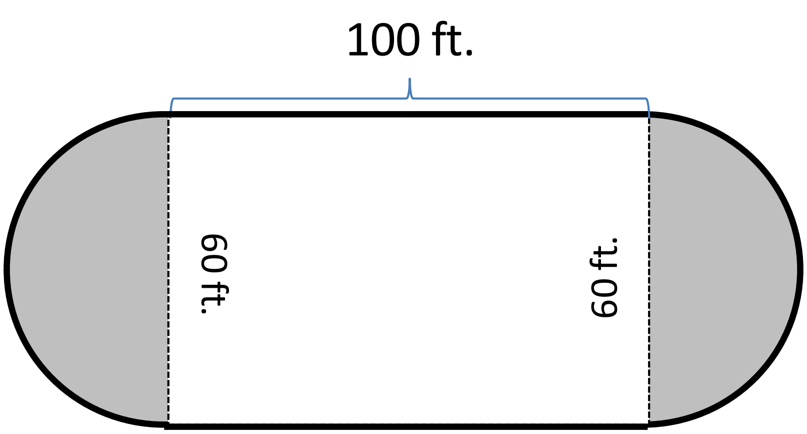
Shaded region area: ______________ Shaded region area: ______________

7. For the racetrack below:

   A. What is the distance around this track? ____________

   B. What is the area of the shaded regions? ____________

   C. What is the area of the complete region inside the track? ____________



1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Differentiated Worksheet

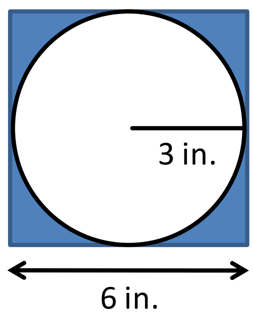
Name: ____________________

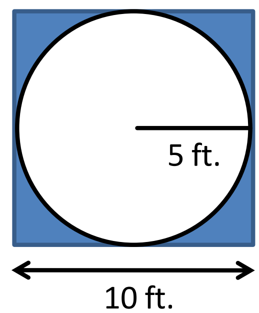
Date: ____________________

Class: ____________________

Worksheet: Circles and Rectangles

1. What are the equations for the circumference of a circle? ________________
2. What is the equation for the area of a circle? ________________
3. What is the value of pi? ________________
4. For each of the following:





A. What is the circumference of the circle inside the square?

B. What is the area of the circle inside the square?

C. What is the total area of the square?

D. What is the area of the shaded region?

Circumference: ______________

Circle area: ______________

Area of square: ______________

Shaded region area: ______________

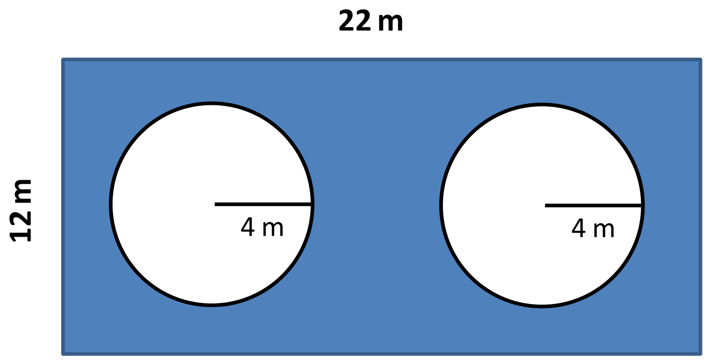
Circumference: ______________

Circle area: ______________

Area of square: ______________

Shaded region area: ______________

5. In the following figure:

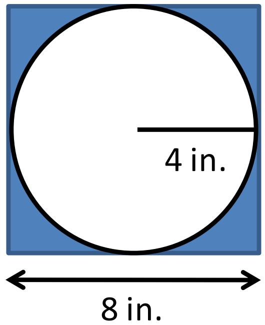


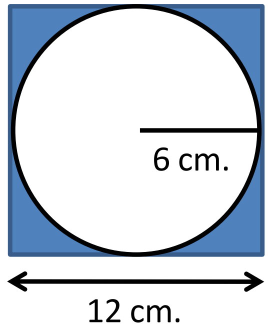
A. What is the circumference of each small circle? ______________________________

B. What is the area of each small circle? ______________________________

C. What is the area of the shaded region? ______________________________

6. For each of the following:





A. What is the circumference of the circle inside the square?

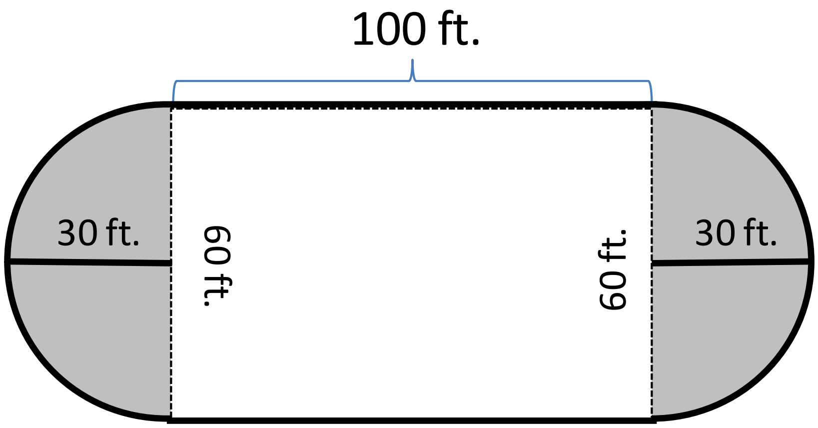
B. What is the area of the circle inside the square?

C. What is the total area of the square?

D. What is the area of the shaded region?

Circumference:	_______________	Circumference:	_______________
Circle area:	_______________	Circle area:	_______________
Area of square:	_______________	Area of square:	_______________
Shaded region area:	_______________	Shaded region area:	_______________

7. For the racetrack below:



A. What is the distance around this track? ____________

B. What is the area of the shaded regions? ____________

C. What is the area of the complete region inside the track? ____________

Circles and Rectangles (Answer Key)

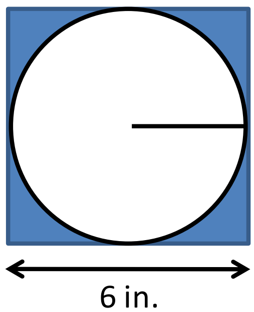
1. What are the equations for the circumference of a circle? $\boldsymbol{2 \times \pi \times r}$ $\boldsymbol{\pi \times d}$

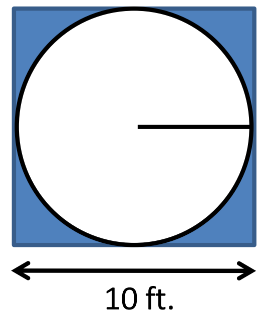
2. What is the equation for the area of a circle? $\boldsymbol{\pi \times r^2}$

3. What is the value of pi? **approx. 3.14**

4. For each of the following:

   A. What is the circumference of the circle inside the square?
   B. What is the area of the circle inside the square?
   C. What is the total area of the square?
   D. What is the area of the shaded region?





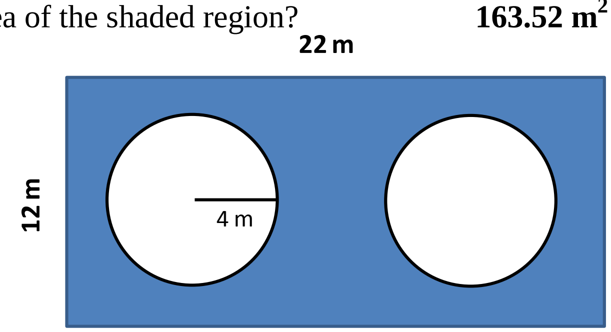
Circumference:	**18.84 in.**	Circumference:	**31.4 ft.**
Circle area:	**28.26 in.^{2}**	Circle area:	**78.5 ft.^{2}**
Area of square:	**36 in.^{2}**	Area of square:	**100 ft.^{2}**
Shaded region area:	**7.74 in.^{2}**	Shaded region area:	**21.5 ft.^{2}**

5. In the following figure:

   A. What is the circumference of each small circle? **25.12 m**

   B. What is the area of each small circle? **50.24 m^2**

   C. What is the area of the shaded region? **163.52 m^2**



6. For each of the following:

   A. What is the circumference of the circle inside the square?
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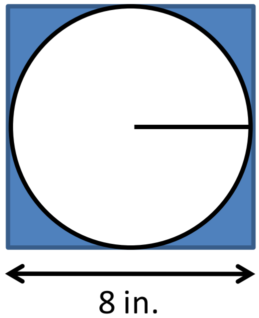
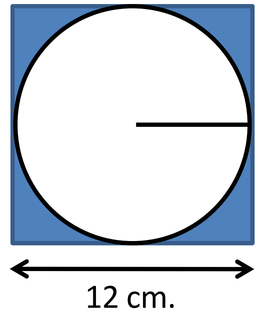




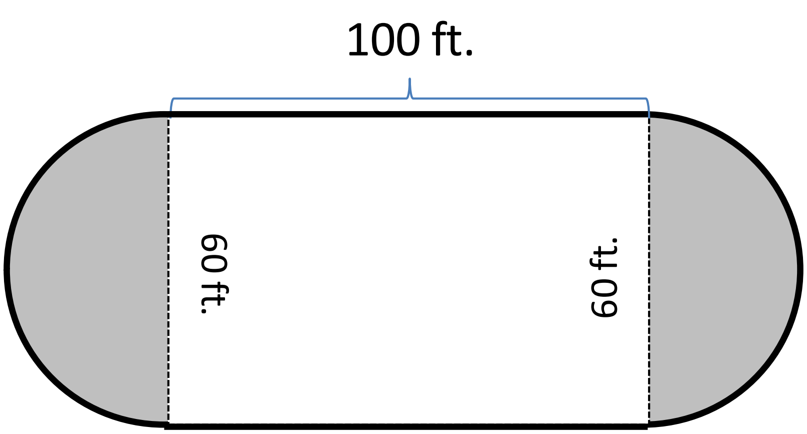
Circumference:	**25.12 in.**	Circumference:	**37.68 cm.**
Circle area:	**50.24 in.^{2}**	Circle area:	**113.04 cm.^{2}**
Area of square:	**64 in.^{2}**	Area of square:	**144 cm.^{2}**
Shaded region area:	**13.76 in^2**	Shaded region area:	**30.96 cm.^{2}**

7. For the racetrack below:

   A. What is the distance around this track? **388.4 ft.**

   B. What is the area of the shaded regions? **2826 ft.^{2}**

   C. What is the area of the complete region inside the track? **8826 ft.^{2}**

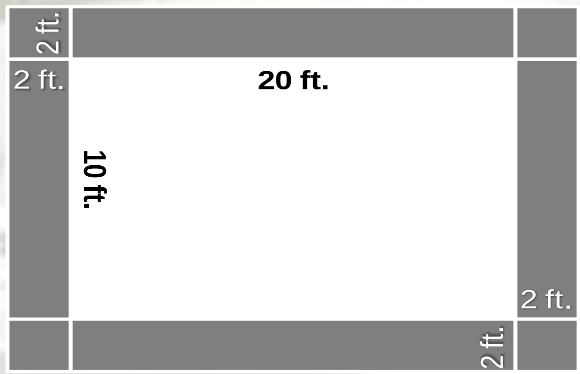


Royal Mail Steamer *Queen Mary 2*

Dinner, September 26th, 2010

Appetizer

Find the total area and the shaded area of the following shape:

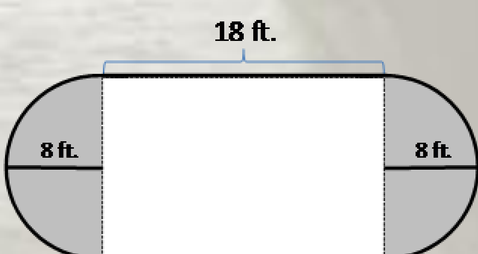


1.a Total Area: _______________ 1.b Shaded Area: _______________

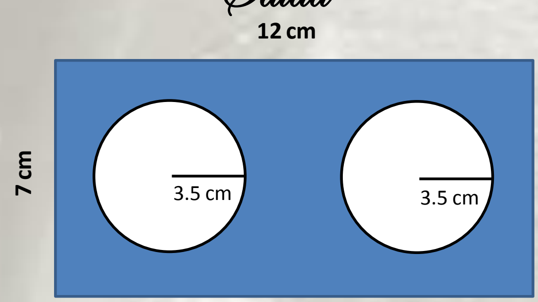
Soup or Salad

Choose just one and find the requested answers:

Soup



Salad



2.a Shaded region area: _______________

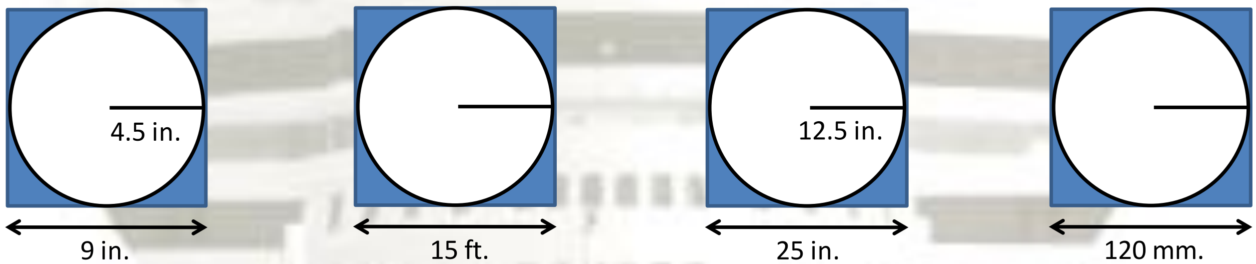
2.b Total Area: _______________

2.c Each circle area: _______________

2.d Shaded area: _______________

Main Course

Choose any two and find the requested answers:



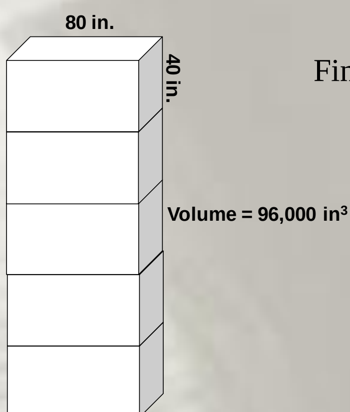
3.a Circle Area: _______ 3.d Circle Area: _______ 3.g Circle Area: _______ 3.j Circle Area: _______

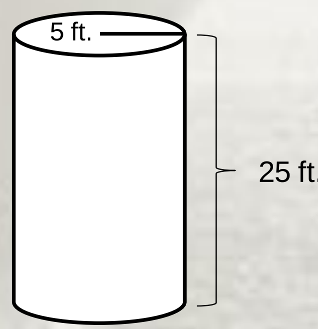
3.b Square Area: _______ 3.e Square Area: _______ 3.h Square Area: _______ 3.k Square Area: _______

3.c Shaded Area: _______ 3.f Shaded Area: _______ 3.i Shaded Area: _______ 3.l Shaded Area: _______

Dessert

Find the requested answers:





4.a Width: _______________

5.a Surface Area: _______________

5.b Volume: _______________

Instructions from the Captain's Table:

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

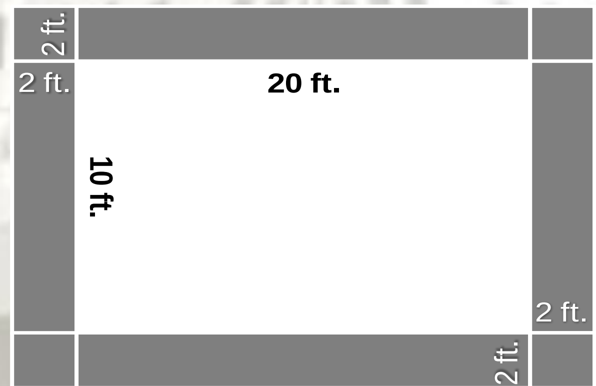
Answer Key

Royal Mail Steamer *Queen Mary 2*

Dinner, February 22nd, 2010

Appetizer

Find the total area and the shaded area of the following shape:

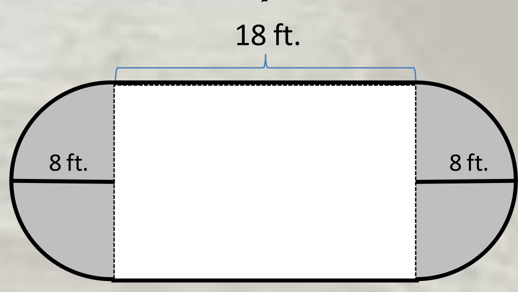


1.a Total Area: **336 ft.^{2}** 1.b Shaded Area: **136 ft.^{2}**

Soup or Salad

Choose just one and find the requested answers:

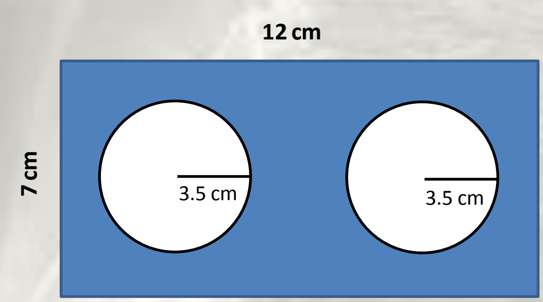
Soup



2.a Shaded region area: **200.96 ft.^{2}**

2.b Total Area: **488.96 ft.^{2}**

Salad

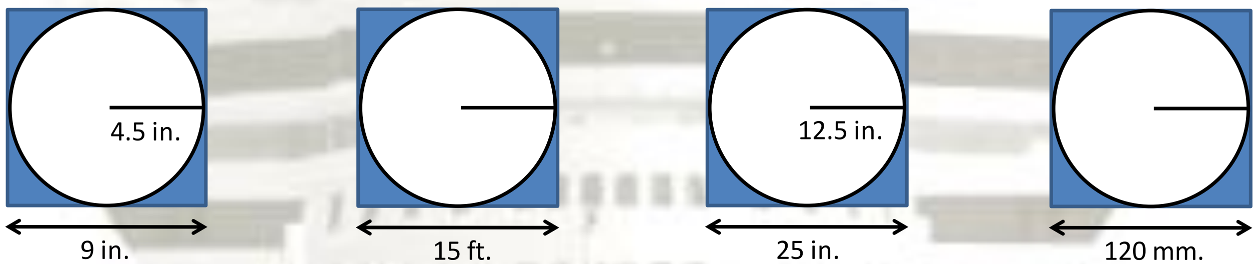


2.c Each circle area: **38.465 cm.^{2}**

2.d Shaded area: **45.535 cm.^{2}**

Main Course

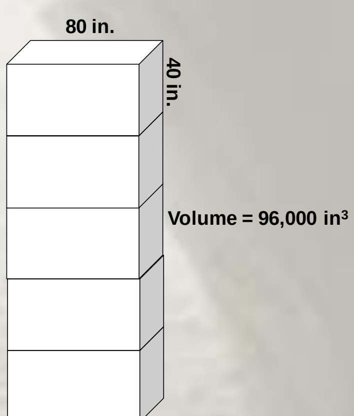
Choose any two and find the requested answers:

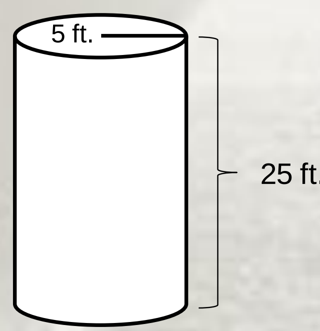


3.a Circle Area: **63.59 in.^{2}**	3.d Circle Area: **176.63 ft.^{2}**	3.g Circle Area: **490.63 ft.^{2}**	3.j Circle Area: **11304 mm.^{2}**
3.b Square Area: **81 in.^{2}**	3.e Square Area: **225 ft.^{2}**	3.h Square Area: **625 ft.^{2}**	3.k Square Area: **14400 mm.^{2}**
3.c Shaded Area: **17.41 in.^{2}**	3.f Shaded Area: **48.37 ft.^{2}**	3.i Shaded Area: **134.37 ft.^{2}**	3.l Shaded Area: **3096 mm.^{2}**

Dessert

Find the requested answers:





4.a Width: **6 ft.**

5.a Surface Area: **942 ft.^{2}**

5.b Volume: **1962.5 ft.^{2}**

Instructions from the Captain's Table:

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________
Date:_____________________
Class:____________________

Circles and Cylinders

1. Definitions.

   a. Circumference: _____ Line segment whose endpoints lie on the circle.

   b. Radius: _____ The sum of the areas of the surfaces of a three-dimensional object.

   c. Diameter: _____ The distance around a closed curve such as a circle.

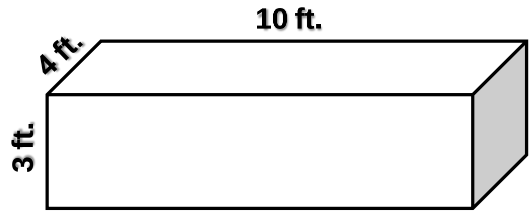
   d. Chord: _____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

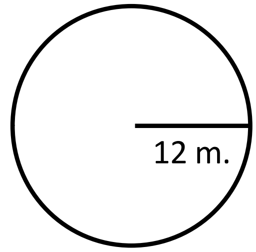
   e. Volume: _____ Any line segment from the center of a circle to its perimeter.

   f. Surface Area: _____ The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? _________________
3. What is the formula for the **area** of a circle? _________________
4. What is the formula for the **volume** of a rectangular prism? _________________
5. What is the formula for the **surface area** of a rectangular prism? _________________
6. What is the formula for the **volume** of a cylinder? _________________
7. What is the formula for the **surface area** of a cylinder? _________________
8. What is a general formula for the **volume** of a prism or cylinder? _________________
9. What is a general formula for the **surface area** of a prism or cylinder? _________________

10. For the following shapes, find the asked-for information:



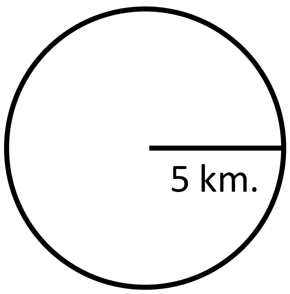


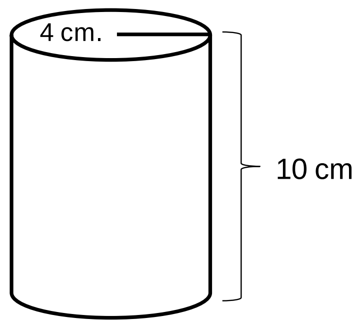
10.a Surface Area: ________________

10.b Volume: ________________

10.c Circumference: ________________

10.d Area: ________________



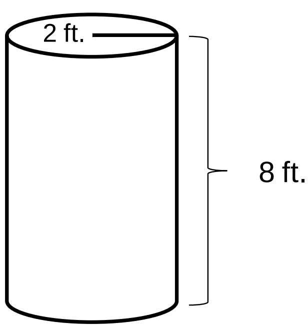


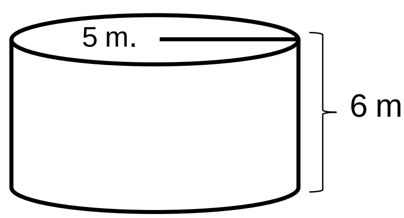
10.e Circumference: ________________

10.f Area: ________________

10.g Surface Area: ________________

10.h Volume: ________________





10.i Surface Area: ________________

10.j Volume: ________________

10.k Surface Area: ________________

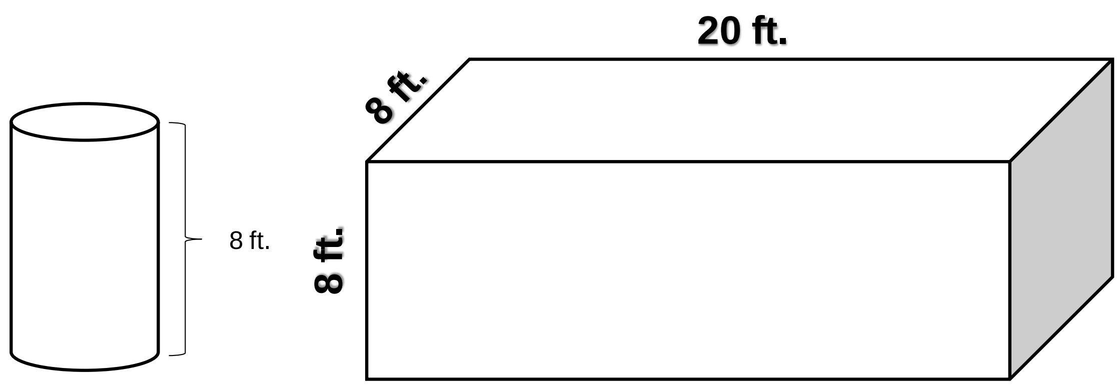
10.l Volume: ________________

11. Remember our cargo container? Dimensions were 20 ft. x 8 ft. x 8 ft.

a. What is the surface area of the container? ________________

b. What is the volume of the container? ________________

c. Suppose we had to ship cylinders each of which is 8 ft. high and has a volume of 25.12 $ft.^3$ How many cylinders do you think we could fit into our container? ***Think!*** ________________



Differentiated Version

Circles and Cylinders

1. Definitions.

   g. Circumference: _____ Line segment whose endpoints lie on the circle.

   h. Radius: _____ The sum of the areas of the surfaces of a three-dimensional object.

   i. Diameter: _____ The distance around a closed curve such as a circle.

   j. Chord: _____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

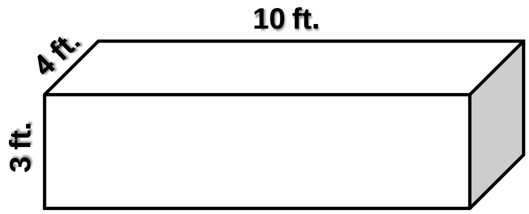
   k. Volume: _____ Any line segment from the center of a circle to its perimeter.

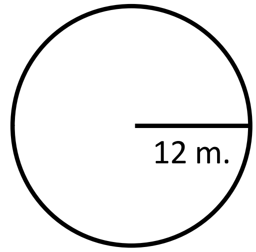
   l. Surface Area: _____ The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? ________________
3. What is the formula for the **area** of a circle? ________________
4. What is the formula for the **volume** of a rectangular prism? ________________
5. What is the formula for the **surface area** of a rectangular prism? ________________
6. What is the formula for the **volume** of a cylinder? ________________
7. What is the formula for the **surface area** of a cylinder? ________________
8. What is a general formula for the **volume** of a prism or cylinder? ________________
9. What is a general formula for the **surface area** of a prism or cylinder? ________________

Differentiated Version

10. For the following shapes, find the asked-for information:



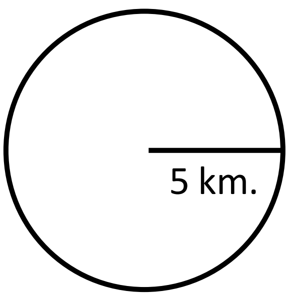


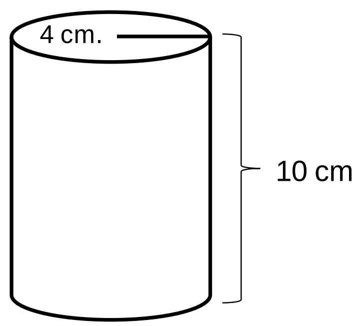
10.a Surface Area: ___________________

10.b Volume: ___________________

10.c Circumference: ___________________

10.d Area: ___________________





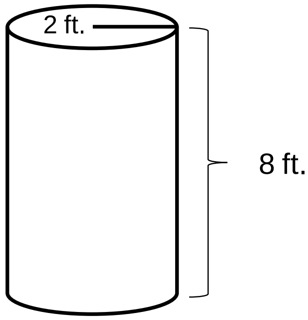
10.e Circumference: ___________________

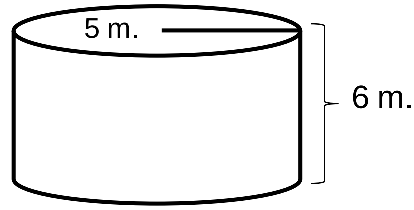
10.f Area: ___________________

10.g Surface Area: ___________________

10.h Volume: ___________________

Differentiated Version





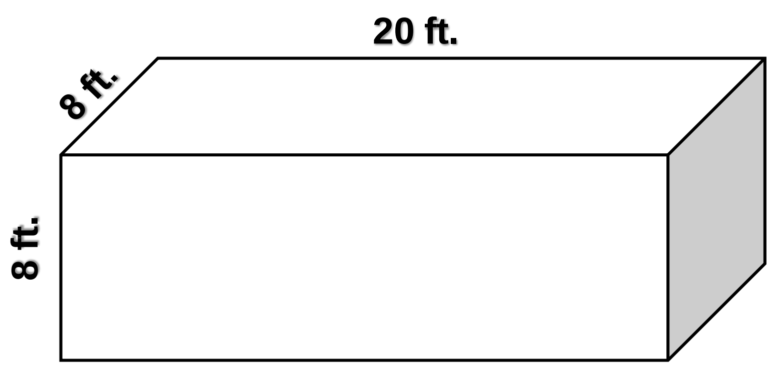
10.i Surface Area: ________________

10.j Volume: ________________

10.k Surface Area: ________________

10.l Volume: ________________

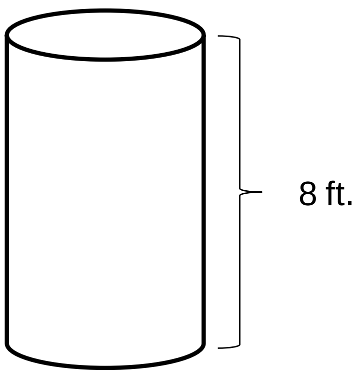
11. Remember our cargo container? Dimensions were 20 ft. x 8 ft. x 8 ft.



a. What is the surface area of the container? ________________

b. What is the volume of the container? ________________

c. Suppose we had to ship cylinders each of which is 8 ft. high and has a volume of 25.12 $ft.^3$ How many cylinders do you think we could fit into our container? ***Think!*** ________________



Worksheet: Circles and Cylinders (Answer Key)

1. Definitions.

a.	Circumference:	**d**	Line segment whose endpoints lie on the circle.
b.	Radius:	**f**	The sum of the areas of the surfaces of a three-dimensional object.
c.	Diameter:	**a**	The distance around a closed curve such as a circle.
d.	Chord:	**c**	Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.
e.	Volume:	**b**	Any line segment from the center of a circle to its perimeter.
f.	Surface Area:	**e**	The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? $2\pi r$

3. What is the formula for the **area** of a circle? πr^2

4. What is the formula for the **volume** of a rectangular prism? Bh

5. What is the formula for the **surface area** of a rectangular prism? $2B + Ph$

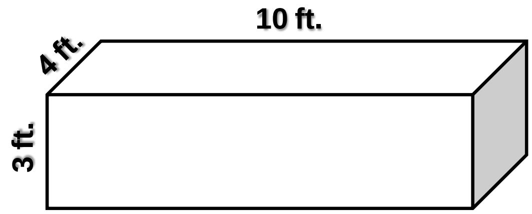
6. What is the formula for the **volume** of a cylinder? $\pi r^2 h$

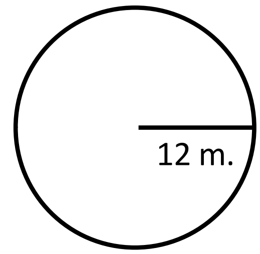
7. What is the formula for the **surface area** of a cylinder? $2\pi r^2 + 2\pi rh$

8. What is a general formula for the **volume** of a prism or cylinder? Bh

9. What is a general formula for the **surface area** of a prism or cylinder? $2B + Ph$

10. For the following shapes, find the asked-for information:



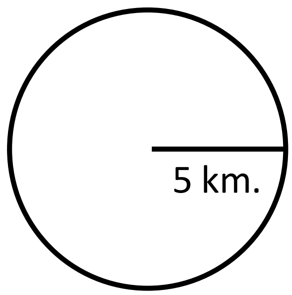


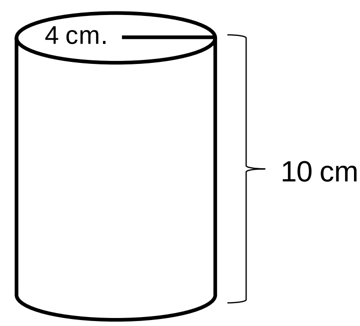
10.a Surface Area: **164 ft^2**

10.b Volume: **120 ft^3**

10.c Circumference: **75.36 m**

10.d Area: **452.16 m^2**



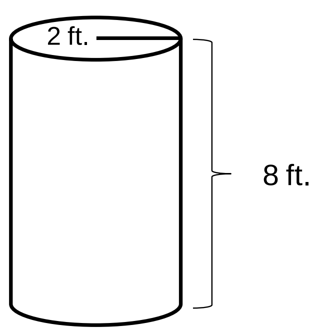


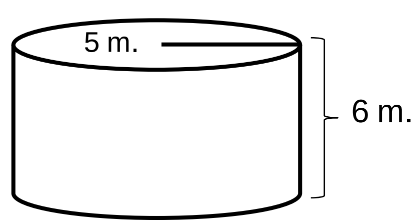
10.e Circumference: **31.4 km.**

10.f Area: **78.5 km.^{2}**

10.g Surface Area: **351.68 cm^2**

10.h Volume: **502.4 cm^3**





10.i Surface Area: **125.6 ft.^{2}**

10.j Volume: **100.48 ft.^{3}**

10.k Surface Area: **345.4 ft.^{2}**

10.l Volume: **471 ft.^{3}**

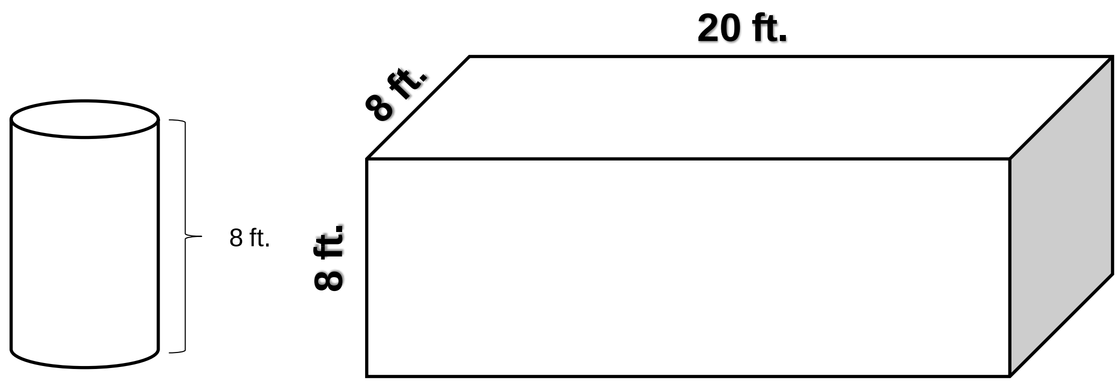
11. Remember our cargo container? Dimensions were 20 ft. x 8 ft. x 8 ft.

a. What is the surface area of the container? **768 ft.^{2}**

b. What is the volume of the container? **1280 ft.^{3}**

c. Suppose we had to ship cylinders each of which is 8 ft. high and has a volume of 25.12 ft.^{3} How many cylinders do you think we could fit into our container? ***Think!***

$\mathbf{25.12 \div 8 = 3.14}\ \boldsymbol{ft.}^{\mathbf{2}}$
$\mathbf{10 \times 4 = 40}$



1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name: ______________________________

Date: ______________________________

Advisor: ____________________________

Geometry of the Ship: Practice 2: Surface Area and Volume of 3 – Dimensional Shapes

Formulas: Use the following formulas to complete this practice exercise:

I. Surface area and volume of a regular prism of base area *B*, base perimeter *P*, and height *H*.

A. **Surface area:** $SA = 2B + PH$

B. **Area:** $V = BH$

II. Square prism with side length *s*:

A. **Base Perimeter:** $P = s + s + s + s = 4s$

B. **Base Area:** $B = s \times s = s^2$

III. Rectangular prism with base length *l* and width *w*:

A. **Perimeter:** $P = l + w + l + w = 2l + 2w$

B. **Area:** $B = l \times w$

IV. Circular prism of radius *r*:

A. **Base Perimeter:** $P = 2 \times \pi \times r = 2\pi r$

B. **Base Area:** $B = \pi \times r \times r = \pi r^2$

V. Triangle of base length *b*, base side length *s*, and base height *h*

A. **Base Perimeter:** $P = 3s$

B. **Base Area:** $A = \left(\frac{1}{2}\right) bh$

Problems: Find the surface area and volume of the following 3 – dimensional shapes. Draw each shape, using ruler and compass, labeling the dimensions. The drawings do not have to match the exact dimensions. Show all work and answers on separate sheets of paper attached to this one.

1. A square prism of side length $s = 7\ cm$.

2. A square prism of side length $s = 12\ mm$.

3. A square prism of side length $s = 22\ ft$.

4. A rectangular prism with base length $l = 6\ in.$, base width $w = 4\ in.$, and height $H = 12\ in.$

5. A rectangular prism with base length $l = 13\ in.$, base width $w = 6\ in.$, and height $H = 7\ in.$

6. A rectangular prism with base length $l = 15\ m.$, base width $w = 7.5\ m.$, and height $H = 7\ m.$

7. A rectangular prism with base length $l = 22\ m.$, base width $w = 110\ cm.$, and height $H = 6.5\ m.$

8. A rectangular with base length $l = 18\ ft.$, base width $w = 108\ in.$, and height $H = 7\ ft.$

9. A circular prism of base radius $r = 2\ in.$ and height $H = 12\ in.$

10. A circular prism of base radius $r = 17\ in.$, and height $H = 40\ in.$

11. A circular prism of base radius $r = 19\ ft.$, and height $H = 360\ in.$

12. A circular prism of base diameter $d = 12\ in.$, and height $H = 12\ ft.$

13. A circular prism of base diameter $d = 42\ cm.$, and height $H = 12\ m.$

14. A circular prism of base diameter $d = 1\ in.$, and height $H = 12\ in.$

15. A regular triangular prism of base length / base side length $b = s = 2\ cm.$, base height $h = 1.73\ cm.$, and prism height $H = 10\ cm.$

16. A regular triangular prism of base length / base side length $b = s = 12\ in.$, base height $h = 10.39\ in.$, and prism height $H = 10\ cm.$

<table>
<tr><td>Grade / Content Area</td><td>8th / 9th Grade Geometry</td></tr>
<tr><td>Lesson Title</td><td>Building a Ship Model (2 weeks)</td></tr>
<tr><td>Guiding Question</td><td>How can we interpret a two dimensional plan and translate it into a three dimensional object.</td></tr>
<tr><td>Content Standards</td><td>State Content Standards:

I. M(G&M)–10–10: Demonstrates conceptual understanding of spatial reasoning and visualization by sketching or using dynamic geometric software to generate three-dimensional objects from two-dimensional perspectives, or to generate two-dimensional perspectives from three-dimensional objects, or by solving related problems.</td></tr>
<tr><td></td><td>Common Core Standards:

I. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td></tr>
<tr><td>Preparation</td><td>I. Classroom Organization: Students will work individually on this project but can interact to assist each other.

II. Materials:

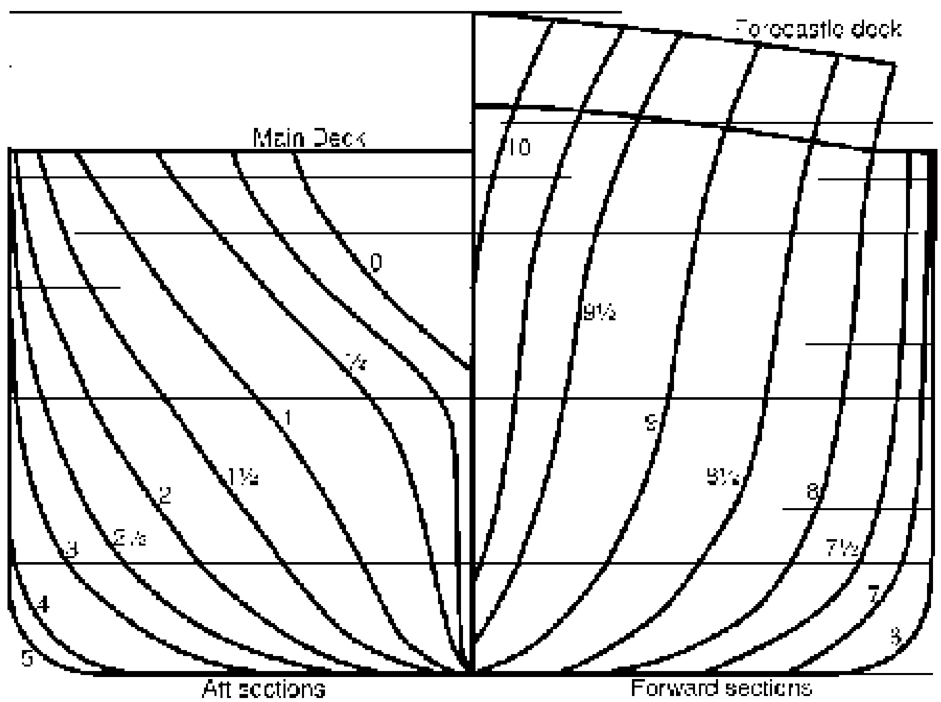
A. “The Geometry of the Ship” task sheets with ship contour drawings copied onto graph paper.
B. Poster board.
C. Wooden kabob skewers.
D. Scissors.
E. Glue guns and glue sticks.
F. Colored construction paper.
G. Newsprint and paste for making papier mache.</td></tr>
<tr><td>Student Learning Objectives</td><td>I. Students will draw a precise reflection of a two dimensional shape on graph paper.

II. Students will translate a two dimensional plan of a three dimensional object into a three dimensional model of that object.</td></tr>
<tr><td>Instruction and Engagement</td><td>I. Warm-up (each day for 10 minutes).

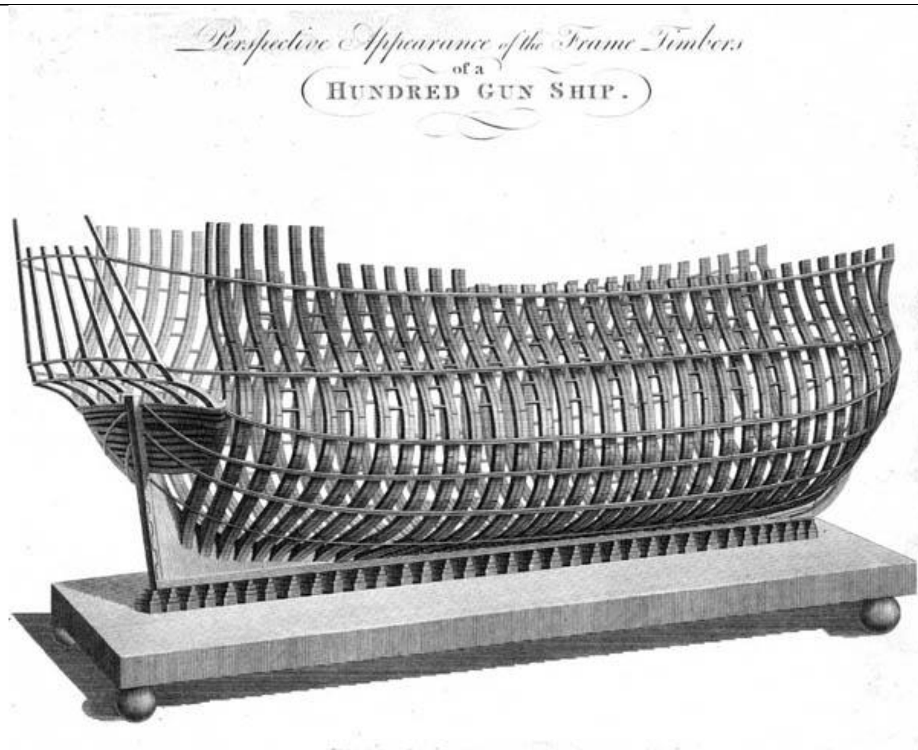
II. Launch (first day – 15 minutes).

A. I will show the following drawing and ask students if they can determine</td></tr>
</table>

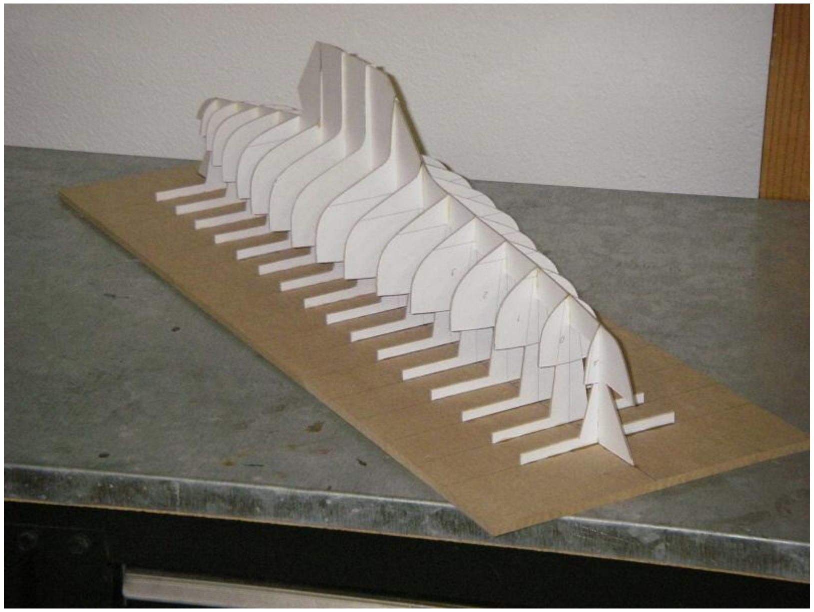
what it is:

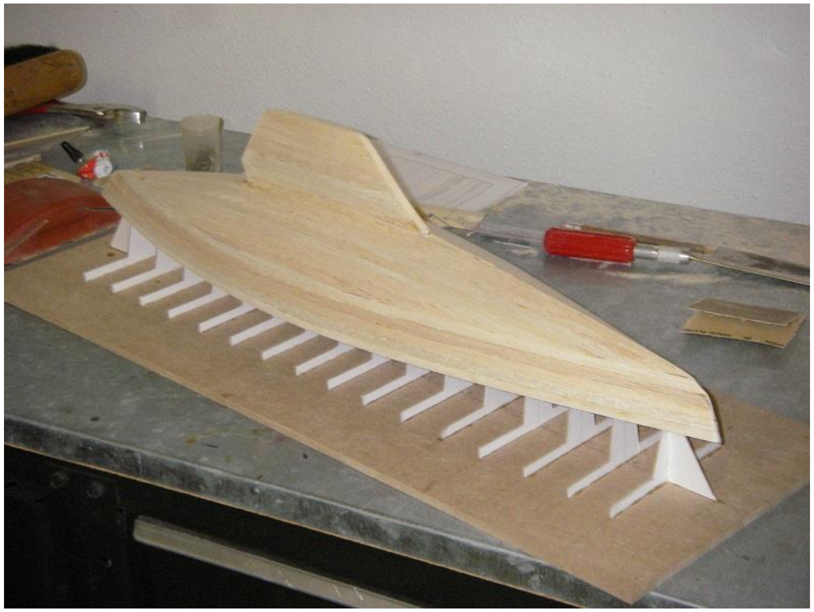


B. I will then show these pictures and tell them these are the three dimensional equivalents of the drawing:

	 C. The drawing is a naval architect’s two dimensional drawing, the other pictures are the scale models he makes from the drawing. From the scale models, he makes the ship. III. *Engagement* (**each day – 60 minutes**). Students will complete their drawings and construct their models in accordance with the attached task sheets.
Assessment	I. Students will meet the standard if they: A. Complete precise reflections of the forward and aft contours in the naval architect’s drawing. B. Trace and cutout precise cutouts of the contours. C. Place the contour cutouts on the wooden skewers precisely equidistant. D. Cover the model with papier mache and paint it so that it looks like a ship model.

Body Plan to Model





Name: ____________________
Class: ____________________
Advisor: ____________________

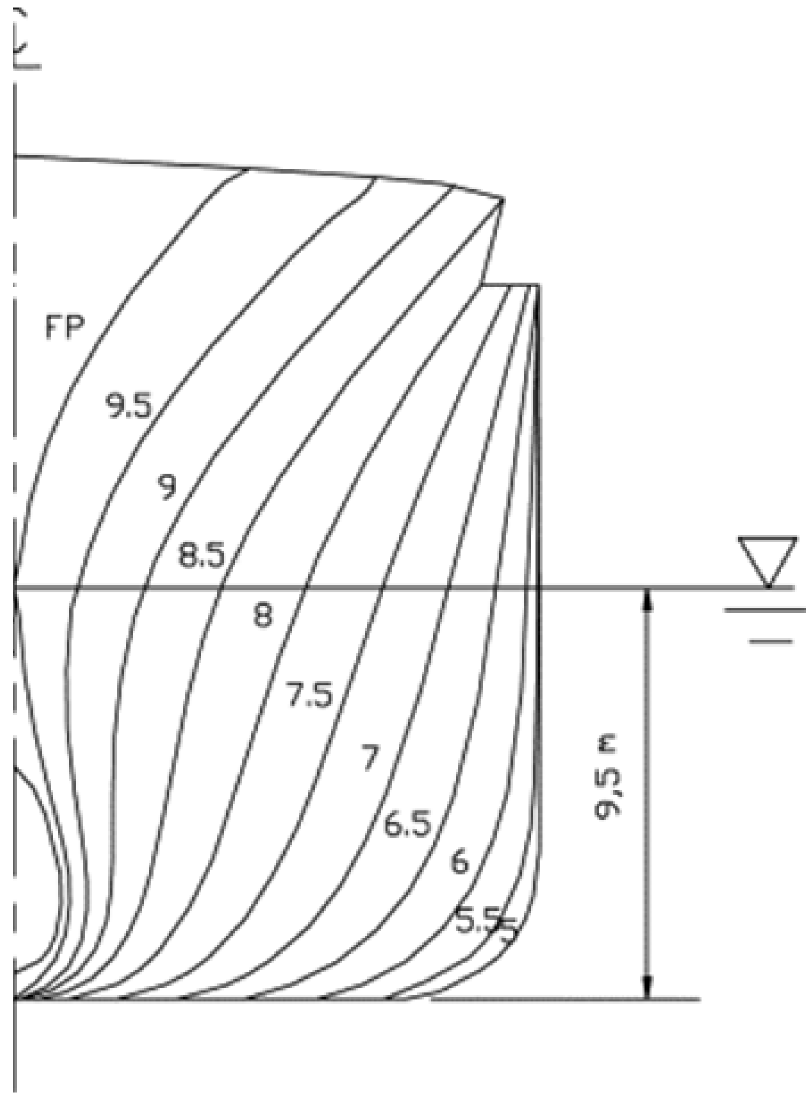
Project: The Geometry of the Ship – Task 1

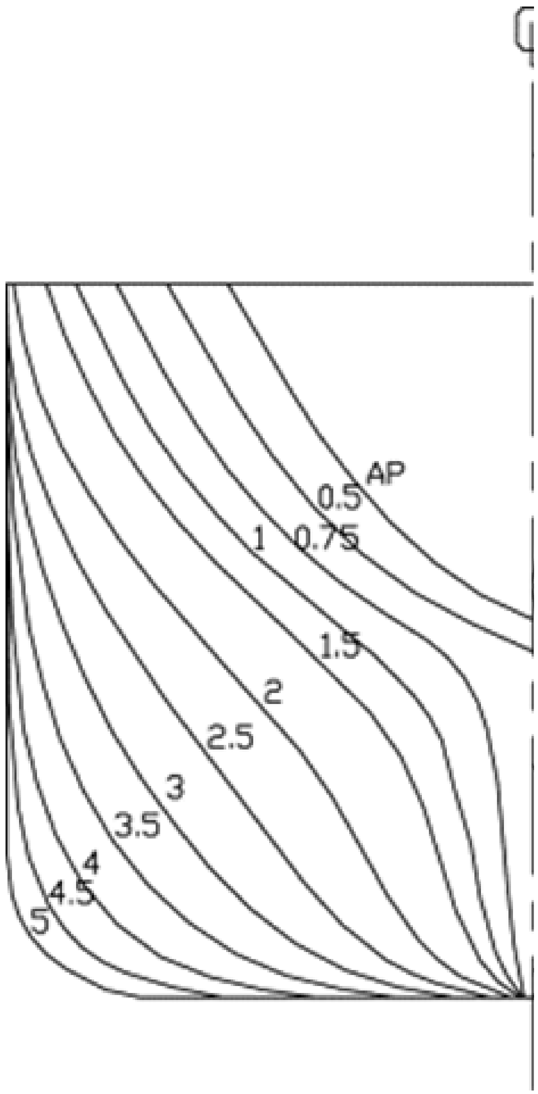
Directions: Each drawing is one half of the body plan of a naval architect's ship drawing. The first drawing depicts ½ of the ship as you would see it if you were standing directly in front of the ship looking at the bow. The second drawing depicts ½ of the ship as you would see it if you were standing directly behind the ship looking at the stern.

Your task today is to produce a precise mirror image of each drawing **of one of the three ships**, connected to the first drawing so that you have a complete view of the plan of the forward part of the ship and the aft part of the ship.

Option 1: Container Ship

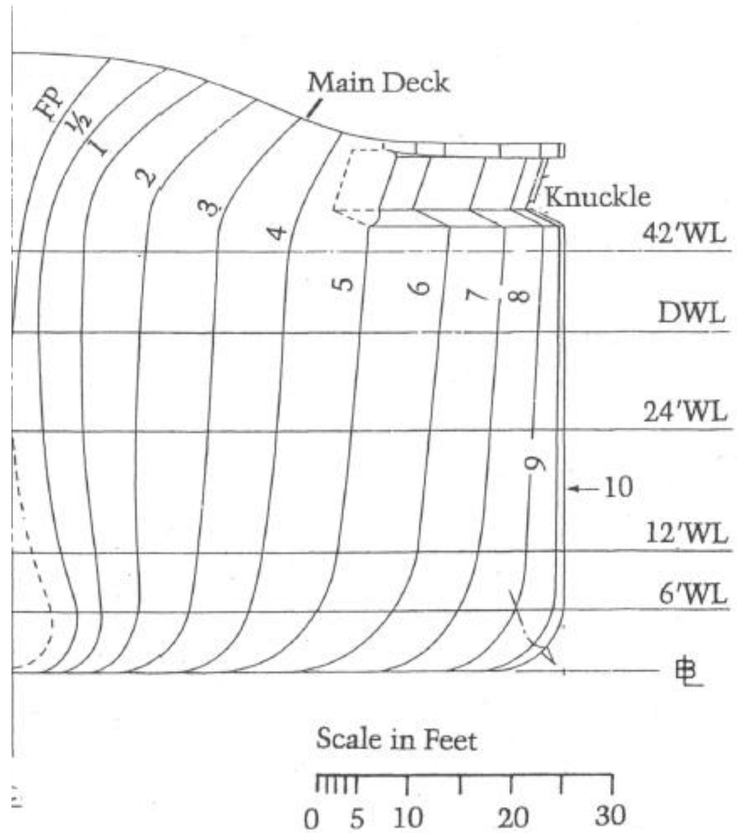
	Actual Dimensions	Model Dimensions
Length Overall	965 ft.	36 in.
Maximum Beam	106 ft.	
Maximum Draft	39 ft. 6 in.	

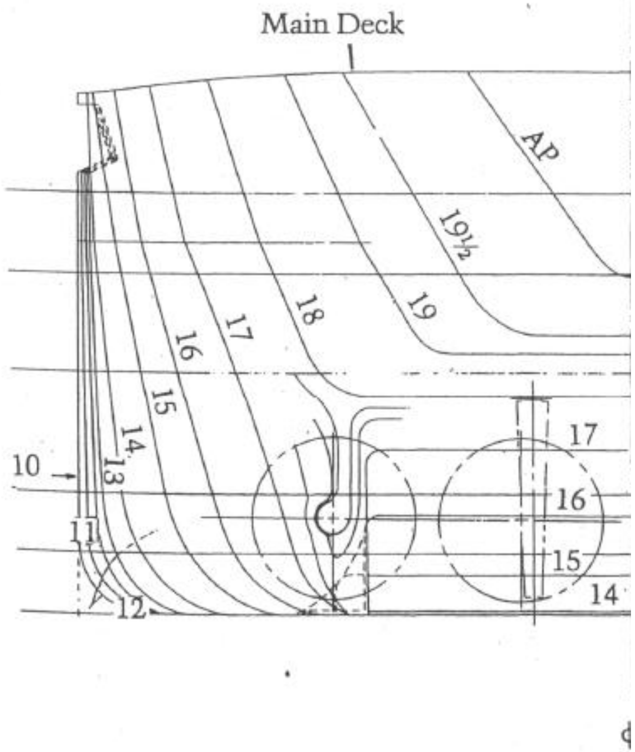



AP
0.5
1
0.75
1.5
2
2.5
3
3.5
4
4.5
5

Option 2: U.S.S. *Massachussets* (Battleship)

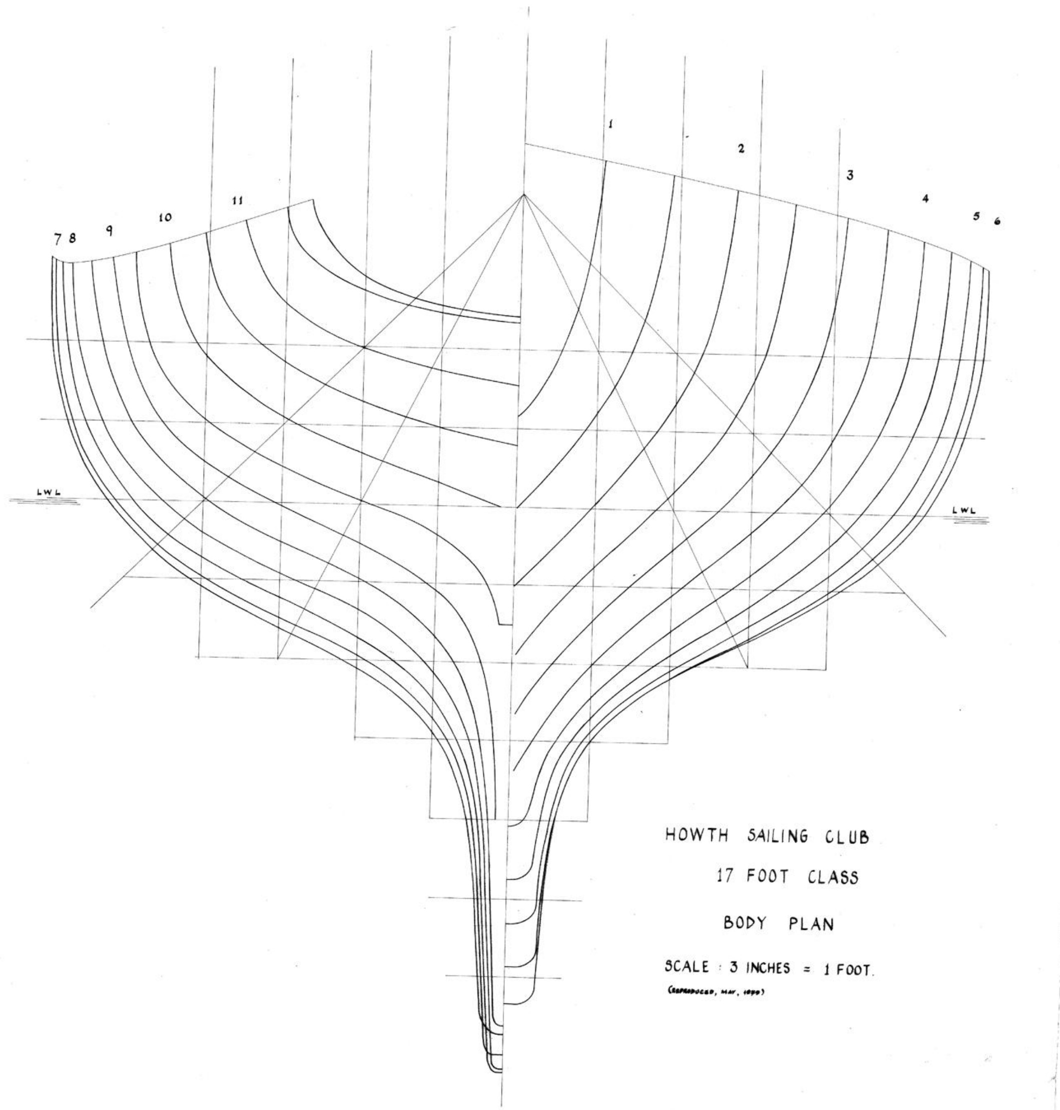
	Actual Dimensions	Model Dimensions
Length Overall	680 ft. 9.813 in.	36 in.
Maximum Beam	108 ft., 2.250 in.	
Maximum Draft	36 ft., 9.000 in.	





Main Deck
AP
19½
19
18
17
16
15
14
13
12
11
10
17
16
15
14

Option 3: 17 foot Sailing Yacht – Model Dimensions: 36 inches in length



Project: The Geometry of the Ship – Task 2

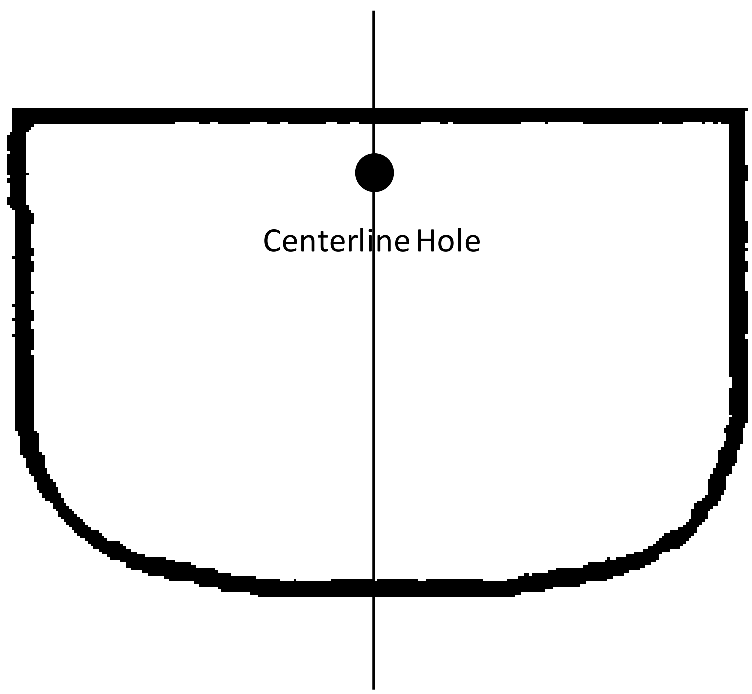
Directions: You are going to construct a builder's model ship from your drawings. When your drawings from task 1 are approved by the instructor:

1. Trace the shape of each contour curve on poster board. ***Be as precise as possible***.

2. Carefully cut each shape from the poster board.

3. Find a point on each piece that is along the centerline, equidistant from the top edge. ***Be as precise as possible.***

    a. Using one of the kabob skewers and / or a scissors, punch a hole in each shape.
    b. You should be able to slide each piece onto the skewer.



    c. Use additional skewers as necessary through different holes to make the ship model sturdy.

    d. Measure and cut a piece of poster board to be glued on top of the model (on the flat side) to act as a top deck.

    e. Cover the model with papier mache, paper, or cardboard and paint it.

Project: The Geometry of the Ship – Task 3

Background: To complete our shipbuilder's models we will construct propellers and rudders for them. This will complete our consideration of the relationship between two-dimensional and three-dimensional shapes which covered the following standards:

1. *Visualize relationships between two-dimensional and three-dimensional objects.*

2. *Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).*

3. *Demonstrate conceptual understanding of spatial reasoning and visualization by sketching or using dynamic geometric software to generate three-dimensional objects from two-dimensional perspectives, or to generate two-dimensional perspectives from three- dimensional objects, or by solving related problems.*

A ship's propeller(s) (called a screw propeller in nautical terminology) and her rudder(s) are what propel and steer the ship through the water. Just as a window fan moves air into a room, a screw propeller moves water causing the ship to move forward. The water moving across the rudder (which is astern of the screw propeller) makes it possible for the rudder to turn the ship.

Several photos are depicted below.

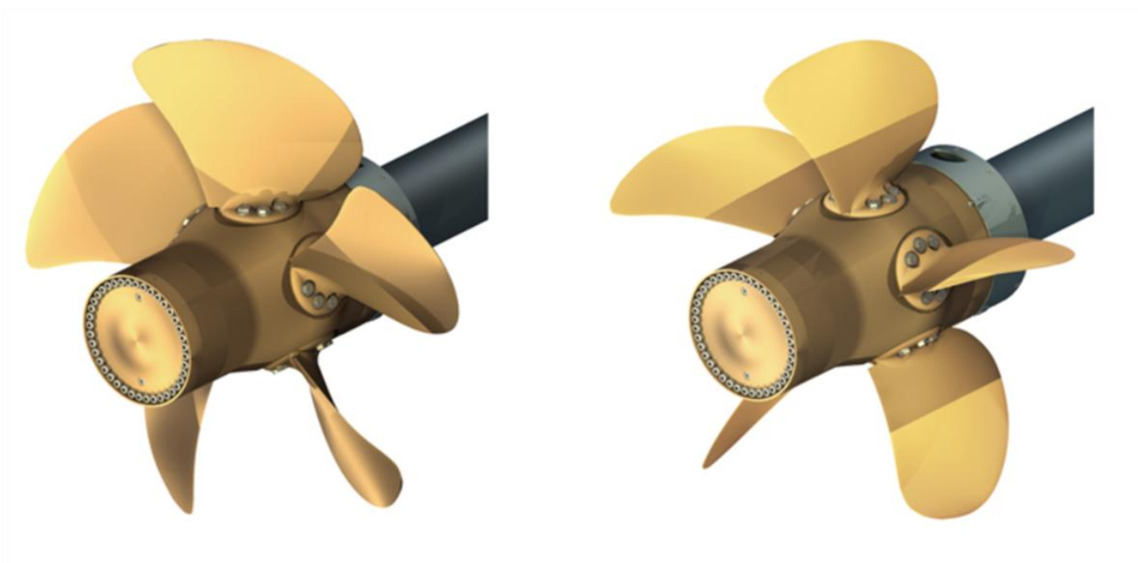
Directions: You will construct a propeller shaft, propeller, and rudder for your ship model.

1. The shaft can be made out of the kabob sticks we have been using.

2. The hub of the shaft should be a cone.

3. The propeller blades should be mounted on the hub 45º from the axis of the shaft as depicted below. There should be four blades, positioned equidistant around the side of the cone.

4. The length of the rudder should be at least the diameter of the circle inscribed by the propeller blades.

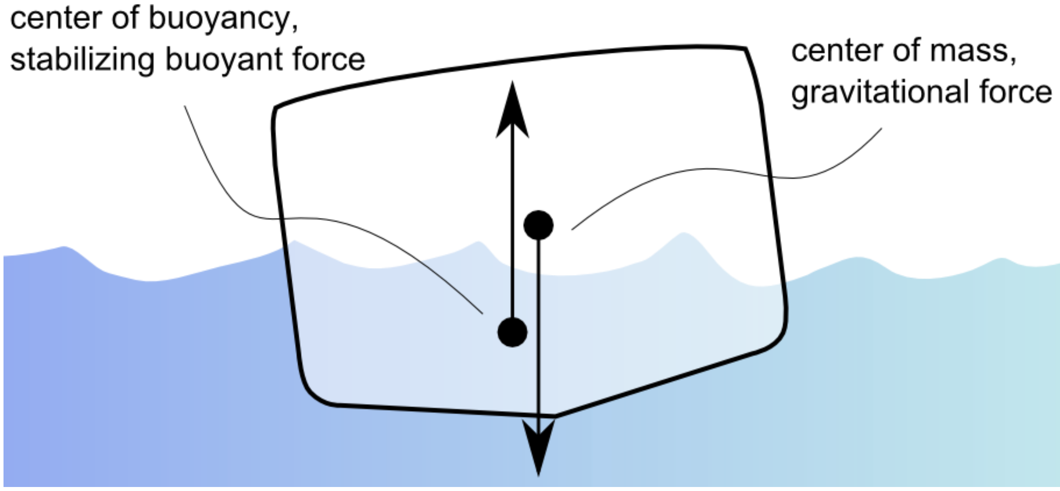


Blades 45 deg offset from axis of shaft

Blades 0 deg offset from axis of shaft

Grade / Content Area	# 8^{th} / 9^{th} Grade Geometry
Lesson Title	**"Why Does a Ship Float?" (1 day)**
Guiding Question	*"Why does a ship float and what do we need to consider when loading it so it won't sink?"*
Content Standards	*State Content Standards:* I. **M(G&M)–8–5:** Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. II. **M(G&M)–8–6:** Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, cylinders, pyramids, or cones. Expresses all measures using appropriate units. III. **M(G&M)-10-2:** Makes and defends conjectures, constructs geometric arguments, uses geometric properties or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g. Pythagorean Theorem, Triangle Inequality Theorem).
	NCTM Standards: Middle and high school students should: I. **Analyze characteristics:** precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship. II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: None for this lesson.
Preparation	I. *Classroom Organization.* Students will work in groups of four in a room with sufficient sinks to provide one for each group.

	II. *Differentiation.* There are many things in this lesson that will appeal to the multiple intelligences of the students: specifically new terminology and hands-on exercises. This lesson also does not involve group work which may be appealing to more introverted students. I will be particularly mindful when grouping students of where I place my English language learners, those with reading comprehension challenges, and those who are already having difficulty with the course material. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials*: A. 4 – 5 sinks filled with water. B. 5 – 7 wooden boards approximately 12" × 4" × 2" (from Home Depot). Each board should have a very long nail hammered into the center to measure the angle of heel. C. *Algeblocks* to represent container boxes and other items stowed on a ship's deck. D. Protractors to measure angle of heel.
Student Learning Objectives	I. Students will apply prior learning about volume and liquid measures to the problem of loading a ship in such a way as not to impair its stability. II. Students will understand that the stability problem involves a geometric relationship and the application of what they have learned in this unit.
Instruction and Engagement	I. *Warm-up* (**10 minutes**): II. *Opening* (**10 minutes**). I will begin with the question, *"Why does a ship float?"* After discussing that for a few minutes, I will introduce some vocabulary before conducting experiments. A. **Center of mass (a.k.a. center of gravity):** The point through which the force due to gravity, that is the weight of the body, acts. B. **Buoyancy:** The upward force acting on a floating or submerged body due to the water pressures on its boundary. C. **Center of buoyancy.** That point through which the buoyancy force acts. I will then show this diagram on a slide and ask the guiding question again. This will lead us into our experiment.



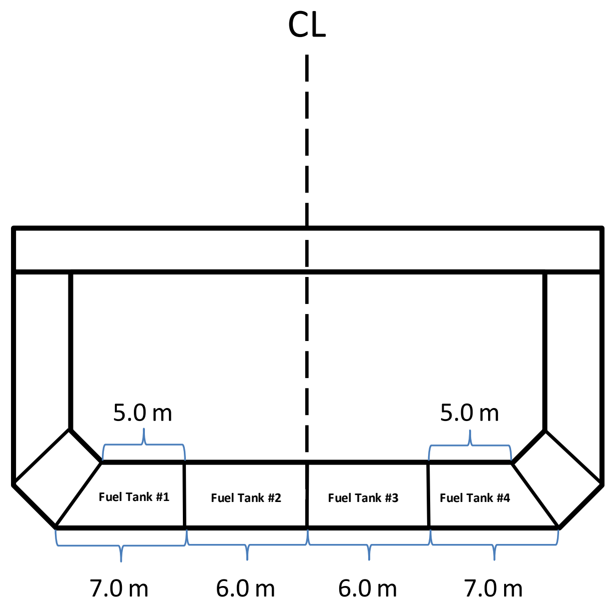
III. *Engagement:* We will perform some experiments in class and see if we can form any hypotheses from them.

A. Each group will work at a sink, equipped with the materials set forth above. They will identify the center of gravity (where the nail is hammered into the board. They will then place *Algeblocks* to determine:

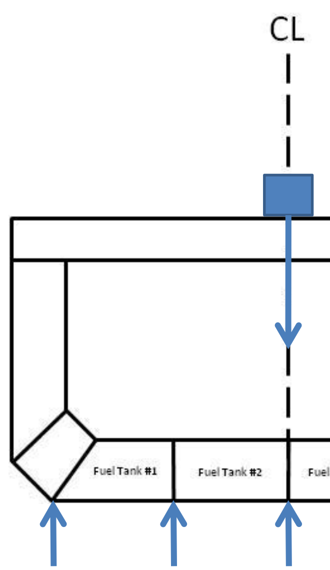
1. How many are necessary to sink the board.

2. How many and where they are placed to capsize the board. Students will measure the angle the board “heels” (using the nail and a protractor), finding the angle at which the board capsizes.

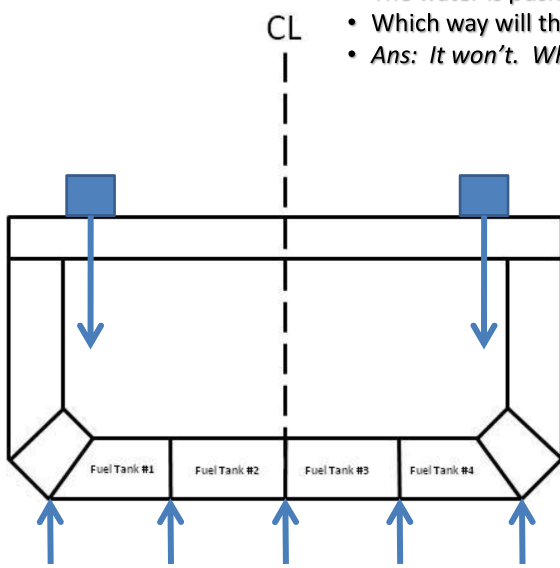
B. I will then ask students to consider the following diagrams (which we saw before in the last lesson) and answer the questions on them:

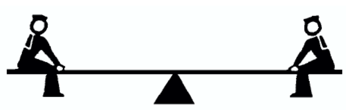


- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?
- *Ans: It won't. Why?*

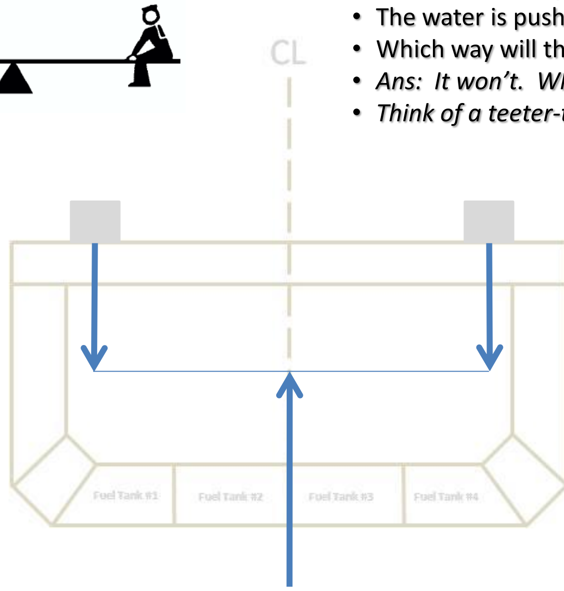


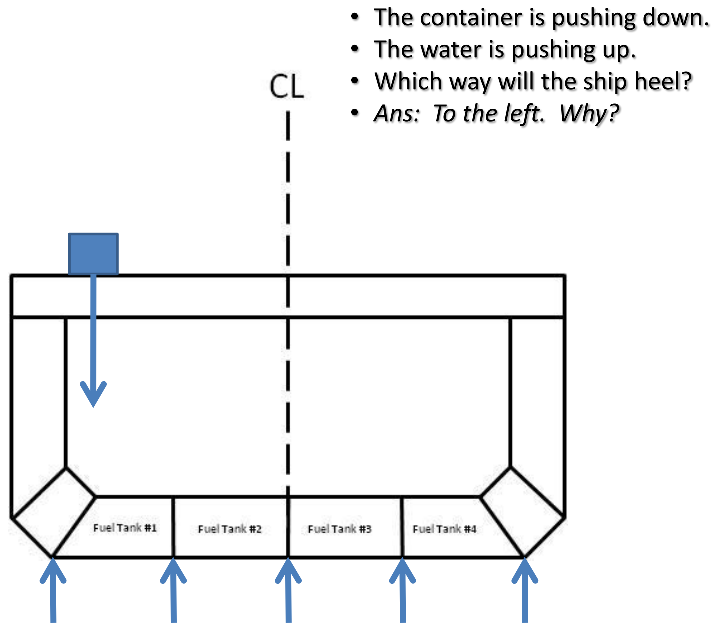
- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?
- *Ans: It won't. Why?*

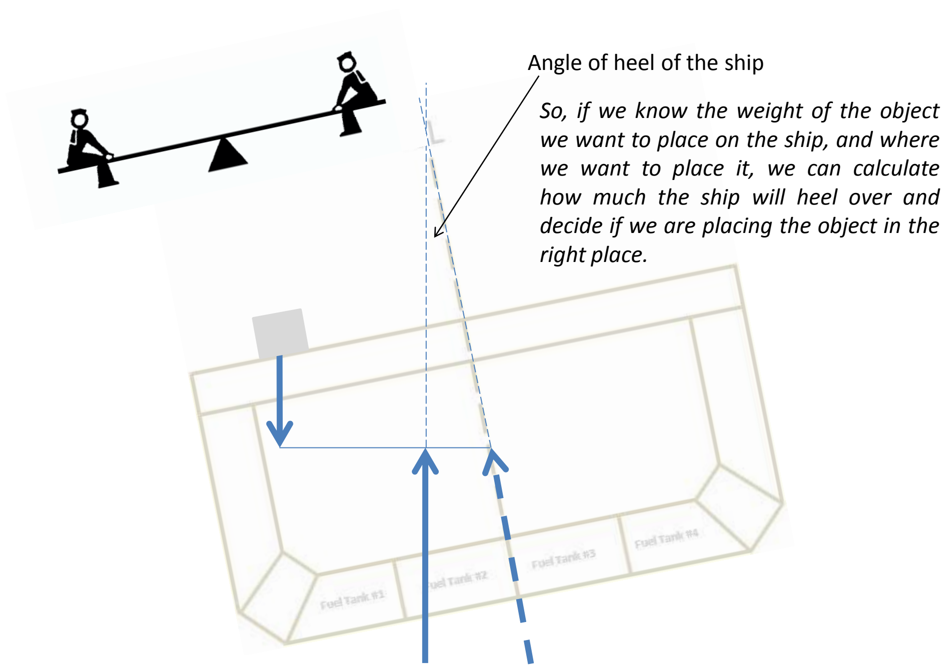




- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?
- *Ans: It won't. Why?*
- *Think of a teeter-totter.*







IV. *Closing.* I will close with the conclusion that what we have is a geometric relationship among the various forces acting on the ship. I will ask the question: *"How can we use what we have learned thus far to help us determine how best to load our ship?"* The answer is that if we can compute the volumes and weights of the objects we put into our packing crates and then into our containers and then onto our ship, we can determine how best to load

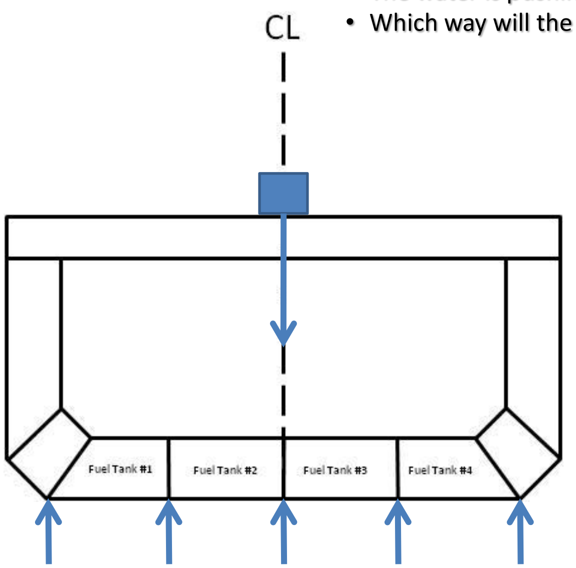
	the ship. *"That is what shippers and sea officers do every day with the hundreds of ships loaded every day."*
Assessment	While this lessons objectives and content are, to some degree, outside of the objectives of this unit, students will be required to draw on their prior learning in this unit to engage in the discussion. During the discussion, I will assess their mastery of the required concepts. I will only assess students on the material relevant to this unit in the summative assessment.

Name: ____________________
Date: ____________________
Class: ____________________

"How Does a Ship Float?"

Directions: Answer the question next to each diagram on the lines to the right of the diagram.

- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?



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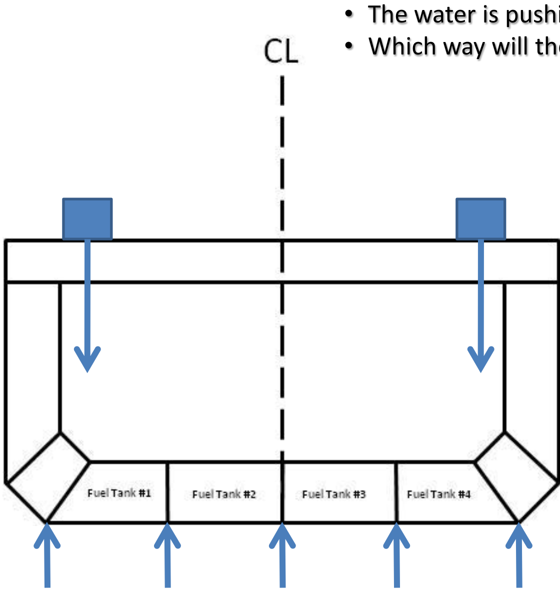
- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?



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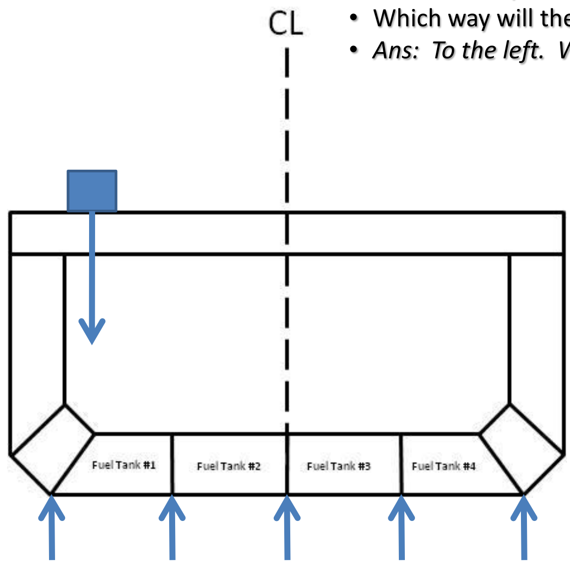
- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?
- *Ans: To the left. Why?*



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1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2. ft^3. etc.)

Name:____________________
Date:_____________________
Class:_____________________

Assessment Review: Surface Area and Volume of 3D Shapes

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. **Definitions:** Place the letter of the term next to the best matching definition.

   a. Circumference: _____ Line segment whose endpoints lie on the circle.

   b. Radius: _____ The sum of the areas of the surfaces of a three-dimensional object.

   c. Diameter: _____ The distance around a closed curve such as a circle.

   d. Chord: _____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

   e. Volume: _____ Any line segment from the center of a circle to its perimeter.

   f. Surface Area: _____ The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? _________________

3. What is the formula for the **area** of a circle _________________

4. What is the formula for the **surface area** of a rectangular prism? _________________

5. What is the formula for the **volume** of a rectangular prism? _________________

6. What is the formula for the **surface area** of a cylinder? _________________

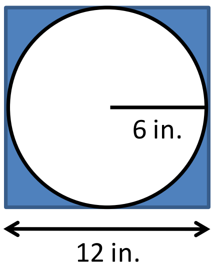
7. What is the formula for the **volume** of a cylinder? __________________

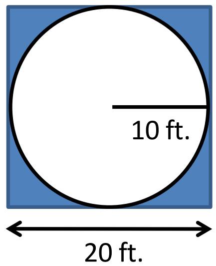
8. What is the general formula for the **volume** of a regular prism or cylinder? __________________

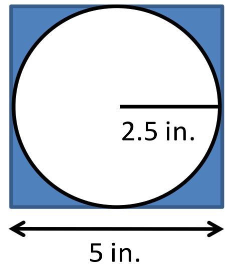
9. What is the general formula for the **surface area** of a regular prism or cylinder?

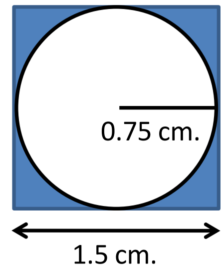
____

10. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)







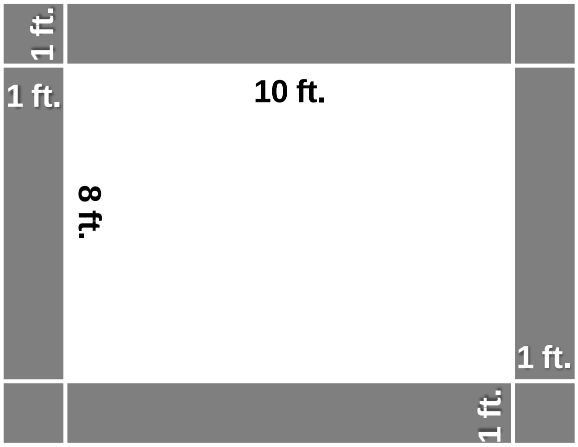


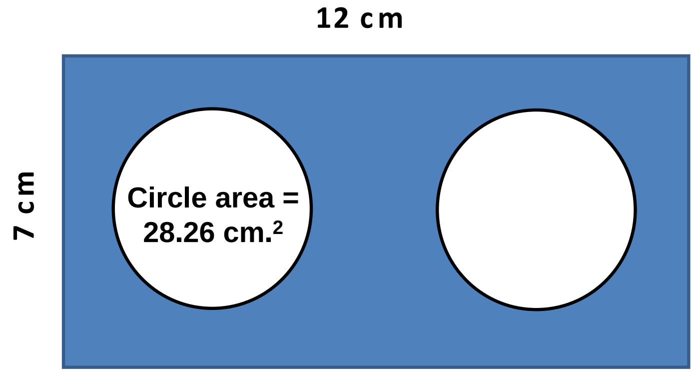
a Circle Area: ________	d Circle Area: ________	g Circle Area: ________	j Circle Area: ________
b Square Area: ________	e Square Area:________	h Square Area:________	k Square Area: _______
c Ratio: ________	f Ratio: ________	i Ratio: ________	l Ratio: ________

m Are c, f, i, and l the same? **Yes** **No** (circle one)

n In your opinion, why do you think the ratios are the same (or different)?

11. Find the area and volume (if asked for) of the following shapes:



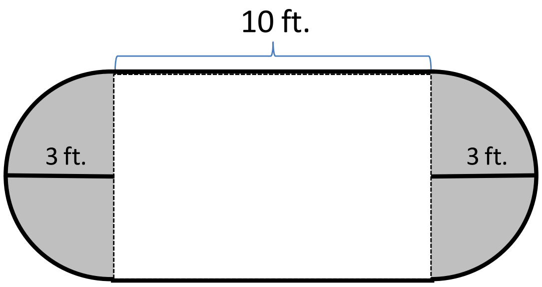


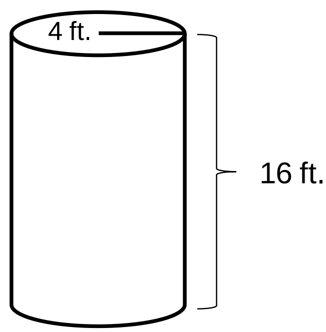
a Total Surface Area: ________

b Area of Shaded Region: ________

c Area of Shaded Region: ________

d Radius of each Circle: ________



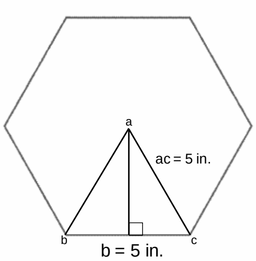


e Area of Shaded Region: ________

f Total Area: ________

g Surface Area: ________

h Volume: ________



Regular hexagonal prism with this hexagon as its base and a height of 40 in.

i Surface Area: ________

j Volume: ________

12. A Coca-Cola can has the following dimensions:

Diameter = 2.25 in.
Height = 5 in.

a. What is the surface area of a Coca-Cola can? ________________

b. What is the volume of a Coca-Cola can? ________________

c. How many Coca-Cola cans fit into a shipping crate that is 27 in. x 18 in. x 10 in. and has a volume of 4860 $in.^3$? ***Think! Remember fitting shapes into the containers. Draw a picture.***

____

d. What is the total volume of all these Coca-Cola cans?

____

e. What is the ratio of the volume of all the Coca-Cola cans to the volume of the container?

____

f. ***Extra Credit:*** *Is this the answer you expected? Why or why not?*

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2. ft^3. etc.)

Differentiated Version

Name:____________________
Date:____________________
Class:____________________

Assessment Review: Surface Area and Volume of 3D Shapes

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

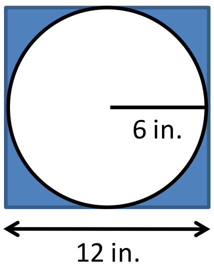
1. **Definitions:** Place the letter of the term next to its definition.

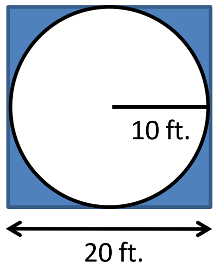
a.	Circumference:	_____	Line segment whose endpoints lie on the circle.
b.	Radius:	_____	The sum of the areas of the surfaces of a three-dimensional object.
c.	Diameter:	_____	The distance around a closed curve such as a circle.
d.	Chord:	_____	Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.
e.	Volume:	_____	Any line segment from the center of a circle to its perimeter.
f.	Surface Area:	_____	The amount of space, inside and out, a solid body occupies.

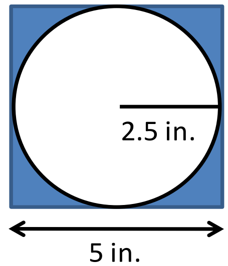
2. What is the formula for the **circumference** of a circle? ________________
3. What is the formula for the **area** of a circle ________________
4. What is the formula for the **surface area** of a rectangular prism? ________________
5. What is the formula for the **volume** of a rectangular prism? ________________
6. What is the formula for the **surface area** of a cylinder? ________________
7. What is the formula for the **volume** of a cylinder? ________________
8. What is the general formula for the **volume** of a regular prism or cylinder? ________________
9. What is the general formula for the **surface area** of a regular prism or cylinder?

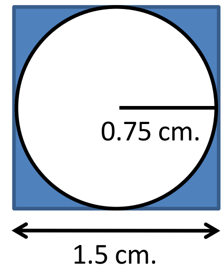
____

10. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)







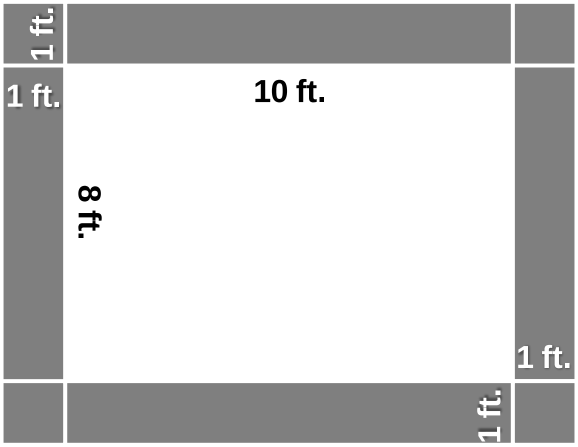


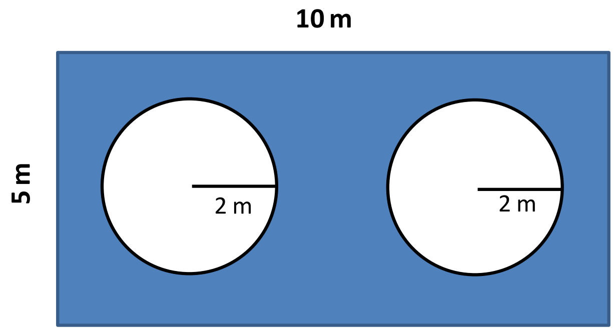
a	Circle Area: ________	d	Circle Area: ________	g	Circle Area: ________	j	Circle Area: ________
b	Square Area: ________	e	Square Area:________	h	Square Area:________	k	Square Area: _______
c	Ratio: ________	f	Ratio: ________	i	Ratio: ________	l	Ratio: ________

m Are c, f, i, and l the same? **Yes** **No** (circle one)

n In your opinion, why do you think the ratios are the same (or different)?

11. Find the area and volume (if asked for) of the following shapes:



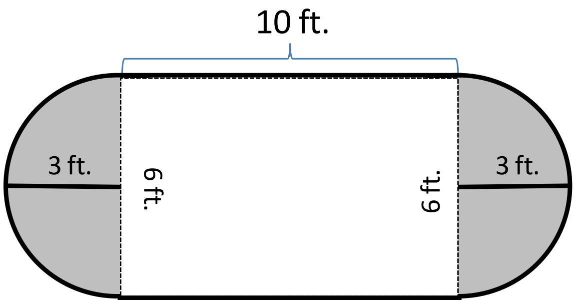


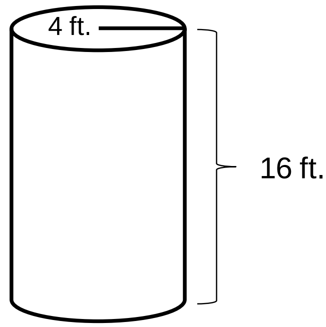
a Total Surface Area: ________

b Area of Shaded Region: ________

c Area of Each Circle: ________

d Area of Shaded Region ________



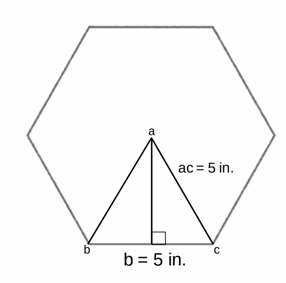


e Area of Shaded Region: ________

f Total Area: ________

g Surface Area: ________

h Volume: ________



Regular hexagonal prism with this hexagon as its base and a height of 40 in.

i Surface Area: ________

j Volume: ________

12. A Coca-Cola can has the following dimensions:

Diameter = 2.25 in.
Height = 5 in.

a. What is the surface area of a Coca-Cola can? ________________

b. What is the volume of a Coca-Cola can? ________________

c. How many Coca-Cola cans fit into a shipping crate that is 27 in. x 18 in. x 10 in. and has a volume of 4860 in.^{3}? ***Think! Remember fitting boxes into the containers. Draw a picture.***

____

d. What is the total volume of all these Coca-Cola cans?

____

e. What is the ratio of the volume of all the Coca-Cola cans to the volume of the container?

____

f. ***Extra Credit:*** Is this the answer you expected? Why or why not?

Answer Key

Assessment Review: Surface Area and Volume of 3D Shapes

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. Definitions.

   a. Circumference: **d**_____ Line segment whose endpoints lie on the circle.

   b. Radius: **f**_____ The sum of the areas of the surfaces of a three-dimensional object.

   c. Diameter: **a**_____ The distance around a closed curve such as a circle.

   d. Chord: **c**_____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

   e. Volume: **b**_____ Any line segment from the center of a circle to its perimeter.

   f. Surface Area: **e**_____ The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? ________________

3. What is the formula for the **area** of a circle ________________

4. What is the formula for the **surface area** of a rectangular prism? ________________

5. What is the formula for the **volume** of a rectangular prism? ________________

6. What is the formula for the **surface area** of a cylinder? ________________

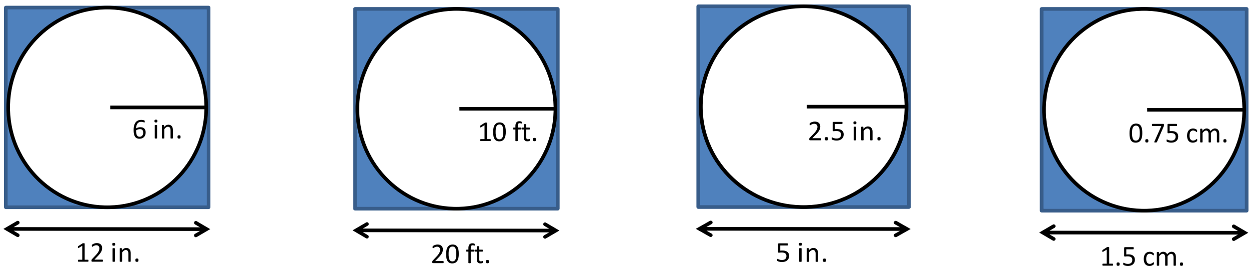
7. What is the formula for the **volume** of a cylinder? _________________

8. What is the general formula for the **volume** of a regular prism or cylinder? _________________

9. What is the general formula for the **surface area** of a regular prism or cylinder?

____

10. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)

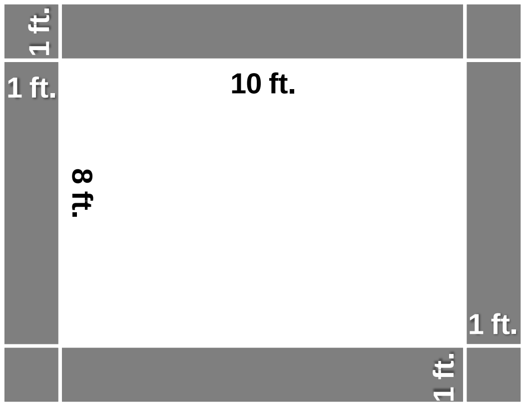


a	Circle Area:	**113.04 in^2**	d	Circle Area:	**314 ft.^{2}**	g	Circle Area:	**19.625 in.^{2}**	j	Circle Area:	**1.77 cm.^{2}**
b	Square Area:	**144 in^2**	e	Square Area:	**400 ft^2**	h	Square Area:	**25 ft.^{2}**	k	Square Area:	**2.25 cm.^{2}**
c	Ratio:	**0.785**	f	Ratio:	**0.785**	i	Ratio:	**0.785**	l	Ratio:	**0.785**

m Are c, f, i, and l the same? **Yes** **No** (circle one)

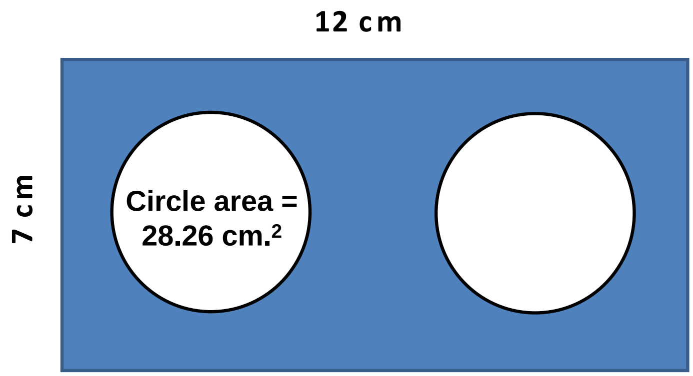
n In your opinion, why do you think the ratios are the same (or different)?

11. Find the area and volume (if asked for) of the following shapes:



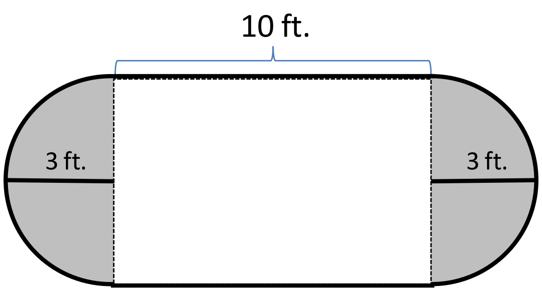
a Total Surface Area: **120 ft.^{2}**

b Area of Shaded Region: **40 ft.^{2}**



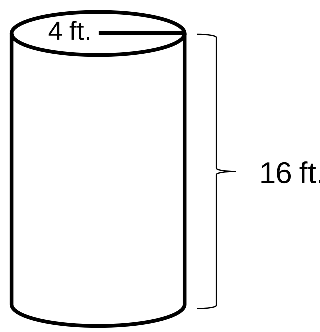
c Area of Shaded Region: **26.8 cm.^{2}**

d Radius of Each Circle **3 cm.**



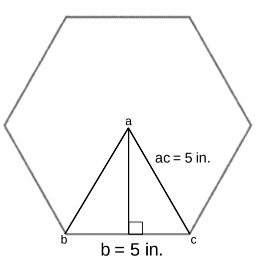
e Area of Shaded Region: **28.26 ft.^{2}**

f Total Area: **88.26 ft.^{2}**



g Surface Area: **502.4 ft.^{2}**

h Volume: **803.84 ft.^{3}**



Regular hexagonal prism with this hexagon as its base and a height of 40 in.

9.i Surface Area:

9.j Volume:

12. A Coca-Cola can has the following dimensions:

Diameter = 2.25 in.
Height = 5 in.

a. What is the surface area of a Coca-Cola can? **43.27 in.^{2}**

b. What is the volume of a Coca-Cola can? **19.87 in.^{2}**

c. How many Coca-Cola cans fit into a shipping crate that is 27 in. x 18 in. x 10 in. and has a volume of 4860 in.^{3}? ***Think! Remember fitting boxes into the containers. Draw a picture.***

192

d. What is the total volume of all these Coca-Cola cans?

3815.04 in.^{3}

e. What is the ratio of the volume of all the Coca-Cola cans to the volume of the container?

0.785

g. ***Extra Credit:*** Is this the answer you expected? Why or why not?

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________
Date:______________________
Advisor:___________________

Assessment: Surface Area and Volume of 3D Shapes

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

* 2 – level Problems *

A. **Definitions (Matching).**

i.	Chord:	_____	Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.
ii.	Diameter:	_____	The distance around a closed curve such as a circle.
iii.	Volume:	_____	Any line segment from the center of a circle to its perimeter.
iv.	Radius:	_____	The sum of the areas of the surfaces of a three-dimensional object.
v.	Surface Area:	_____	The amount of space, inside and out, a solid body occupies.
vi.	Circumference:	_____	Line segment whose endpoints lie on the circle.

2. **Formulas.**

i. What is the formula for the **circumference** of a circle? _________________

b. What is the formula for the **area** of a circle? _________________

c. What is the formula for the **surface area** of a rectangular prism? _________________

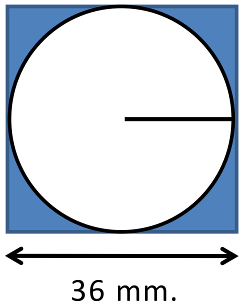
d. What is the formula for the **volume** of a rectangular prism? _________________

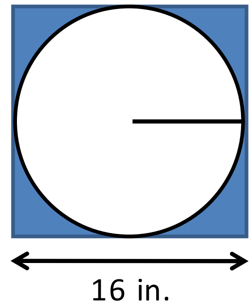
e. What is the formula for the **surface area** of a cylinder? _________________

f. What is the general formula for the **surface area** of a regular prism? _________________

Round all answers to the nearest hundredth

3. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)





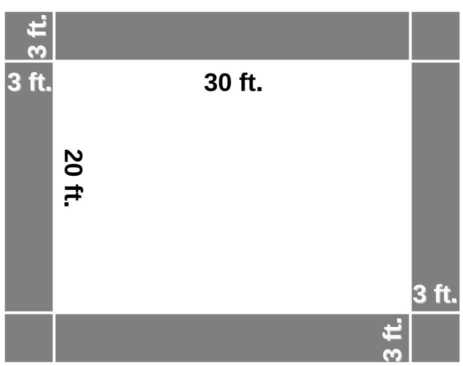
3.a Circle Area: _______ 3.d Circle Area: _______

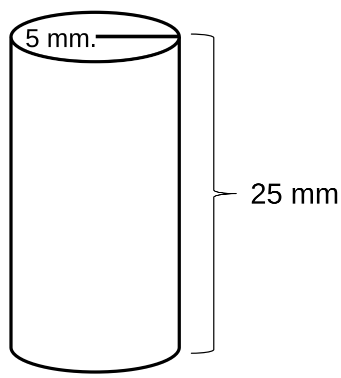
3.b Square Area: _______ 3.e Square Area: _______

3.c Ratio: _______ 3.f Ratio: _______

* 3 – level Problems *

4. Find the area and volume (if asked for) of the following shapes:

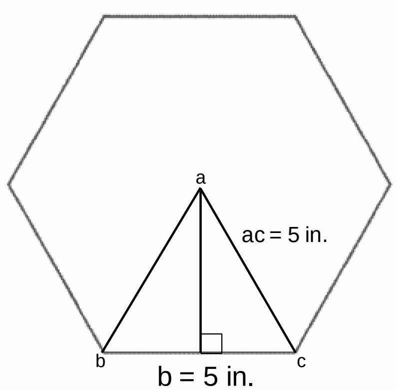




4.a Total Area: _______ 4.c Surface Area: _______

4.b Area of Shaded Region: _______ 4.d Volume: _______

Regular hexagonal prism with this hexagon as its base and a height of 40 in.



4.e Surface Area: _______

4.f Volume: _______

Round all answers to the nearest hundredth

* 4 – level Problems *

5. A tennis ball can has the following dimensions:

   Diameter = 6.6 cm.
   Height = 20 cm.

   a. What is the surface area of a tennis ball can? ________________

   b. What is the volume of a tennis ball can? ________________

   c. How many tennis ball cans fit into a shipping box that is 26.4 cm. x 13.2 cm. x 40 cm. and has a volume of 13,939.2 cm.^{3}?

      Draw a picture before you try to figure it out.

      ____

   d. What is the total volume of all these tennis ball cans?

      ____

   e. What is the ratio of the volume of all the tennis ball cans to the volume of the shipping box?

      ____

   f. Is this the answer you expected? Why or why not?

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Differentiated Version

Name:____________________
Date:_____________________
Advisor:__________________

Test: Circle Area and Circumference / Surface Area and Volume of Cylindrical Prisms
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. **Definitions.**

   a. Chord: _____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

   b. Diameter: _____ The distance around a closed curve such as a circle.

   c. Volume: _____ Any line segment from the center of a circle to its perimeter.

   d. Radius: _____ The sum of the areas of the surfaces of a three-dimensional object.

   e. Surface Area: _____ The amount of space, inside and out, a solid body occupies.

   f. Circumference: _____ Line segment whose endpoints lie on the circle.

2. **Formulas.**

   a. What is the formula for the **circumference** of a circle? _________________

   b. What is the formula for the **area** of a circle? _________________

   c. What is the formula for the **surface area** of a rectangular prism? _________________

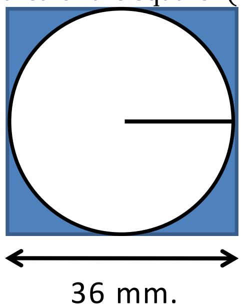
   d. What is the formula for the **volume** of a rectangular prism? _________________

   e. What is the formula for the **surface area** of a cylinder? _________________

   f. What is the general formula for the **surface area** of a regular prism? _________________

Round all answers to the nearest hundredth

6. Find the area of the square and the inscribed circle. What is the **ratio** of the area of the circle to the area of the square $(A_{\text{circle}} \div A_{\text{square}} = ?)$

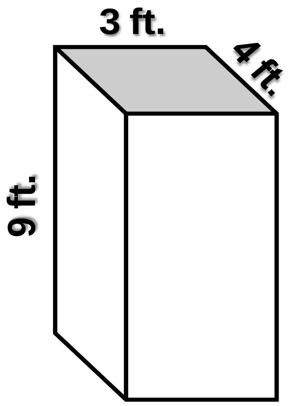


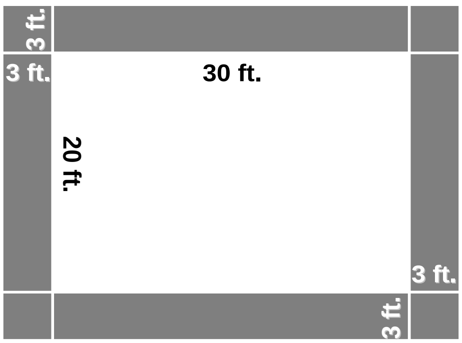
3.a Circle Area: _______

3.b Square Area: _______

3.c Ratio: **100%** **75%** **50%** **(circle one)**

7. Find the area and volume (if asked for) of the following shapes:



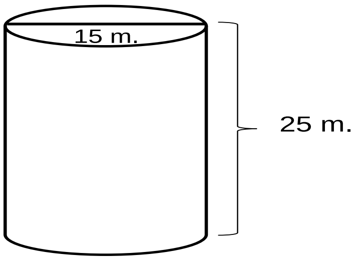


4.a Surface Area: _______

4.b Volume: _______

4.c Total Surface Area: _______

4.d Area of Shaded Region: _______



4.e Surface Are: _______

4.f Volume: _______

Round all answers to the nearest hundredth

8. A tennis ball can has the following dimensions:

Diameter = 6.6 cm.
Height = 20 cm.

f. What is the surface area of a tennis ball can? ________________

g. What is the volume of a tennis ball can? ________________

************* Do c, d, e, and f for extra credit ***********

h. How many tennis ball cans fit into a shipping box that is 26.4 cm. x 13.2 cm. x 40 cm. and has a volume of 13,939.2 cm.^{3}? ***Think! Remember fitting boxes into the containers. Draw a picture.***

____

i. What is the total volume of all these tennis ball cans?

____

j. What is the ratio of the volume of all the tennis ball cans to the volume of the shipping box?

____

g. Is this the answer you expected? Why or why not?

Round all answers to the nearest hundredth

Assessment: Circle Area and Circumference / Surface Area and Volume of Cylindrical Prisms
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. **Definitions.**

   a. Chord: **b**_____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

   b. Diameter: **f**_____ The distance around a closed curve such as a circle.

   c. Volume: **d**_____ Any line segment from the center of a circle to its perimeter.

   d. Radius: **e**_____ The sum of the areas of the surfaces of a three-dimensional object.

   e. Surface Area: **c**_____ The amount of space, inside and out, a solid body occupies.

   f. Circumference: **a**_____ Line segment whose endpoints lie on the circle.

2. **Formulas.**

   a. What are the two formulas for the **circumference** of a circle? $\boldsymbol{2 \times \pi \times r}$ or $\boldsymbol{\pi \times d}$

   b. What is the formula for the **area** of a circle? $\boldsymbol{\pi \times r^2}$

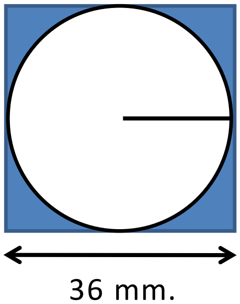
   c. What are the two formulas for the **volume** of a rectangular prism? $\boldsymbol{l \times w \times h}$ or $\boldsymbol{B \times h}$ where $\boldsymbol{B = l \times w}$

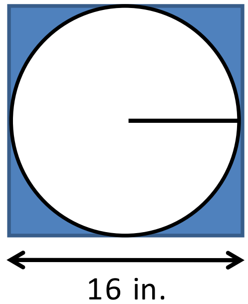
   d. What is the formula for the **surface area** of a cylinder? $\boldsymbol{(2 \times \pi \times r^2) + (2 \times \pi \times h)}$

   e. What are the two formulas for the **volume** of a cylinder? $\boldsymbol{B \times h}$ **or** $\boldsymbol{\pi \times r^2 \times h}$

Round all answers to the nearest hundredth

3. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)





3.a Circle Area: **1017.36 mm^2**

3.b Square Area: **1296 mm^2**

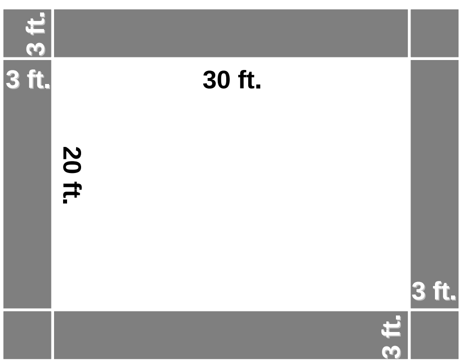
3.c Ratio: **0.79**

3.d Circle Area: **200.96 in^2**

3.e Square Area: **256 in^2**

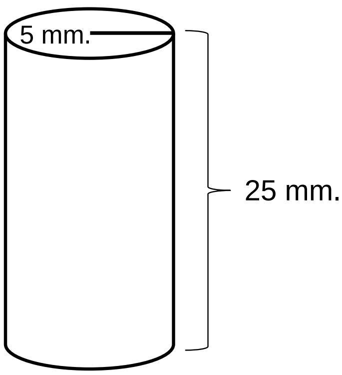
3.f Ratio: **0.79**

4. Find the area and volume (if asked for) of the following shapes:



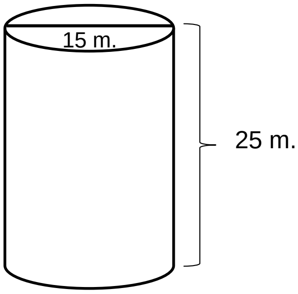
4.a Total Surface Area: **936 ft.^{2}**

4.b Area of Shaded Region: **336 ft.^{2}**



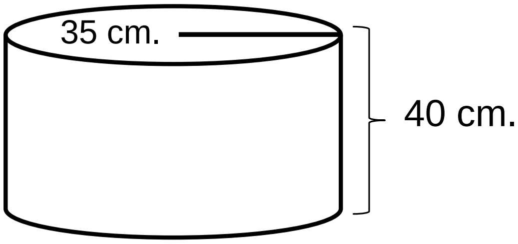
4.c Surface Area: **942 mm.^{2}**

4.d Volume: **1962.5 mm.^{3}**



4.e Surface Area: **1530.75 m.^{2}**

4.f Volume: **4415.6 m^3**



4.g Surface Area: **16,485 cm.^{2}**

4.h Volume: **153,860 cm.^{3}**

5. A tennis ball can has the following dimensions:

Diameter = 6.6 cm.
Height = 20 cm.

a. What is the surface area of a tennis ball can? **482.87 cm.^{2}**

b. What is the volume of a tennis ball can? **683.89 cm.^{3}**

c. How many tennis ball cans fit into a shipping box that is 26.4 cm. x 13.2 cm. x 40 cm. and has a volume of 13,939.2 cm.^{3}? ***Think! Remember fitting boxes into the containers. Draw a picture.***

16 cans

d. What is the total volume of all these tennis ball cans?

10,942.3 cm.^{3}

e. What is the ratio of the volume of all the tennis ball cans to the volume of the shipping box?

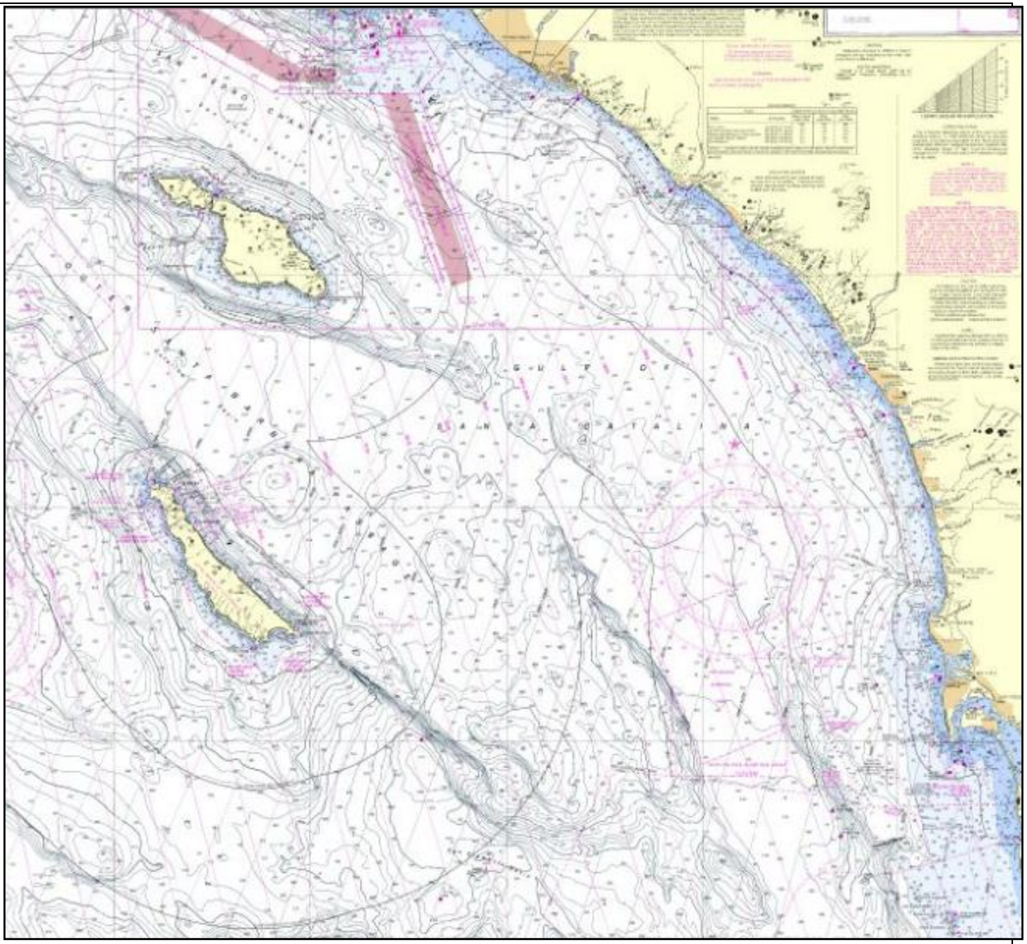
0.79

h. ***Extra Credit:*** Is this the answer you expected? Why or why not?

Grade / Content Area	8^{th} / 9^{th} Grade Geometry
Lesson Title	**Navigation and a Rescue at Sea (3 days)**
Guiding Question	*"How is Geometry used to assist mariners in navigating at sea?"*
Content Standards	*State Content Standards*: I. **M(N&O)–8–4**: Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (Local) II. **M(G&M)–8–2:** Applies the Pythagorean Theorem to find a missing side of a right triangle, or in problem solving situations. (Local) III. **M(F&A)–8–3**: Demonstrates conceptual understanding of algebraic expressions by evaluating and simplifying algebraic expressions (including those with square roots, whole number exponents, or rational numbers); or by evaluating an expression within an equation (e.g., determine the value of y when x = 4 given). (Local) IV. **M(F&A)–8–4:** Demonstrates conceptual understanding of equality by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving formulas for a variable requiring one transformation (e.g., d = rt; d/r = t); by solving multi-step linear equations with integer coefficients; by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of linear equations in a context. (Local)
	NCTM Standards: In middle and high school students should: I. **Specify locations:** use coordinate geometry to represent and examine the properties of geometric shapes; use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides. II. **Use visualization:** draw geometric objects with specified properties, such

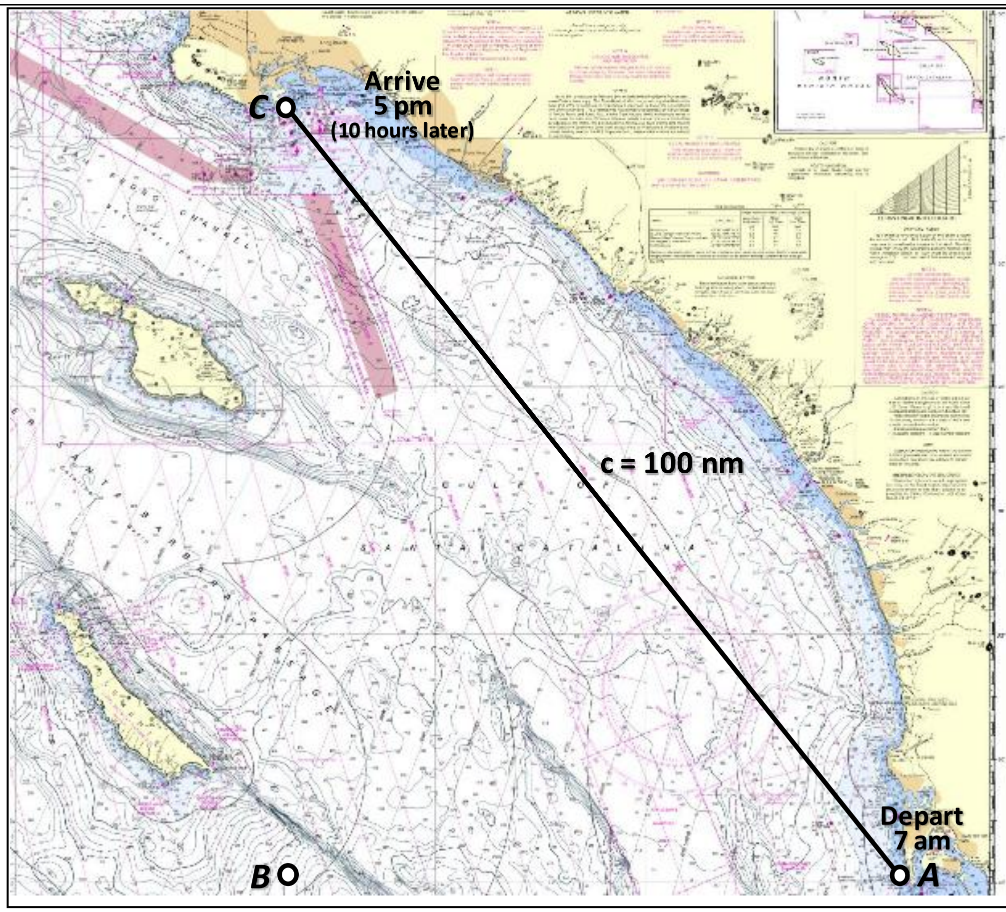
	as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Preparation	I. *Classroom Organization*: Students will work in groups of three or four. Desks will be rearranged to permit four students to work together with a common writing surface. II. *Differentiation*. As before, there are many things in this lesson that will appeal to the multiple intelligences of the students: including new terminology and hands-on exercises. Further working in groups promotes inclusion provided students are organized such that they can leverage off each others' strengths and minimize their individual weaknesses. The culminating exercise is particularly challenging. Mindful of my special needs students, I may (depending on the class and the grouping) prepare worksheets that have some supports (partially filled out answers or hints) for these students. I will only use these if I and the collaborating special needs teacher feel it is appropriate. As always, I will be mindful of those students who have difficulty with comprehension. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials.* A. **Day 1:** For each group of students: 1. Four congruent right triangles cut from construction paper, two red, two orange. The measures of the sides should be 6 inches, 6 inches, and 8.5 inches. 2. One right triangle with sides measuring 5 inches, 12 inches, 13 inches. 3. Calculator. B. **Day 2:** For each group of students:

	1. Pythagorean Review worksheet (one per student). 2. Calculator (one per student). 3. Computer with internet access. C. **Day 3:** For each student: 1. Problem sheet "Navigating a Ship at Sea." 2. Calculator. 3. Ruler.
Student Learning Objectives	I. Given a track laid out on a navigational chart, time and distance, students will determine the velocity a ship needs to achieve to meet a scheduled port arrival. II. Given the original destination and a new intermediate point, students will apply the Pythagorean Theorem to determine the revised distance and required velocity to meet the same scheduled port arrival.
Instructional Procedures	I. *Warm-up* **Day 1** (**10 minutes**): II. *Launch.* **Day 1:** I will begin with the question, *"So, now our ship is at sea enroute to Long Beach. It's a long voyage. How do you think ships navigate at sea?"* a. To answer, I will show them this picture of a nautical cart of the coastal waters of Southern California, describing it as analogous to a road map:



b. *"Navigators plot the paths of their ships (called "tracks") on these maps (called "charts"). For example, let's say we are part of the crew of U.S.S.* Rentz*, a Navy ship stationed in San Diego California.* Rentz *is scheduled to get underway from San Diego at 7 AM and travel 100 nautical miles to the Port of Long Beach. The navigator's track would look something like this* (I will show the chart pictured below with the track laid out).

c. My next question will be, "Rentz *is scheduled to arrive at Long Beach at 5 PM, ten hours later. How fast must she travel?"* The answer is 10 nautical miles per hour. This is a fairly straightforward time – distance problem, similar to the rate problems the students solved during the linear relationships unit in the fall semester. This question serves to review some of the concepts learned in this unit.



d. Now I will propose the problem. *"Our container ship is approaching Long Beach when one of our crew becomes ill. We need to get him off as soon as possible. We call the Coast Guard and they divert* Rentz*, leaving San Diego, to intercept us.* Rentz *will intercept us at Point B, 50 nautical miles west of San Diego, pick up the sick man and take him to Long Beach.*

e. Now comes the question we will answer at the conclusion of the unit, *"If we travel to Point B, stop for one hour, and then turn toward Point C, how fast must the cutter travel to get there by 5 PM? How do we solve this problem?"*

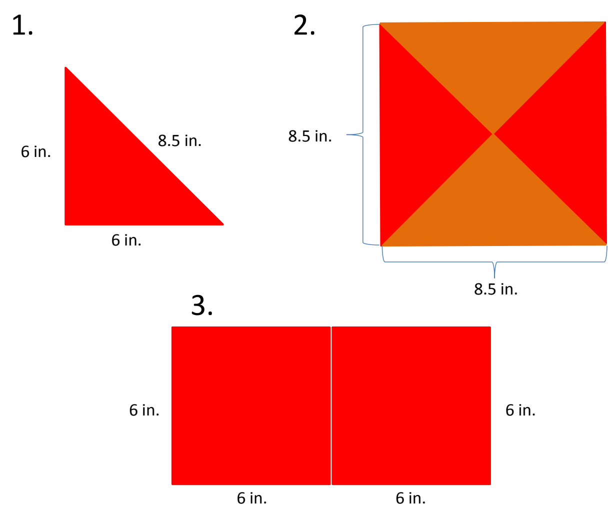
f. Of course, we could simply measure the distances and do the time-distance problem as before but we will explore the triangular relationship to determine an easier way to solve the problem. At this point, I will draw the right triangle ΔCBA and ask what kind of triangle is this. After we identify this as a right triangle, I will ask, *"Does anyone know of a relationship between the sides? Let's explore this relationship."*

	III. *Engagement.* **Day 1:** First, we will examine the sides of such a triangle and speculate on the relationship among the sides. We will conduct an activity that is derived from the following interactive activity found on the NCTM *Illuminations* site: http://illuminations.nctm.org/ActivityDetail.aspx?id=30.

A. Each group of students will have four congruent triangles cut from construction paper, two red and two yellow. They will first measure and record the sides of the triangles: 6 inches, 6 inches, and 5 inches.

B. Given the measurements, students will be asked to speculate on the relationship between the sides. At this point, I would not expect that they will discover the relationship: $6^2 + 6^2 = 8.5^2$.

C. Students will now conduct a group activity. Students will be asked to arrange their triangles as in the following illustration and measure the sides of the new shapes. I will ask prompting questions as I walk around monitoring each group.



1. Question: *"What is the area of the second shape?* Students should be able to determine that it is approximately 72 square inches.

2. Question: *"What is the area of the third shape?* Students should be able to determine that it is also 72 square inches (36 sq. in. + 36 sq. in.).

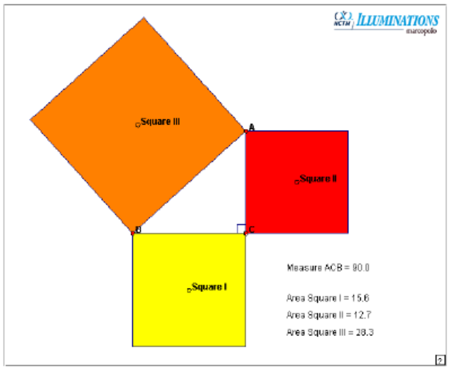
3. Question: *"So, is it correct to say that:*

	$$36 + 36 = 72$$ or $$6^2 + 6^2 = 8.5^2$$ 4. Question: *"So, what is the relationship among the sides of the triangle?* At this point, one or more students will likely get that the some of the squares of the sides equals the square of the hypotenuse or: $$a^2 + b^2 = c^2$$ IV. *Closing.* **Day 1:** We will conclude by going back to the *Illuminations* activity page (http://illuminations.nctm.org/ActivityDetail.aspx?id=30) where I will demonstrate that the relationship holds for all right triangles regardless of the lengths of the sides. I will then leave them with the question: *"Now, let's go back to our example of the Navy ship* Rentz *and our container ship. Is this the same kind of triangle? If we know the measures of two of the sides, can we determine the third and then solve the time-distance problem? We'll look at this over the next couple of days."* V. *Warm-up:* **Day 2** (**10 minutes**): VI. *Launch:* **Day 2:** We will spend a second day on the concept of the Pythagorean Theorem. I will begin this lesson with a review of what we learned on the previous day. VII. *Engagement:* **Day 2:** After I review the basic concepts, student groups will work at computers with internet access, logging on to the NCTM *Illuminations* to the following applet: http://illuminations.nctm.org/ActivityDetail.aspx?ID=164. Using the applet, the following worksheet, and a calculator, students will refamiliarize themselves with the relationship among the sides of a right triangle.

Pythagorean Review

NAME ____________________

You know that the Pythagorean theorem relates the lengths of the sides of a right triangle. But did you also know it is a relationship about areas? On the Illuminations site, open the Pythagorean Review applet. You should see the following right triangle with a square attached to each side.



1. Use your calculator to see how the areas of Squares I and II relate to the area of Square III. You can change the size of the triangle by dragging point A or point B. Investigate the relationship, and write your observations below.

The area of Square III is ______________________________

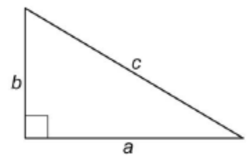
____

____

______________________________.

The Pythagorean theorem states:

> **In a right triangle with legs *a* and *b* and hypotenuse *c*, the sum of the square of the legs is equal to the square of the hypotenuse. That is, $a^2 + b^2 = c^2$.**



On the previous page, you should have noticed that the area of Square I plus the area of Square II is equal to the area of Square III. (If you did not get this result, go back and verify it with your calculator.)

2. How do a^2, b^2, and c^2 relate to the area of each square? Briefly explain your thinking.

3. What is meant by the statement, "The Pythagorean theorem is a relationship of areas?"

VIII. *Closing:* **Day 2:** We will now share our findings in this exploratory exercise. Once we complete this activity, we will conclude the lesson with some fairly straightforward equations, solving exercises to ensure students are comfortable with manipulating the Pythagorean equation. This will set the stage for the next day's activity. We will discuss how to solve for each side of the triangle, i.e.:

$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$
$$b^2 = c^2 - a^2$$

	I will then remind students of our original problem, solving the time-distance problem for the medical rescue at sea and leaving them to consider, with their new learning, how they might solve it. IX. *Warm-up:* **Day 3** (**10 minutes**): X. *Launch:* **Day 3:** After a brief review of the concept of the Pythagorean Theorem using the *Illuminations* applet students worked with on the previous day, we will embark on the main exercise of the lesson: solving the time distance problem. XI. *Engagement:* **Day 3:** Students, working in groups, will solve the problems on the worksheet "Navigating a Ship at Sea." This is a fairly challenging task because it integrates the concepts of the Pythagorean Theorem and linear relationships (both learned this year) into a real-world problem with new terminology. This is not unlike the lesson I taught earlier in the year on linear relationships, using the space shuttle as my example. However, I will be mindful of this challenge as I walk around the classroom, observing and helping with student work. XII. *Closing:* **Day 3:** We will share our findings and review the answers. Depending on how long it takes students to solve these problems, I may allow them to finish the exercise as homework, reviewing it the following day. I will close the lesson with the following question, *"OK, so now we have seen how we can use the Pythagorean Theorem to compute the lengths of the sides of a right triangle – using that skill to solve a real-world problem...but what can we do if the triangle is NOT a right triangle?"* We will not cover this topic this year but I will discuss it with interested students.
Assessment	The completed worksheets from days 2 and 3 will serve as informal assessments. For the day 3 exercise, I will assess if the: 1. Student correctly assesses that velocity is a factor of distance and time and applies this knowledge to determine the speed necessary for the ship to achieve an on-time port arrival. I will also assess if the time-distance problem is correctly set up, arithmetic is correct resulting in a correct answer. 2. Response has no flaws in reasoning or computation. Student understands how to use the Pythagorean Theorem to get correct solutions. It contains

	solid explanations on how the student determined the length of segements "a" and "b" and used that information to determine the length of the hypotenuse "c". Response communicates student's thoughts clearly and effectively. 3. Student correctly assesses that application of the Pythagorean Theorem is appropriate to determine distances. Student demonstrates understanding of the relationship among distance, time, and velocity in a practical voyage planning problem. All work is shown, arithmetic is correct and correct answers are arrived at.

Coastal Piloting of a U. S. Navy Ship

____

Name

____

Class

____

Date

Coastal Piloting of a U. S. Navy Ship

I. You are the Navigator of the U. S. Navy ship USS *Rentz* (pictured below). *Rentz* is scheduled to depart the U. S. Naval Station at San Diego, California (point A on the *navigation chart* on page 3 below) at 7 AM tomorrow morning and travel to the U. S. Naval Weapons Station at Long Beach, California (point C on the navigation chart) to take on ammunition for an upcoming deployment to the Arabian Gulf. You must arrive at the Weapons Station no later than 5 PM tomorrow afternoon. Long Beach is 100 *nautical miles* from San Diego.

II. Background Information.

A. **Nautical Mile**: The most common measure of distance at sea. 1 nautical mile (nm) = 1.15 "land miles."

B. **Knots**: A measure of speed used by ships. One knot = one nautical mile per hour.

C. **Navigation Chart**: A map used by Navigators to find where they are going and how to get there at sea.

III. At what speed (in *knots*) do you recommend *Rentz* travel to get from San Diego to Long Beach on time?

- *Ans:* $\frac{100nm}{10hrs} = \frac{x\ nm}{1hr} = 10\frac{nm}{hr} = 10\ nautical\ miles\ per\ hour = 10\ knots$

IV. At 6 AM, one hour before you sail, the Captain tells you that *Rentz* has been ordered to travel due West to point B (just south of San Clemente Island) to pick up a sick man on a containership that is traveling to Long Beach. Point B is 50 nautical miles West of San Diego.

A. Once you pick up the sick crewman at point B you turn 90 degrees to the North. How far (in nautical miles) must *Rentz* travel due North to get to Long Beach (point C)?

- *Ans: Since you traveled due West to point B and then turned 90 degrees to due North, you can use Pythagorean Theorem.*

$$a^2 + b^2 = c^2$$

$$50^2 + b^2 = 100^2$$

$$2500 + b^2 = 10000$$

$$b^2 = 10000 - 2500 = 7500$$

$$b = \sqrt{7500} = 86.6\,nm$$

B. What is the total distance (in nautical miles) *Rentz* will need to travel to pick up the sick man and then proceed to Long Beach?

- *Ans:*

$$50\,nm + 86.6\,nm = 136.6\,nm$$

C. If you leave San Diego on time (7 AM), at what speed (in knots) will *Rentz* need to travel to get from San Diego to point B and then from point B to Long Beach on time (by 5 PM) if you stop at point B for one hour to pick up the sick man?

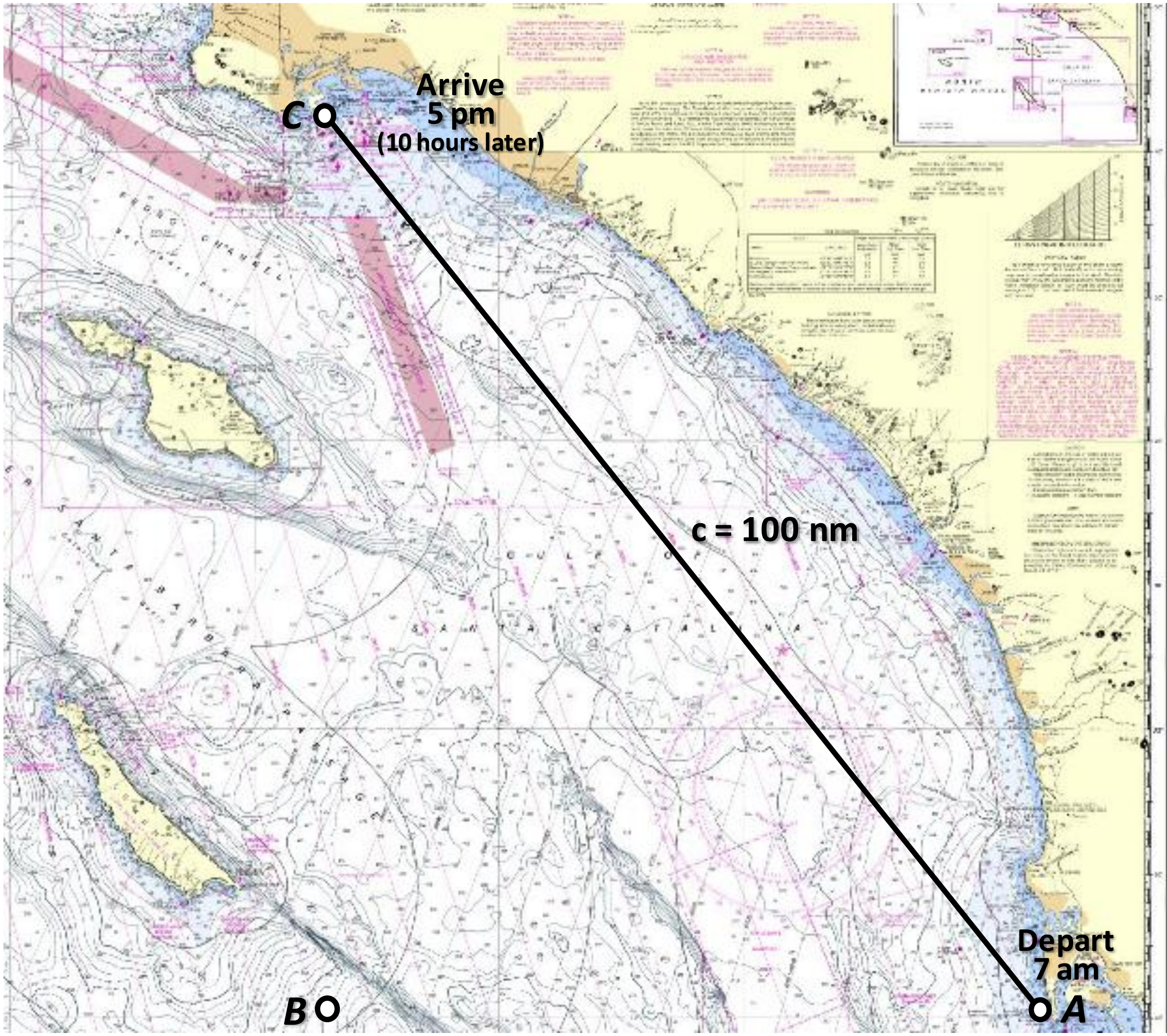
- *Ans:*

  - *First, determine how many hours you will actually be traveling.*

$$10\ hours\ (7\ AM - 5\ PM) - \ 1\ hour\ (to\ pick\ up\ the\ fishermen) = \ 9\ hours$$

  - *Next, compute the speed Rentz will need to make to get to Long Beach on time.*

$$\frac{136.6\,nm}{9\ hours} = \frac{x\ nm}{1\ hour} = 15.2\ nautical\ miles\ per\ hour = 15.2\ knots$$

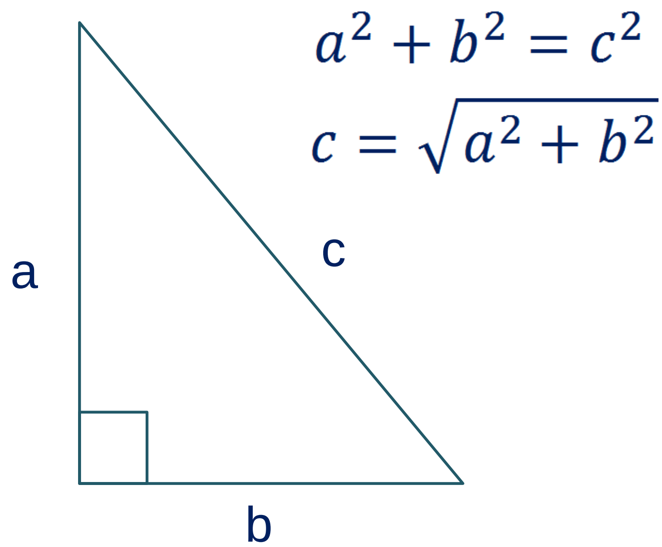

Arrive
5 pm
(10 hours later)
C
c = 100 nm
Depart
7 am
A
B

Grade / Content Area	**Grade 8 / 9 Algebra and Geometry**
Lesson Title	**A Rescue At Sea Using the Distance Formula**
Guiding Question	*"How is Geometry used to assist mariners in navigating at sea?"*
Content Standards	*State Content Standards:* I. **M(N&O)–8–4**: Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (Local) II. **M(G&M)–8–2:** Applies the Pythagorean Theorem to find a missing side of a right triangle, or in problem solving situations. (Local) III. **M(F&A)–8–3**: Demonstrates conceptual understanding of algebraic expressions by evaluating and simplifying algebraic expressions (including those with square roots, whole number exponents, or rational numbers); or by evaluating an expression within an equation (e.g., determine the value of y when x = 4 given). (Local) IV. **M(F&A)–8–4:** Demonstrates conceptual understanding of equality by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving formulas for a variable requiring one transformation (e.g., d = rt; d/r = t); by solving multi-step linear equations with integer coefficients; by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of linear equations in a context. (Local)
	NCTM Standards: In middle and high school students should: I. **Specify locations:** use coordinate geometry to represent and examine the properties of geometric shapes; use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides.

	II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Preparation	I. *Classroom Organization*: Students will work in groups of three to five. Desks will be rearranged to permit students to work together with a common writing surface. II. *Differentiation*. Working in groups promotes inclusion provided students are organized such that they can leverage off each others' strengths and minimize their individual weaknesses. I will be particularly mindful when grouping students of where I place my English language learners, those with reading comprehension challenges, and those who are already having difficulty with the course material. III. *Materials*. One worksheet for each student.
Student Learning Objectives	I. Correctly identify all inputs to a function from a given word problem. II. Understand that maps such as nautical charts are used to graphically portray special coordinates and distances. III. Apply the Pythagorean Theorem to determine distances in a spatial coordinate system. IV. Derive the Distance Formula from application of the Pythagorean Theorem in a spatial coordinate system.
Instruction and Engagement	*Launch (Opening):* I. *How do ships navigate at sea – where everything looks the same? If a ship needs help at sea, how do we find him?* I'll give students an opportunity to think about this and offer ideas. II. *Today we are going to solve a navigation problem involving a rescue at sea.* III. *Who can tell me what the Pythagorean Theorem is?* I will give students an

opportunity to offer answers. I will then ensure the following concept is clear to all students:

A. *Given a right triangle, the length of the hypotenuse is equal to the square root of the sum of the squares of the two sides.*



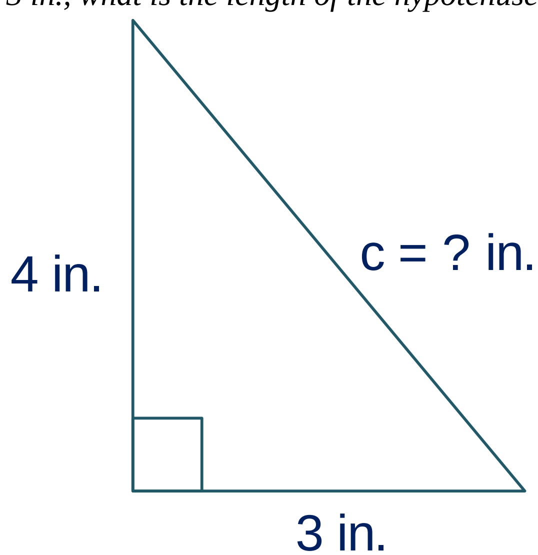
B. *Normally we write this as:*

$$a^2 + b^2 = c^2$$

C. *But it can also be written as:*

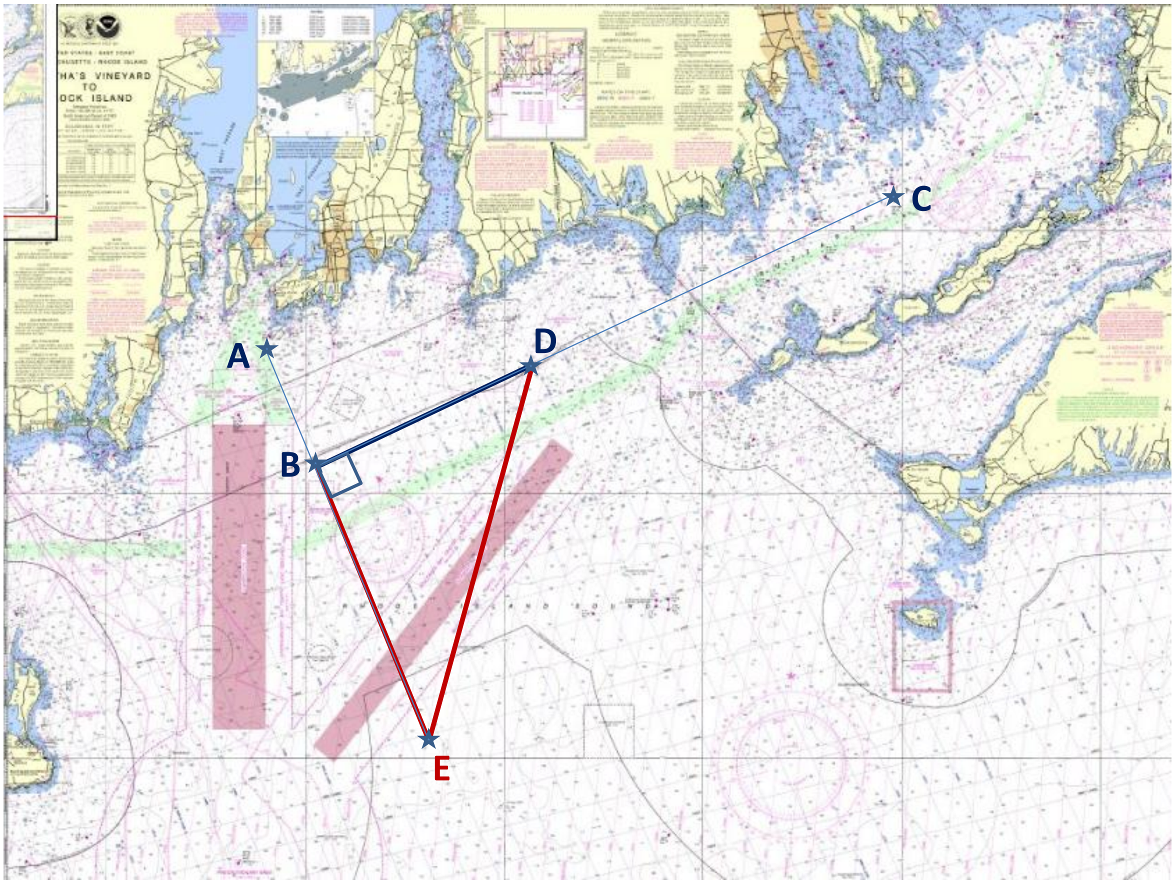
$$c = \sqrt{a^2 + b^2}$$

D. *Here is an example: Given a right triangle with side a = 4 in. and side b = 3 in., what is the length of the hypotenuse c?*



	$$c = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$ IV. *Now let's look at our rescue at sea.* Pass out performance task.
	Explore (Engagement): I. I will hand out the worksheet and allow students to begin working on the problems in groups while I walk around, monitor, and ask questions such as: A. *To answer the first two questions, how would you organize the information that you do know?* B. *What information about this triangle helps you decide how to answer the third question? What is the equation we would use to solve for this distance?* C. *What is the relationship between x_2 and x_1? How about y_2 and y_1? How would you substitute x_1, x_2, y_1, y_2 into are equation to solve for the distance between Point D and Point E?*
	Summarize / Share (Closure): To conclude, we will consider the following question: *What does a Cartesian coordinate system allow us to do? Think of a circle. If we plot it in a Cartesian coordinate system, how could we express that circle?*
Assessment	I. I will assess student understanding by monitoring progress in completion

Name: ______________________________
Date: ______________________________



At 2 PM on Tuesday, June 8th, U. S. Coast Guard Southeastern New England headquarters receives a distress call from a fishing vessel located 16 nautical miles south, southeast of the entrance to Narragansett Bay (Point E). The vessel is sinking and will not remain afloat for more than four hours. The Coast Guard headquarters dispatches two cutters, USCGC *Tiger Shark* from Newport, Rhode Island and USCGC *Hammerhead* from New Bedford, Massachusetts to find and rescue the fishing boat.

At 5 PM:

A. *Tiger Shark* is at Point B which is 26 nautical miles from the entrance to New Bedford Harbor (Point C) and 5 nautical miles from the entrance to Narragansett Bay (Point A).

B. *Hammerhead* is at Point D which is 17 nautical miles from the entrance to New Bedford Harbor (Point C).

C. The fishing boat is at point E which is 16 nautical miles from the entrance to Narragansett Bay (Point A).

You must rendezvous with the fishing boat by 6 PM in order to have time to get the crew off before the boat sinks.

1. How far is *Hammerhead* (Point D) from *Tiger Shark* (Point B)?

$$26\ nm - 17\ nm = 9\ nm$$

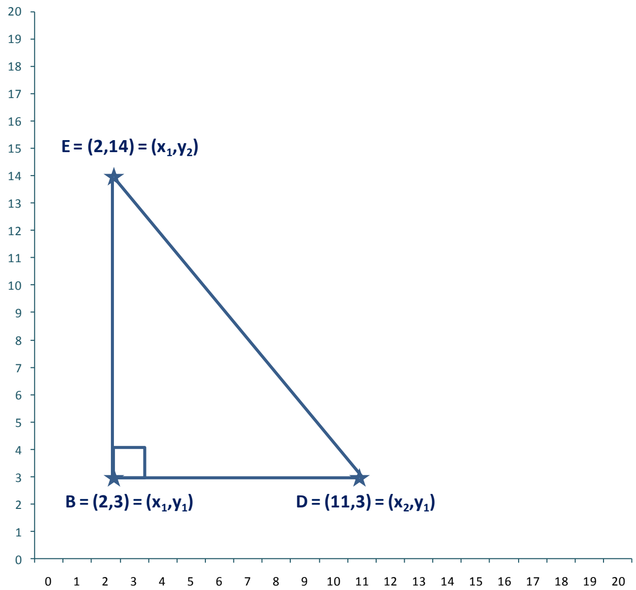
2. How far is *Tiger Shark* (Point B) from the fishing boat (Point E)?

$$16\ nm - 5\ nm = 11\ nm$$

3. How far is *Hammerhead* (Point D) from the fishing boat (Point E)?

$$c = \sqrt{9^2 + 11^2} = \sqrt{81 + 121} \cong 14.21$$

4. Now, let's put that triangle on a coordinate grid so that it looks like the picture below. Can you derive a formula for the distance between Point D and Point E expressed in terms of x_i and y_i?



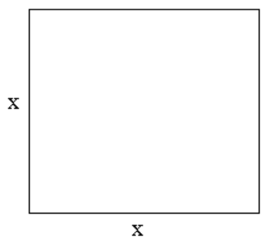
$$c = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$$

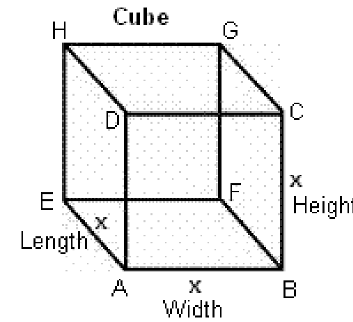
5. *Bonus:* If both *Tiger Shark* and *Hammerhead* have one hour to get to the fishing boat, how fast does each ship need to go?

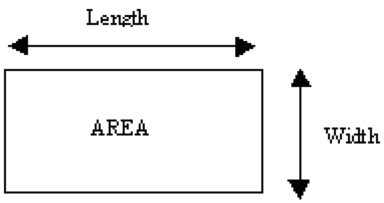
Hammerhead: $\frac{14.21nm}{1hr} = 14.21\frac{nm}{hr}$

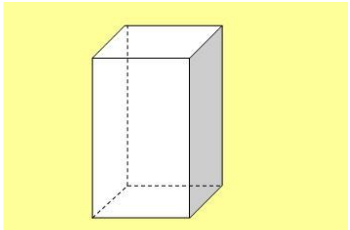
Tiger Shark: $\frac{11nm}{1hr} = \frac{11nm}{hr}$

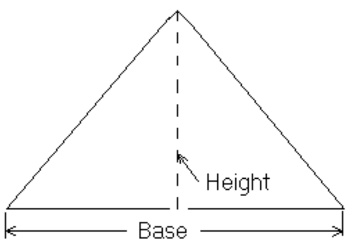
Surface Area and Volume of 3D Shapes – *Save this forever!!!!!!*

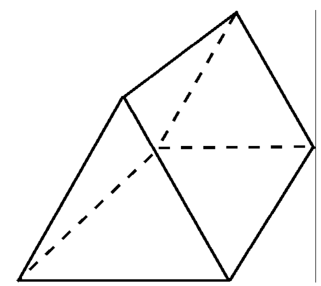




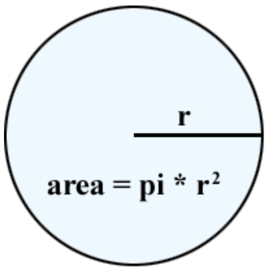


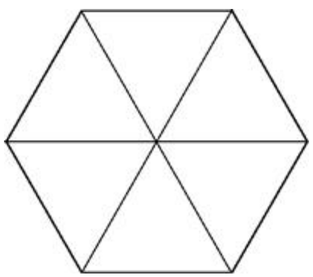




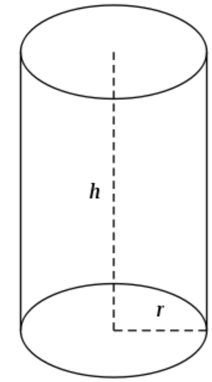


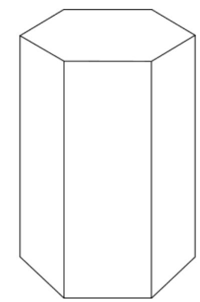
Area of a Square: $\boldsymbol{A = s^2}$ where s = the length of a side.	***Right Square Prism:*** • $\boldsymbol{SA = 2(s^2) + 4(sH)}$ • $\boldsymbol{V = s^2 H}$ ...where s = the length of a side and H = height of the prism. ***Right Square Pyramid:*** • $\boldsymbol{SA = s^2 + 4\left(\frac{1}{2}b \times sh\right)}$ • $\boldsymbol{V = \frac{1}{3}sH}$ ...where s = the length of a side and H = height of the prism.
Area of a Rectangle: $\boldsymbol{A = l \times w}$ where l = the length of the rectangle and w = the width.	***Right Rectangular Prism:*** • $\boldsymbol{SA = 2\big((lw) + (lH) + (wH)\big) = 2B + PH}$ • $\boldsymbol{V = l \times w \times H = PH}$ ...where l = the length, w = the width, P = perimeter of the base, H = height of the prism.
Area of a Triangle: $\boldsymbol{A = \frac{1}{2}bh}$ where b = the base of the triangle and h = the height of the triangle.	***Right Triangular Prism:*** • $\boldsymbol{SA = 2\left(\frac{1}{2}bh\right) + (S_1 + S_2 + S_2)H}$ • $\boldsymbol{V = \left(\frac{1}{2}bh\right)H}$ ...where S_i is the length of side i, b is the length of the base of the base triangle, h is the height of the base triangle, and H is the height of the prism. ***Right Triangular Pyramid:*** • $\boldsymbol{SA = \frac{1}{2}bh + 3(\frac{1}{2}b \times sh)}$ • $\boldsymbol{V = \frac{1}{3}\left(\frac{1}{2}bh\right)H}$ ...b is the length of the base of the base triangle, h is the height of the base triangle, sh is the height of each side triangle, and H is the height of the pyramid.

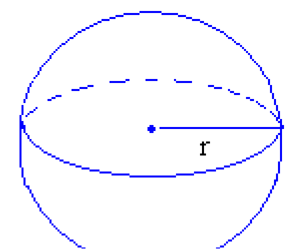


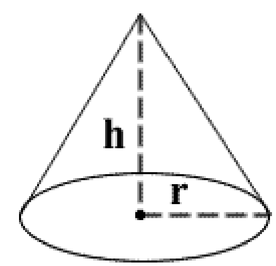


Area of a Circle:	$\boldsymbol{A = \pi r^2}$ where r = the radius of the circle.	***Right Cylinder:*** • $\boldsymbol{SA = 2\pi r^2 + 2\pi rH}$ • $\boldsymbol{V = \pi r^2 H}$ ...where r is the radius of the base circle and H is the height of the cylinder. ***Right Cone:*** • $\boldsymbol{SA = \pi r^2 + \pi r \times \sqrt{r^2 + H^2}}$ • $\boldsymbol{V = \frac{1}{3}\pi r^2 H}$ ...where r is the radius of the base circle and H is the height of the cylinder. ***Sphere:*** • $\boldsymbol{SA = 4\pi r^2}$ • $\boldsymbol{V = \frac{4}{3}\pi r^3}$ ...where r is the radius of the sphere.
Area of a Regular Hexagon:	$\boldsymbol{A = \frac{1}{2}aP}$ where a = the apothem and P = the perimeter of the base.	***Right Regular Hexagonal Prism:*** • $\boldsymbol{SA = 2 \times (\frac{1}{2}aP) + PH}$ • $\boldsymbol{V = \frac{1}{2}aP \times H}$ ...where a = the apothem, P = the perimeter of the base, and H = the height of the prism.
General Formulas		***Prism and Cylinder:*** • $\boldsymbol{SA = 2B + PH}$ • $\boldsymbol{V = BH}$ ...where B is the area of the base, P is the perimeter of the base, and H is the height of the prism. ***Pyramid:*** • $\boldsymbol{SA = B + N(\frac{1}{2}b \times sh)}$ • $\boldsymbol{V = \frac{1}{3}BH}$ ...where B is the area of the base, N is the number of sides, b is the length of the base of a side triangle, and sh is the slant height of the side triangle, and H is the height of the pyramid.









PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth

The Naval Battle of Midway

Operations Research and Mathematical War Gaming

Thomas R. Beall
Captain, U. S. Navy (Ret.)

I. **Background.** The final project will be in multiple parts and will be completed over the next three months. The project will fall under a historical scenario that will unfold as each part is given to you.

II. **Format.** Each part will include instructions, some of the required materials, and all the information you will need to complete that part. It will be necessary to complete each part to move on to the next part because the answers to each part will be required when working on the next part.

III. **Grading.** This project will make up one half of your final course grade. If you do not complete all parts, you will not pass the third trimester of 9th grade mathematics.

Dear Paul Cuffee 9th Grade Parent,

With the start of the Third Trimester, your son or daughter will begin the 9th Grade Final Mathematics Project. This project is in several parts, called tasks, each of which is an assignment designed to challenge him or her to put the mathematics skills learned this year to practical use. It is also designed to help your son or daughter build the self – confidence and resiliency to take on and complete challenging tasks on a fixed deadline. It is, therefore, very important that your son or daughter complete each part by its due date.

The Final Mathematics Project brings together history and mathematics by exploring the geography, ships, and men involved in the World War II Naval Battle of Midway. In June of 1942, two great fleets, one Japanese and one American, clashed near the small island of Midway in the Central Pacific Ocean. Both sides realized that the outcome of the Pacific War might depend on the outcome of this battle. To gain an advantage, each side applied mathematical techniques to all aspects of the battle, techniques which your son or daughter will have the opportunity to practice. The project concludes with a four – day war game played by all students to refight the battle. We also commemorate the anniversary of the battle (June 4th) with a guest speaker and a celebratory cake.

Attached to this letter is a list of all project tasks and the dates they are due to me. Late submission will result in assignment of a grade one level lower than otherwise would have been assigned for the work completed. Please ensure your son or daughter completes assigned tasks on time.

Please feel free to contact me with any questions or concerns. My e-mail address is tbeall@paulcuffee.org.

Sincerely,

Thomas R. Beall
Captain, U. S. Navy (Ret.)

Estimado Padre/Encargado de 9no Grado del Escuela Paul Cuffee,

Con el inicio del tercer trimestre, su hijo o hija comenzará el Proyecto Final de Matemáticas de 9no Grado. Este proyecto consta de varias partes, llamadas tareas, cada una de las cuales está diseñada para retarlo(a) a poner las habilidades matemáticas aprendidas este año en uso práctico. El proyecto también está diseñado para ayudar a su hijo(a) a desarrollar confianza en sí mismo(a) y flexibilidad para asumir y completar tareas difíciles en un plazo determinado. Es, por lo tanto, muy importante que su hijo(a) complete cada parte en la fecha de entrega.

El Proyecto Final de Matemáticas reúne la historia y las matemáticas mediante la exploración de la geografía, los barcos y los personajes involucrados en la Batalla Naval de Midway en la Segunda Guerra Mundial. En junio de 1942, dos grandes flotas, una japonesa y una estadounidense, se enfrentaron cerca de la pequeña isla de Midway, en el Océano Pacífico Central. Ambas partes se dieron cuenta de que el resultado de la Guerra del Pacífico podría depender del resultado de esta batalla. Para obtener una ventaja, cada parte aplicó técnicas matemáticas en todos los aspectos de la batalla, técnicas que su hijo(a) tendrá la oportunidad de practicar. El proyecto concluye con un juego de guerra de cuatro días interpretado por todos los estudiantes para revivir la batalla. También conmemoramos el aniversario de la batalla (4 de junio) con un invitado y un pastel de celebración.

Adjunto, encontrará una lista de todas las tareas del proyecto y las fechas en que deben ser entregadas. La entrega tarde de cualquier tarea resultará en un grado más bajo del que le hubiera sido asignado por el trabajo realizado. Por favor asegúrese de que su hijo(a) termine las tareas asignadas a tiempo.

Por favor, no dude en ponerse en contacto conmigo con cualquier pregunta o preocupación. Mi dirección de correo electrónico es tbeall@paulcuffee.org.

Atentamente,

Thomas R. Beall
Capitán, U. S. Marina (Ret.)

Historical Background[13]

The Battle of Midway, fought over and near the tiny U.S. mid-Pacific base at Midway atoll, represents the strategic high water mark of Japan's Pacific Ocean war. Prior to this action, Japan possessed general naval superiority over the United States and could usually choose where and when to attack. After Midway, the two opposing fleets were essentially equals, and the United States soon took the offensive.

Japanese Combined Fleet commander Admiral Isoroku Yamamoto moved on Midway in an effort to draw out and destroy the U.S. Pacific Fleet's aircraft carrier striking forces, which had embarrassed the Japanese Navy in the mid-April Doolittle Raid on Japan's home islands and at the Battle of Coral Sea in early May. He planned to quickly knock down Midway's defenses, follow up with an invasion of the atoll's two small islands and establish a Japanese air base there. He expected the U.S. carriers to come out and fight, but to arrive too late to save Midway and in insufficient strength to avoid defeat by his own well-tested carrier air power.

Yamamoto's intended surprise was thwarted by superior American communications intelligence, which deduced his scheme well before battle was joined. This allowed Admiral Chester W. Nimitz, the U.S. Pacific Fleet commander, to establish an ambush by having his carriers ready and waiting for the Japanese. On 4 June 1942, in the second of the Pacific War's great carrier battles, the trap was sprung. The perseverance, sacrifice and skill of U.S. Navy aviators, plus a great deal of good luck on the American side, cost Japan four irreplaceable fleet carriers, while only one of the three U.S. carriers present was lost. The base at Midway, though damaged by Japanese air attack, remained operational and later became a vital component in the American trans-Pacific offensive.

[13] Source: Naval History and Heritage Command. Accessed online 15 April 2012 at http://www.history.navy.mil/photos/events/wwii-pac/midway/midway.htm.

Name: ___________________________________

The Final Project Journal

Attach this sheet to the inside front cover of your journal.

The final project journal is the single most important graded item of the project. ***It is worth one fourth of your final course grade.*** Although your journal will be kept in the classroom, you will be personally and solely responsible for it at all times. If it is lost, you must replace it. If it is damaged, you must replace it. If it is stolen, you must replace it. You will be responsible for its neatness and its completeness. It is yours to maintain and complete. I will accept no responsibility for it.

In completing your journal, you will have two main tasks:

1. **Inserting completed work.** You will attach the following completed tasks in your journal once each task has been graded:

    a. Breaking the Code
    b. Staying Power / Firepower System of Equations (Algebra only)
    c. 16 inch Gun Data Analysis (Geometry only)
    d. Warship Survivability Data Analysis
    e. Ship Stability Problems (Geometry only)
    f. Battleship Massachusetts Scaled Drawing
    g. Understanding the Engineering Plant
    h. Battleship Massachusetts Field Trip Assignment and Reflection
    i. Maneuvering Board Exercise (Geometry only)
    j. Communications Signals Exercise

2. **Daily written entries.** Each day you will make a written entry which, at a minimum, will include the following:

   a. The daily warm – up. These daily exercises are designed to improve your ability to complete assigned tasks and, ultimately, play the war game.

   b. Daily navigation information to include:

      i. Sun's declination: (www.wsanford.com/~wsanford/exo/sundials/DEC_Sun.html)

      ii. The following solar transit information for Honolulu, Hawaii: (http://aa.usno.navy.mil/data/docs/RS_OneDay.php)

         - Begin civil twilight
         - Sunrise
         - Sun transit
         - Sunset
         - End civil twilight

      iii. Hawaiian coastal waters weather forecast (http://www.prh.noaa.gov/hnl/pages/CWF.php)

   c. Reflections on the project work you are doing to include what you find challenging, what you find easy, what ideas you have to solve problems and complete work.

3. **Grading.** I will grade your journal each week on Friday on the standard “1-4” scale. I will pay particular attention to whether or not you completed the minimum entries and how neatly you have kept your journal. Your final journal grade (one fourth of your final course grade) will be the average of these weekly grades.

Part I: Numbers and Operations – Breaking the Code

Teacher Notes

Objectives:

1. Recognize patterns in number sequences.
2. Identify arithmetic operations performed on a sequence of numbers.
3. Develop self-confidence and resiliency by completing a challenging and unfamiliar task in which all steps may not be clearly identified.

Teacher Support:

1. The encoded message, when decoded, reads as follows:

   FM COMMANDER COMBINED FLEET
   TO COMMANDER CARRIER STRIKE FLEET
   SUBJECT: MIDWAY OPERATION
   MIDWAY ISLAND WILL BE ATTACKED ON JUNE FOURTH
   MIDWAY ISLAND IS LOCATED AT 28°13' N 177°22'W.
   THE CARRIER STRIKE FLEET WILL LEAD THE ATTACK.
   YOU WILL HAVE FOUR AIRCRAFT CARRIERS.
   KAGA
   AKAGI
   HIRYU
   SORYU
   YOU WILL BE PREPARED TO ATTACK AND DESTROY
   UNITED STATES NAVAL FORCES DEFENDING MIDWAY.

2. Each student has received an encoded message in which each letter has been translated to its number equivalent and then subjected to another arithmetic operation which is the same for all letters in the message. For example:

   a. $Letter\ \text{“}B\text{”} = 2$
   b. $2^2 = 4$ so the number that corresponds to “B” in the encoded message is 4.

3. There are seven versions of the encoded message. Each student received one version. The multiple versions are designed to discourage student copying.

4. In each of the seven versions, I performed a single arithmetic operation on all of the numbers which correspond to letters. You can identify the version because it is listed on the page with the code in the bottom right-hand corner. The operations in the different versions are as follows:

   a. Version 1 (v.1): Numbers raised to the second power.
   b. Version 2 (v.2): Numbers raised to the third power.
   c. Version 3 (v.3): Numbers reduced to their square roots.
   d. Version 4 (v.4): Numbers multiplied by “5”.
   e. Version 5 (v.5): Numbers divided by “2”.
   f. Version 6 (v.6): The number “7” added to each number.
   g. Version 7 (v.7): The number “13” added to each number.

5. Since one of the primary objectives in this task and all tasks of this project is to develop student self-confidence and resiliency, teachers should be judicious in the amount of support provided to students. Many will undoubtedly ask for as much as they can get. Teachers should ensure that the help provided is only in accordance with documented needs.

Name: ________________________

Part I: Numbers and Operations – Breaking the Code

This task is due no later than March 7th, 2014

1. **Historical Background.** In the Second World War, all the combatant powers translated all of their radio and written messages into a code (a process called encryption) to keep the contents of their messages secret from their enemies.

   Of course, each combatant tried to break his enemy's codes and all combatants succeeded to some degree. One of the most famous code breaking efforts was conducted by the United States armed forces against the Empire of Japan during the Pacific War (1941 - 1945). American code breakers managed to read enough of the Japanese codes to know, after early 1942, what the Japanese military was planning against American forces.

2. **Code - breaking Techniques.** All code - breakers (called cryptanalysts) used common techniques for breaking codes. Among those techniques which you might find useful in performing this task are:

   a. Look for numbers that occur most frequently. The letter "E" is the most frequently occurring letter in the English language (and many other languages). The most frequently occurring number often (though not always) represents the letter "E".

   b. Look for repeated number sequences. Many words are used more than once in most written messages. Look for identical number sequences. They will often represent the same word.

   c. One number occurs most frequently but look carefully at where it occurs. It should indicate that it is used for only one thing.

3. **Scenario.** It is May 1st, 1942. Last December, the Imperial Japanese Navy's Combined Fleet attacked the U.S. Pacific Fleet at its base in Pearl Harbor, Hawaii, destroying many ships and killing many men. Since then, the United States has been at war with the Empire of Japan. During

these months, the Japanese have conquered many islands and territories in the Pacific Ocean. Admiral Chester W. Nimitz, the Commander in Chief of the U.S. Pacific Fleet, is trying to figure out where Japan's next major attack will be. American naval code - breakers are assisting him by breaking Japanese secret codes and providing the information in them to members of Nimitz' staff.

4. **Assignment.** On the next page is a Japanese coded message with some hints about it. You are to break the code and turn in the decoded message, ***typewritten***, no later than **March 7th, 2014.**

5. **Note:** Not all students received the same encoded message. If you do not do your own work, you may get an incorrect answer and will get no credit.

Coded Message

1. Each number corresponds to one and only one letter of the alphabet.
2. In developing this code I started with the basic table below and then made some further arithmetic changes (in other words, I may have added to, subtracted from, raised to a power, multiplied or divided the numbers from the basic table). I made the same change to each number.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

3. The number which appears most often has a special significance.
4. Numbers in quotation marks should be read as the actual number, not a coded letter.
5. Each line represents a line of text.

Getting Started:

1. Find the number that occurs the second most often. That number most likely represents "A" or "E".
2. Determine what mathematical operation you need to perform on that number to make it equal "1" (for "A") or "5" (for "E").
3. Perform the same operation on all other numbers except the one that occurs most often.
4. See if the numbers now represent the letters in the table above. If they do, you have broken the code.

36	169	0	9	225	169	169	1	196	16	25	324	0	9	225	169	4	81	196	25	16	0	36	144	25	25	400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
400	225	0	9	225	169	169	1	196	16	25	324	0	9	1	324	324	81	25	324	0	361	400	324	81	121	25	0	36	144	25	25	400	0	0	0	0	0	0	0	0	0	0	0	0	0
361	441	4	100	25	9	400	:	0	169	81	16	529	1	625	0	225	256	25	324	1	400	81	225	196	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
169	81	16	529	1	625	0	81	361	144	1	196	16	0	529	81	144	144	0	4	25	0	1	400	400	1	9	121	25	16	0	225	196	0	100	441	196	25	0	36	225	441	324	400	64	.
169	81	16	529	1	625	0	81	361	144	1	196	16	0	81	361	0	144	225	9	1	400	25	16	0	1	400	0	28°13'	196	0	177°22'	529	.	0	0	0	0	0	0	0	0	0	0	0	0
400	64	25	0	9	1	324	324	81	25	324	0	361	400	324	81	121	25	0	36	144	25	25	400	0	529	81	144	144	0	144	25	1	16	0	400	64	25	0	1	400	400	1	9	121	.
625	225	441	0	529	81	144	144	0	64	1	484	25	0	36	225	441	324	0	1	81	324	9	324	1	36	400	0	9	1	324	324	81	25	324	361	0	0	0	0	0	0	0	0	0	0
121	1	49	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	121	1	49	81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	81	324	625	441	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
361	225	324	625	441	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
625	225	441	0	529	81	144	144	0	4	25	0	256	324	25	256	1	324	25	16	0	400	225	0	1	400	400	1	9	121	0	1	196	16	0	16	25	361	400	324	225	625	0	0	0	0
441	196	81	400	25	16	0	361	400	1	400	25	361	0	196	1	484	1	144	0	36	225	324	9	25	361	0	16	25	36	25	196	16	81	196	49	0	169	81	16	529	1	625	.	0	0

v.1

Coded Message

1. Each number corresponds to one and only one letter of the alphabet.
2. In developing this code I started with the basic table below and then made some further arithmetic changes (in other words, I may have added to, subtracted from, raised to a power, multiplied or divided the numbers from the basic table). I made the same change to each number.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

3. The number which appears the most has a special significance.
4. Numbers in quotation marks should be read as the actual number, not a coded letter.
5. Each line represents a line of text.

Getting Started:

1. Find the number that occurs the second most often. That number most likely represents "A" or "E".
2. Determine what mathematical operation you need to perform on that number to make it equal "1" (for "A") or "5" (for "E").
3. Perform the same operation on all other numbers except the one that occurs most often.
4. See if the numbers now represent the letters in the table above. If they do, you have broken the code.

216	2197	0	27	3375	2197	2197	1	2744	64	125	5832	0	27	3375	2197	8	729	2744	125	64	0	216	1728	125	125	8000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8000	3375	0	27	3375	2197	2197	1	2744	64	125	5832	0	27	1	5832	5832	729	125	5832	0	6859	8000	5832	729	1331	125	0	216	1728	125	125	8000	0	0	0	0	0	0	0	0	0	0	0	0	0
6859	9261	8	1000	125	27	8000	:	0	2197	729	64	12167	1	15625	0	3375	4096	125	5832	1	8000	729	3375	2744	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2197	729	64	12167	1	15625	0	729	6859	1728	1	2744	64	0	12167	729	1728	1728	0	8	125	0	1	8000	8000	1	27	1331	125	64	0	3375	2744	0	1000	9261	2744	125	0	216	3375	9261	5832	8000	512	.
2197	729	64	12167	1	15625	0	729	6859	1728	1	2744	64	0	729	6859	0	1728	3375	27	1	8000	125	64	0	1	8000	0	28°13'	2744	0	177°22'	12167	.	0	0	0	0	0	0	0	0	0	0	0	0
8000	512	125	0	27	1	5832	5832	729	125	5832	0	6859	8000	5832	729	1331	125	0	216	1728	125	125	8000	0	12167	729	1728	1728	0	1728	125	1	64	0	8000	512	125	0	1	8000	8000	1	27	1331	.
15625	3375	9261	0	12167	729	1728	1728	0	512	1	10648	125	0	216	3375	9261	5832	0	1	729	5832	27	5832	1	216	8000	0	27	1	5832	5832	729	125	5832	6859	0	0	0	0	0	0	0	0	0	0
1331	1	343	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1331	1	343	729	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
512	729	5832	15625	9261	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6859	3375	5832	15625	9261	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15625	3375	9261	0	12167	729	1728	1728	0	8	125	0	4096	5832	125	4096	1	5832	125	64	0	8000	3375	0	1	8000	8000	1	27	1331	0	1	2744	64	0	64	125	6859	8000	5832	3375	15625	0	0	0	0
9261	2744	729	8000	125	64	0	6859	8000	1	8000	125	6859	0	2744	1	10648	1	1728	0	216	3375	5832	27	125	6859	0	64	125	216	125	2744	64	729	2744	343	0	2197	729	64	12167	1	15625	.	0	0

v.2

Coded Message

1. Each number corresponds to one and only one letter of the alphabet.
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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

3. The number which appears the most has a special significance.
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3. Perform the same operation on all other numbers except the one that occurs most often.
4. See if the numbers now represent the letters in the table above. If they do, you have broken the code.

2.449	3.606	0	1.732	3.873	3.606	3.606	1	3.742	2	2.236	4.243	0	1.732	3.873	3.606	1.414	3	3.742	2.236	2	0	2.449	3.464	2.236	2.236	4.472	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4.472	3.873	0	1.732	3.873	3.606	3.606	1	3.742	2	2.236	4.243	0	1.732	1	4.243	4.243	3	2.236	4.243	0	4.359	4.472	4.243	3	3.317	2.236	0	2.449	3.464	2.236	2.236	4.472	0	0	0	0	0	0	0	0	0	0	0	0	0
4.359	4.583	1.414	3.162	2.236	1.732	4.472	:	0	3.606	3	2	4.796	1	5	0	3.873	4	2.236	4.243	1	4.472	3	3.873	3.742	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3.606	3	2	4.796	1	5	0	3	4.359	3.464	1	3.742	2	0	4.796	3	3.464	3.464	0	1.414	2.236	0	1	4.472	4.472	1	1.732	3.317	2.236	2	0	3.873	3.742	0	3.162	4.583	3.742	2.236	0	2.449	3.873	4.583	4.243	4.472	2.828	.
3.606	3	2	4.796	1	5	0	3	4.359	3.464	1	3.742	2	0	3	4.359	0	3.464	3.873	1.732	1	4.472	2.236	2	0	1	4.472	0	28°13'	3.742	0	177°22'	4.796	.	0	0	0	0	0	0	0	0	0	0	0	0
4.472	2.828	2.236	0	1.732	1	4.243	4.243	3	2.236	4.243	0	4.359	4.472	4.243	3	3.317	2.236	0	2.449	3.464	2.236	2.236	4.472	0	4.796	3	3.464	3.464	0	3.464	2.236	1	2	0	4.472	2.828	2.236	0	1	4.472	4.472	1	1.732	3.317	.
5	3.873	4.583	0	4.796	3	3.464	3.464	0	2.828	1	4.69	2.236	0	2.449	3.873	4.583	4.243	0	1	3	4.243	1.732	4.243	1	2.449	4.472	0	1.732	1	4.243	4.243	3	2.236	4.243	4.359	0	0	0	0	0	0	0	0	0	0
3.317	1	2.646	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	3.317	1	2.646	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.828	3	4.243	5	4.583	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4.359	3.873	4.243	5	4.583	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	3.873	4.583	0	4.796	3	3.464	3.464	0	1.414	2.236	0	4	4.243	2.236	4	1	4.243	2.236	2	0	4.472	3.873	0	1	4.472	4.472	1	1.732	3.317	0	1	3.742	2	0	2	2.236	4.359	4.472	4.243	3.873	5	0	0	0	0
4.583	3.742	3	4.472	2.236	2	0	4.359	4.472	1	4.472	2.236	4.359	0	3.742	1	4.69	1	3.464	0	2.449	3.873	4.243	1.732	2.236	4.359	0	2	2.236	2.449	2.236	3.742	2	3	3.742	2.646	0	3.606	3	2	4.796	1	5	.	0	0

v.3

Coded Message

1. Each number corresponds to one and only one letter of the alphabet.
2. In developing this code I started with the basic table below and then made some further arithmetic changes (in other words, I may have added to, subtracted from, raised to a power, multiplied or divided the numbers from the basic table). I made the same change to each number.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

3. The number which appears the most has a special significance.
4. Numbers in quotation marks should be read as the actual number, not a coded letter.
5. Each line represents a line of text.

Getting Started:

1. Find the number that occurs the second most often. That number most likely represents "A" or "E".
2. Determine what mathematical operation you need to perform on that number to make it equal "1" (for "A") or "5" (for "E").
3. Perform the same operation on all other numbers except the one that occurs most often.
4. See if the numbers now represent the letters in the table above. If they do, you have broken the code.

30	65	0	15	75	65	65	5	70	20	25	90	0	15	75	65	10	45	70	25	20	0	30	60	25	25	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	75	0	15	75	65	65	5	70	20	25	90	0	15	5	90	90	45	25	90	0	95	100	90	45	55	25	0	30	60	25	25	100	0	0	0	0	0	0	0	0	0	0	0	0	0
95	105	10	50	25	15	100	:	0	65	45	20	115	5	125	0	75	80	25	90	5	100	45	75	70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	45	20	115	5	125	0	45	95	60	5	70	20	0	115	45	60	60	0	10	25	0	5	100	100	5	15	55	25	20	0	75	70	0	50	105	70	25	0	30	75	105	90	100	40	.
65	45	20	115	5	125	0	45	95	60	5	70	20	0	45	95	0	60	75	15	5	100	25	20	0	5	100	0	28°13'	70	0	177°22'	115	.	0	0	0	0	0	0	0	0	0	0	0	0
100	40	25	0	15	5	90	90	45	25	90	0	95	100	90	45	55	25	0	30	60	25	25	100	0	115	45	60	60	0	60	25	5	20	0	100	40	25	0	5	100	100	5	15	55	.
125	75	105	0	115	45	60	60	0	40	5	110	25	0	30	75	105	90	0	5	45	90	15	90	5	30	100	0	15	5	90	90	45	25	90	95	0	0	0	0	0	0	0	0	0	0
55	5	35	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	55	5	35	45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	45	90	125	105	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
95	75	90	125	105	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
125	75	105	0	115	45	60	60	0	10	25	0	80	90	25	80	5	90	25	20	0	100	75	0	5	100	100	5	15	55	0	5	70	20	0	20	25	95	100	90	75	125	0	0	0	0
105	70	45	100	25	20	0	95	100	5	100	25	95	0	70	5	110	5	60	0	30	75	90	15	25	95	0	20	25	30	25	70	20	45	70	35	0	65	45	20	115	5	125	.	0	0

v.4

Coded Message

1. Each number corresponds to one and only one letter of the alphabet.
2. In developing this code I started with the basic table below and then made some further arithmetic changes (in other words, I may have added to, subtracted from, raised to a power, multiplied or divided the numbers from the basic table). I made the same change to each number.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

3. The number which appears the most has a special significance.
4. Numbers in quotation marks should be read as the actual number, not a coded letter.
5. Each line represents a line of text.

Getting Started:

1. Find the number that occurs the second most often. That number most likely represents "A" or "E".
2. Determine what mathematical operation you need to perform on that number to make it equal "1" (for "A") or "5" (for "E").
3. Perform the same operation on all other numbers except the one that occurs most often.
4. See if the numbers now represent the letters in the table above. If they do, you have broken the code.

3	6.5	0	1.5	7.5	6.5	6.5	0.5	7	2	2.5	9	0	1.5	7.5	6.5	1	4.5	7	2.5	2	0	3	6	2.5	2.5	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	7.5	0	1.5	7.5	6.5	6.5	0.5	7	2	2.5	9	0	1.5	0.5	9	9	4.5	2.5	9	0	9.5	10	9	4.5	5.5	2.5	0	3	6	2.5	2.5	10	0	0	0	0	0	0	0	0	0	0	0	0	0
9.5	10.5	1	5	2.5	1.5	10	:	0	6.5	4.5	2	11.5	0.5	12.5	0	7.5	8	2.5	9	0.5	10	4.5	7.5	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6.5	4.5	2	11.5	0.5	12.5	0	4.5	9.5	6	0.5	7	2	0	11.5	4.5	6	6	0	1	2.5	0	0.5	10	10	0.5	1.5	5.5	2.5	2	0	7.5	7	0	5	10.5	7	2.5	0	3	7.5	10.5	9	10	4	.
6.5	4.5	2	11.5	0.5	12.5	0	4.5	9.5	6	0.5	7	2	0	4.5	9.5	0	6	7.5	1.5	0.5	10	2.5	2	0	0.5	10	0	28°13'	7	0	177°22'	11.5	.	0	0	0	0	0	0	0	0	0	0	0	0
10	4	2.5	0	1.5	0.5	9	9	4.5	2.5	9	0	9.5	10	9	4.5	5.5	2.5	0	3	6	2.5	2.5	10	0	11.5	4.5	6	6	0	6	2.5	0.5	2	0	10	4	2.5	0	0.5	10	10	0.5	1.5	5.5	.
12.5	7.5	10.5	0	11.5	4.5	6	6	0	4	0.5	11	2.5	0	3	7.5	10.5	9	0	0.5	4.5	9	1.5	9	0.5	3	10	0	1.5	0.5	9	9	4.5	2.5	9	9.5	0	0	0	0	0	0	0	0	0	0
5.5	0.5	3.5	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	5.5	0.5	3.5	4.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	4.5	9	12.5	10.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9.5	7.5	9	12.5	10.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12.5	7.5	10.5	0	11.5	4.5	6	6	0	1	2.5	0	8	9	2.5	8	0.5	9	2.5	2	0	10	7.5	0	0.5	10	10	0.5	1.5	5.5	0	0.5	7	2	0	2	2.5	9.5	10	9	7.5	12.5	0	0	0	0
10.5	7	4.5	10	2.5	2	0	9.5	10	0.5	10	2.5	9.5	0	7	0.5	11	0.5	6	0	3	7.5	9	1.5	2.5	9.5	0	2	2.5	3	2.5	7	2	4.5	7	3.5	0	6.5	4.5	2	11.5	0.5	12.5	.	0	0

v.5

Coded Message

1. Each number corresponds to one and only one letter of the alphabet.
2. In developing this code I started with the basic table below and then made some further arithmetic changes (in other words, I may have added to, subtracted from, raised to a power, multiplied or divided the numbers from the basic table). I made the same change to each number.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

3. The number which appears the most has a special significance.
4. Numbers in quotation marks should be read as the actual number, not a coded letter.
5. Each line represents a line of text.

Getting Started:

1. Find the number that occurs the second most often. That number most likely represents "A" or "E".
2. Determine what mathematical operation you need to perform on that number to make it equal "1" (for "A") or "5" (for "E").
3. Perform the same operation on all other numbers except the one that occurs most often.
4. See if the numbers now represent the letters in the table above. If they do, you have broken the code.

13	20	7	10	22	20	20	8	21	11	12	25	7	10	22	20	9	16	21	12	11	7	13	19	12	12	27	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
27	22	7	10	22	20	20	8	21	11	12	25	7	10	8	25	25	16	12	25	7	26	27	25	16	18	12	7	13	19	12	12	27	7	7	7	7	7	7	7	7	7	7	7	7	7
26	28	9	17	12	10	27	:	7	20	16	11	30	8	32	7	22	23	12	25	8	27	16	22	21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
20	16	11	30	8	32	7	16	26	19	8	21	11	7	30	16	19	19	7	9	12	7	8	27	27	8	10	18	12	11	7	22	21	7	17	28	21	12	7	13	22	28	25	27	15	.
20	16	11	30	8	32	7	16	26	19	8	21	11	7	16	26	7	19	22	10	8	27	12	11	7	8	27	7	28°13'	21	7	177°22'	30	.	7	7	7	7	7	7	7	7	7	7	7	7
27	15	12	7	10	8	25	25	16	12	25	7	26	27	25	16	18	12	7	13	19	12	12	27	7	30	16	19	19	7	19	12	8	11	7	27	15	12	7	8	27	27	8	10	18	.
32	22	28	7	30	16	19	19	7	15	8	29	12	7	13	22	28	25	7	8	16	25	10	25	8	13	27	7	10	8	25	25	16	12	25	26	7	7	7	7	7	7	7	7	7	7
18	8	14	8	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
8	18	8	14	16	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
15	16	25	32	28	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
26	22	25	32	28	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
32	22	28	7	30	16	19	19	7	9	12	7	23	25	12	23	8	25	12	11	7	27	22	7	8	27	27	8	10	18	7	8	21	11	7	11	12	26	27	25	22	32	7	7	7	7
28	21	16	27	12	11	7	26	27	8	27	12	26	7	21	8	29	8	19	7	13	22	25	10	12	26	7	11	12	13	12	21	11	16	21	14	7	20	16	11	30	8	32	.	7	7

v.6

Coded Message

1. Each number corresponds to one and only one letter of the alphabet.
2. In developing this code I started with the basic table below and then made some further arithmetic changes (in other words, I may have added to, subtracted from, raised to a power, multiplied or divided the numbers from the basic table). I made the same change to each number.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

3. The number which appears the most has a special significance.
4. Numbers in quotation marks should be read as the actual number, not a coded letter.
5. Each line represents a line of text.

Getting Started:

1. Find the number that occurs the second most often. That number most likely represents "A" or "E".
2. Determine what mathematical operation you need to perform on that number to make it equal "1" (for "A") or "5" (for "E").
3. Perform the same operation on all other numbers except the one that occurs most often.
4. See if the numbers now represent the letters in the table above. If they do, you have broken the code.

19	26	13	16	28	26	26	14	27	17	18	31	13	16	28	26	15	22	27	18	17	13	19	25	18	18	33	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
33	28	13	16	28	26	26	14	27	17	18	31	13	16	14	31	31	22	18	31	13	32	33	31	22	24	18	13	19	25	18	18	33	13	13	13	13	13	13	13	13	13	13	13	13	13
32	34	15	23	18	16	33	:	13	26	22	17	36	14	38	13	28	29	18	31	14	33	22	28	27	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
26	22	17	36	14	38	13	22	32	25	14	27	17	13	36	22	25	25	13	15	18	13	14	33	33	14	16	24	18	17	13	28	27	13	23	34	27	18	13	19	28	34	31	33	21	.
26	22	17	36	14	38	13	22	32	25	14	27	17	13	22	32	13	25	28	16	14	33	18	17	13	14	33	13	28°13'	27	13	177°22'	36	.	13	13	13	13	13	13	13	13	13	13	13	13
33	21	18	13	16	14	31	31	22	18	31	13	32	33	31	22	24	18	13	19	25	18	18	33	13	36	22	25	25	13	25	18	14	17	13	33	21	18	13	14	33	33	14	16	24	.
38	28	34	13	36	22	25	25	13	21	14	35	18	13	19	28	34	31	13	14	22	31	16	31	14	19	33	13	16	14	31	31	22	18	31	32	13	13	13	13	13	13	13	13	13	13
24	14	20	14	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
14	24	14	20	22	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
21	22	31	38	34	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
32	28	31	38	34	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
38	28	34	13	36	22	25	25	13	15	18	13	29	31	18	29	14	31	18	17	13	33	28	13	14	33	33	14	16	24	13	14	27	17	13	17	18	32	33	31	28	38	13	13	13	13
34	27	22	33	18	17	13	32	33	14	33	18	32	13	27	14	35	14	25	13	19	28	31	16	18	32	13	17	18	19	18	27	17	22	27	20	13	26	22	17	36	14	38	.	13	13

v.7

Name:____________________________

Part II: Data Analysis, Mathematical Modeling, and Prediction: Determining the Warfare Requirements

1. **Historical Background.** Warships such as U.S.S. *Massachusetts* (BB 59) and those that fought at the Battle of Midway were built as a result of compromises. An ideal warship would be faster, more powerful, and more resistant to damage than any other ship of her type. Unfortunately, such a ship would be too expensive to build. Builders, therefore, had to work within a budget, striking the right balance between what we will call ***staying power*** (a function of the ship's resistance to damage from attack and including armor and speed) and ***firepower*** (a function of how much, how fast, how far, and how accurately she could deliver explosive power to an enemy ship).

Task II A – A – Finding the Right Mix of Staying Power and Firepower
Teacher Notes

Objectives:

1. Correctly identify the variables of a system of equations embedded in a word problem.
2. Express the system as described in a word problem as two equations with two unknown variables.
3. Solve the system using all five methods.
4. Develop self-confidence and resiliency by completing a challenging and unfamiliar task in which all steps may not be clearly identified.

Teacher Support:

1. The correct variables are (students may use whatever symbols they choose as long as they are used consistently):

   $sp = staying\ power\ units$
   $fp = firepower\ units$

2. The correct equations and the solutions are:

   $sp + fp = 1000$
   $\$200{,}000 * sp + \$50{,}000 * fp = \$100{,}000{,}000$
   $sp = 333\ staying\ power\ units$
   $fp = 666\ staying\ power\ units$

3. The five methods of solving the system are:

   i. Elimination
   ii. Substitution
   iii. Graphing
   iv. Microsoft EXCEL Program
   v. Matrix Algebra

Name:____________________________

Task II A – A – Finding the Right Mix of Staying Power and Firepower

This task is due no later than March 14th, 2014

Solve the following system of equations problem using all five methods (elimination, substitution, graphing, Microsoft EXCEL program, Matrix Algebra):

To build U.S.S. *Massachusetts*, the Navy Department has offered the builder (Bethlehem Steel Corporation) $100,000,000. Building costs are determined by how many units of staying power and how many units of firepower he builds into the ship. The sum of staying power and fire power units (called the total combat power) must equal at least 1000. Each unit of staying power costs $200,000, each unit of firepower costs $50,000. How many units of staying power and how many units of firepower should the builder build into the ship to stay within the budgeted figure of $100,000,000?

1. Why do you think a unit of staying power costs more than a unit of firepower?

2. Identify the variables:

3. Write the equations:

4. Solve the system using all five methods (attach work on a separate sheets of paper).[14]

[14] *Answer:* 333 staying power units, 667 firepower units.

Task II B – G
U.S.S. *Massachusetts* 16”/45 Gun Data
Teacher Notes

Objectives:

1. Correctly apply a trigonometric function to the solution of a problem.
2. Develop self-confidence and resiliency by completing a challenging and unfamiliar task in which all steps may not be clearly identified.

Teacher Support:

1. Students find the solution by applying the formula on the task sheet:

$$d = \left(\frac{v^2}{g}\right) sin2\theta$$

2. The first problem is set up as follows:

A. $d = \left(\frac{v^2}{g}\right) sin2\theta$

$$d = (\frac{\left(2600\left(\frac{ft}{sec}\right)\right)^2}{32\frac{f}{s^2}}) \times \sin(2 \times 20°)$$

$$d = 135{,}788.88\, ft = 45{,}262.96\, yards$$

B. $\% = 1 - \left(\frac{28{,}000}{45{,}262.96}\right) = 38.14\%$

Name: ______________________________

Task II B – G
U.S.S. *Massachusetts* 16"/45 Gun Data
This task is due no later than March 14th, 2014

1. The range of a projectile such as a shell fired from a gun, neglecting air resistance, is calculated by the following formula:

$$d = \left(\frac{v^2}{g}\right) sin2\theta$$

g = gravitational acceleration of $32\frac{f}{s^2}$.
v = the initial velocity of the projectile in feet per second.
θ = the angle of elevation of the gun.
d = the horizontal distance travelled by the projectile.

A. If the initial velocity of an armor piercing shell fired from one of *Massachusetts'* 16 – inch guns is 2600 feet per second and the angle of elevation of the gun is 20°, how far will the shell travel?

B. If air resistance reduces the range to 28,000 yards by what percent does air resistance reduce the range?

C. Perform the same calculations for the following ranges by completing this table:

Gun Elevation	**Range with no air resistance**	**Range with air resistance**	**% Reduction**
10°		18,000 yards	
30°		34,000 yards	
40°		38,100 yards	

Task II C – A / G – Analyzing Staying Power

Teacher Notes

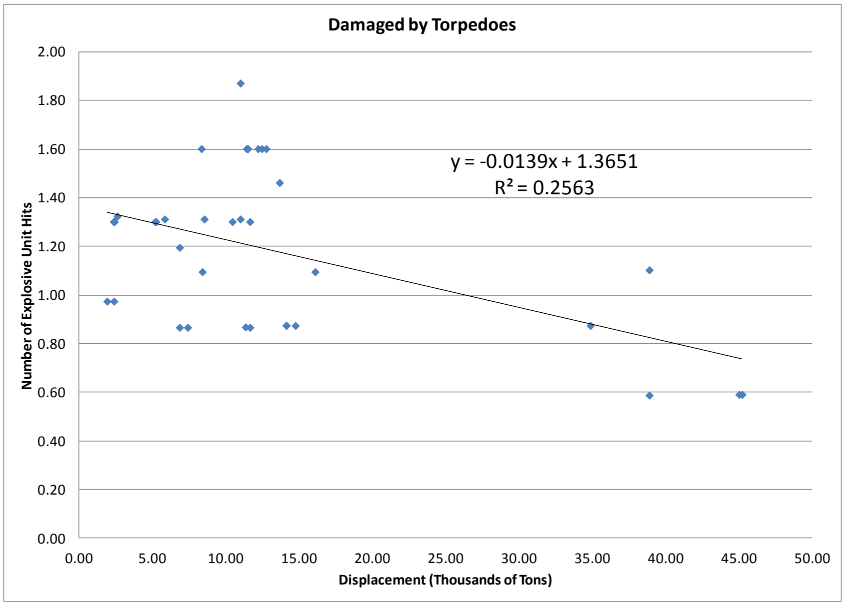
Objectives:

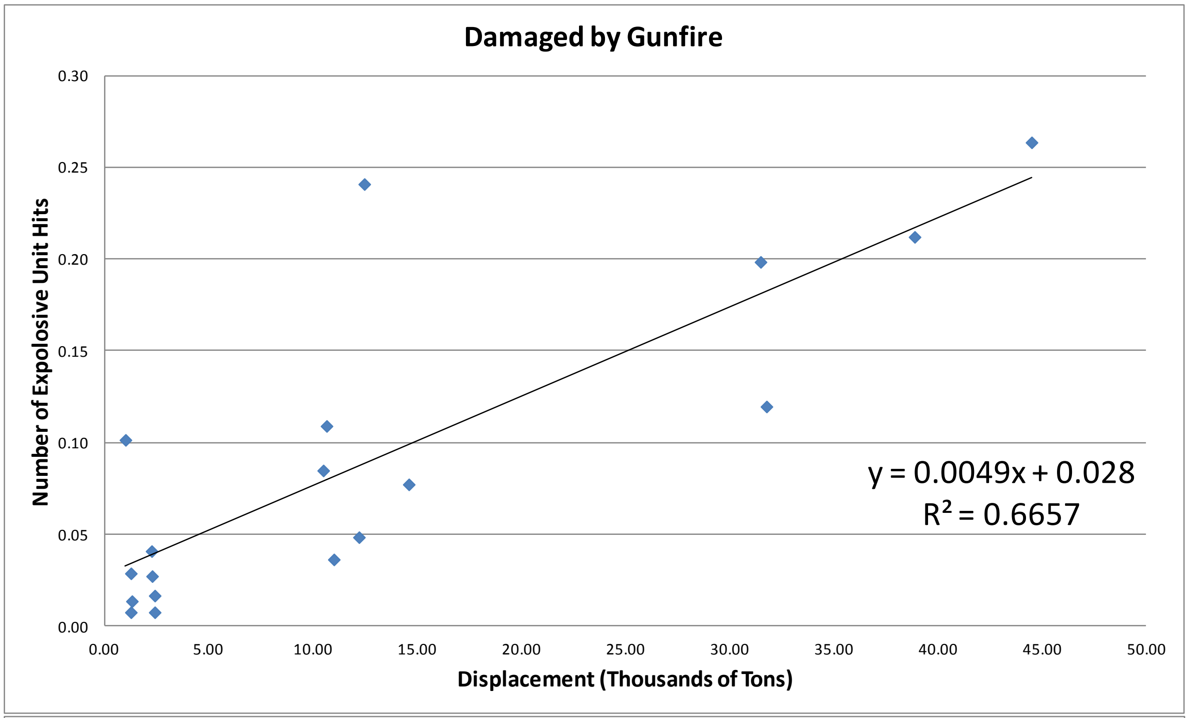
1. Accurately compile a complex data set in Microsoft EXCEL.
2. Create linear model graphs from that data, properly formatted and labeled.
3. Identify the slope and y-intercept of each linear model and use that information to conduct data analysis.
4. Understand and answer questions concerning the outcome and findings of the data analysis.
5. Develop self-confidence and resiliency by completing a challenging and unfamiliar task in which all steps may not be clearly identified.

Teacher Support:

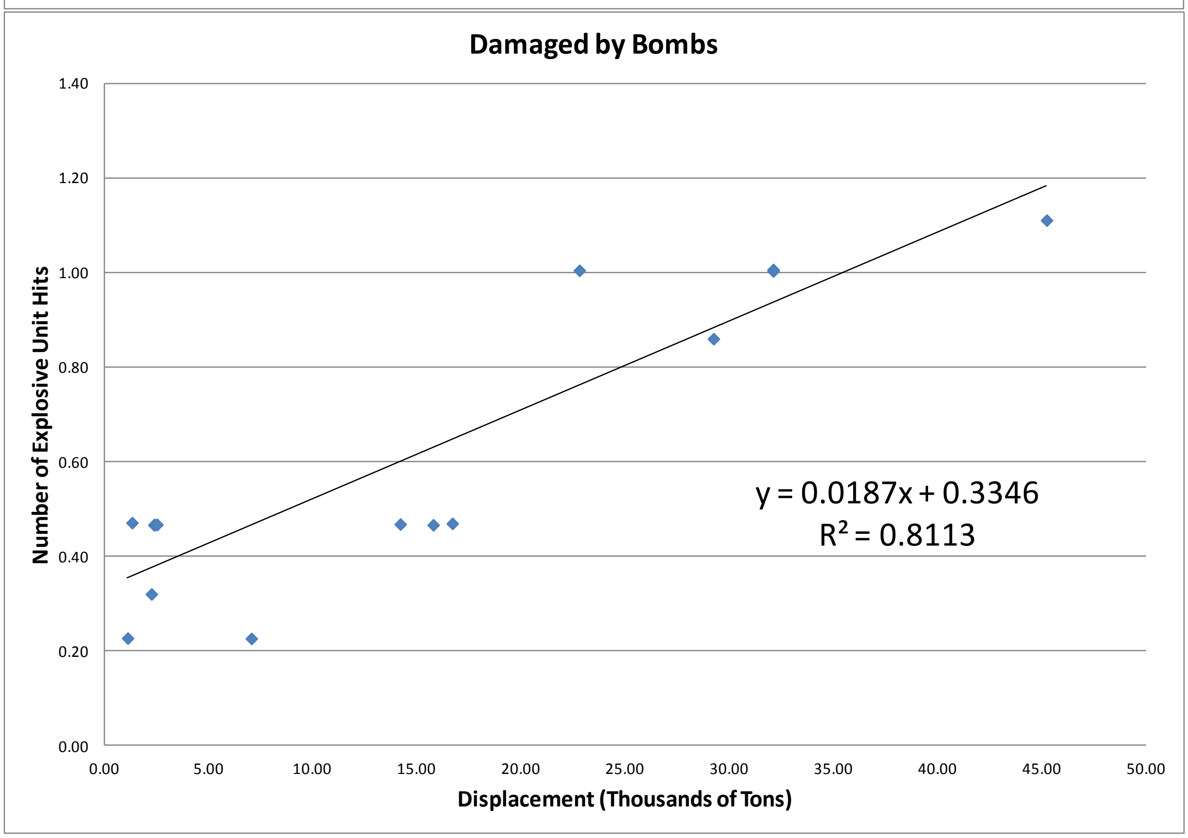
1. Students should be encouraged to fill in the table, assigning their own due dates for each milestone.

2. The five graphs should appear as below. If the equations or R^2 values differ then the student has not entered the data set correctly and should be encouraged to find the error.

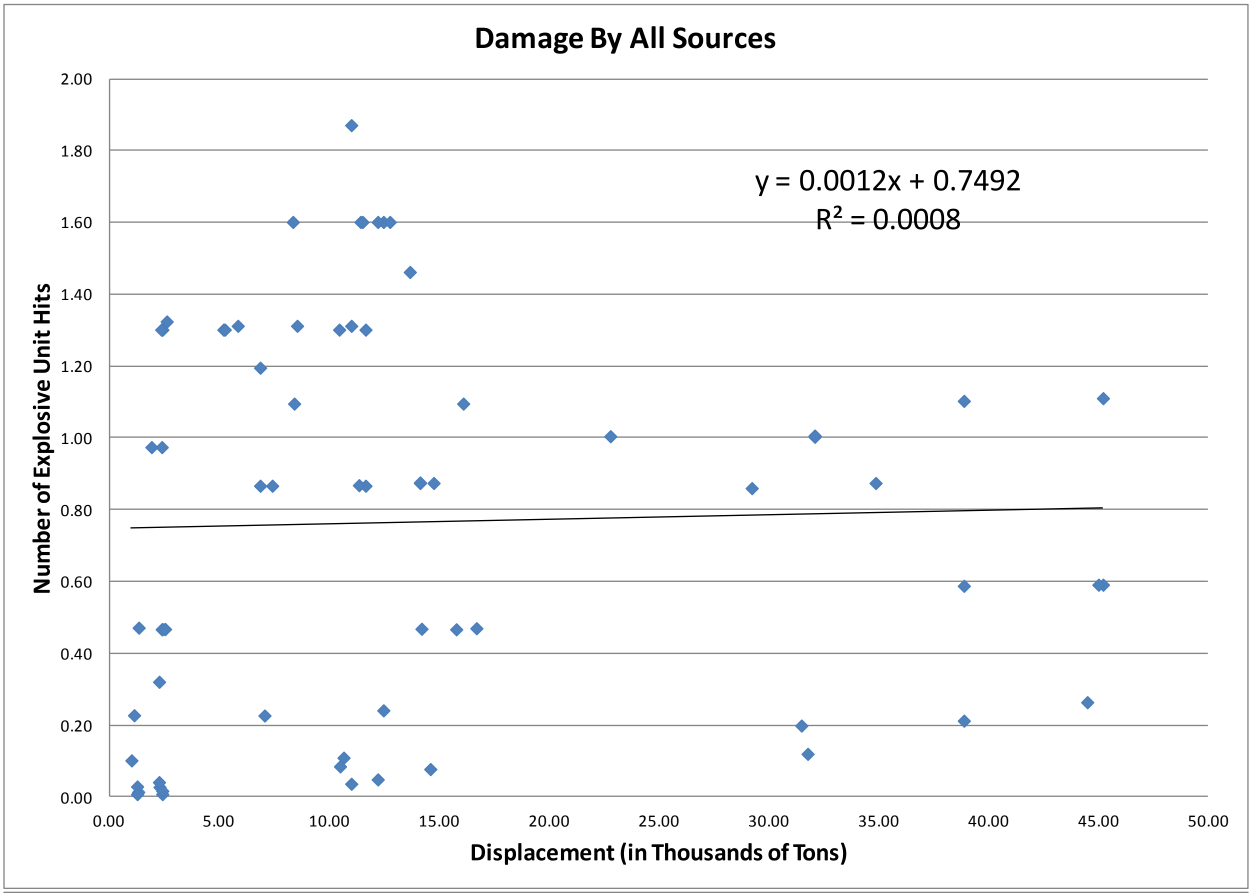


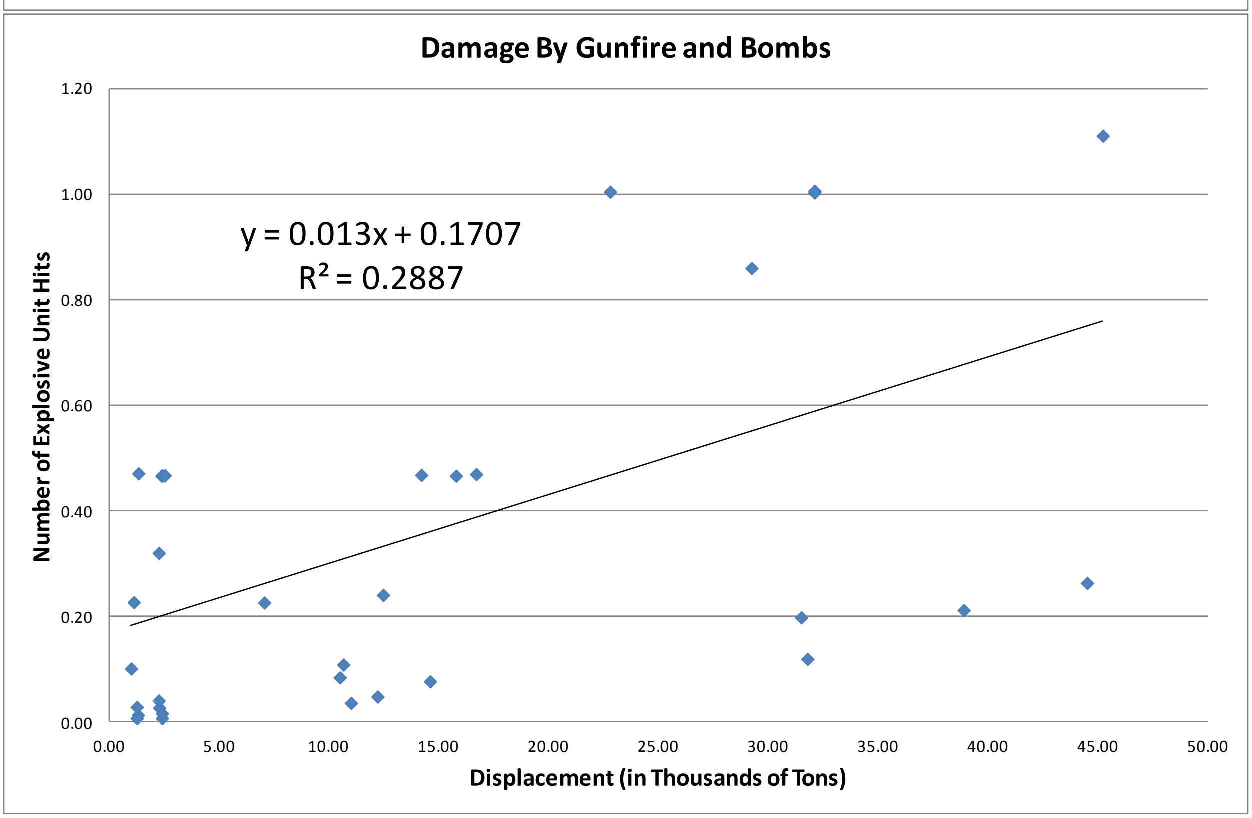


Damaged by Gunfire
Number of Expolosive Unit Hits
Displacement (Thousands of Tons)
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00
y = 0.0049x + 0.028
R² = 0.6657



Damaged by Bombs
Number of Explosive Unit Hits
Displacement (Thousands of Tons)
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00
y = 0.0187x + 0.3346
R² = 0.8113


Damage By All Sources
y = 0.0012x + 0.7492
R² = 0.0008
Number of Explosive Unit Hits
Displacement (in Thousands of Tons)
2.00
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00


Damage By Gunfire and Bombs
y = 0.013x + 0.1707
R² = 0.2887
Number of Explosive Unit Hits
Displacement (in Thousands of Tons)
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00

3. To achieve the objectives, the student must correctly enter the data into EXCEL, graph it, determine the linear models and R^2 values, and prove or disprove the hypothesis: **The greater the full load displacement of a ship (a measure of how big the ship is) the more shells, bombs, or torpedoes it takes to put her out of action.**

    a. The first thing the student should look for is whether the slope of the fitted line is positive or negative. A positive slope suggests a positive correlation between displacement and the number of weapons of the given type needed to put a ship out of action.

    b. The second thing the student should look for is whether the R^2 value is greater than 0.5. If it is, the linear model is a good fit for the data. If not, the linear model is not a good fit and is, therefore, not useful as a predictor.

    c. The conclusion the student should draw is that the two models, "Damaged by Gunfire" and "Damaged by Bombs" are useful. The others are not.

    d. To exceed the standard, the student should conclude that the reason "Damaged by Torpedoes" has a slightly negative correlation is because a single torpedo will put a ship out of action regardless of the ship's size.

Name: ________________________________

Task II C – A / G – Analyzing Staying Power
This task is due no later than March 21st, 2014

1. **Task Checklist**

Task	Date Due	Completed?
Enter data into spreadsheet. Print and attach spreadsheet.		
Compete 1st graph: Damaged by Torpedoes		
Complete 2nd graph: Damaged by Gunfire		
Complete 3rd graph: Damaged by Bombs		
Complete 4th graph: Damaged by Guns and Bombs		
Complete 5th graph: Damaged by Guns, Bombs, and Torpedoes		
Print all five graphs and attach to this paper.		
Complete table of linear equations and R^2 values on the previous page.		
Answer the question: "Of the five linear equation models, which should not be used?" in the space provided on the previous page.		
Complete Final Analysis	Mar 21st	

2. **Historical Background.** Even before a ship like *Massachusetts* is built, the builder needs to understand how much armor and how much speed he needs to build into the ship to give her a reasonable chance of surviving in a battle. In other words, he needs to decide the minimum amount of staying power that must be built into the ship.

   To do this, he might employ mathematical analysts such as you to evaluate damage to ships from past battles in order to determine just what it took to put a ship like *Massachusetts* out of action. In World War II, the weapons used against ships were gun shells, bombs, and torpedoes. As the analyst, therefore, you might want to look at historical data to determine how many shells, bombs, or torpedoes put ships in past battles put ships out of action.

3. **Hypothesis:** Since we are trying to figure out just what it took in terms of gun shells, bombs, or torpedoes to put a ship out of action, we need to develop a hypothesis of the relationship between a ship and the amount of damage she can take. Since it makes sense that the bigger the ship is, the greater number of shells, bombs, or torpedoes it takes to put her out of action, our hypothesis could be:

The greater the full load displacement of a ship (a measure of how big the ship is) the more shells, bombs, or torpedoes it takes to put her out of action.

We will test this hypothesis by analyzing data.

4. **Data.**[15] Below is the data set you will enter into a spreadsheet. It includes data on what it took to put warships out of action in World Wars I and II. It is very detailed and must be entered carefully. You can read it as follows:

   a. **Column 1** gives the names of ships that were damaged in World War I or World War II.
   b. **Column 2** gives the nationality of each ship.
   c. **Column 3** gives the full load displacement of the ship in tons. This is a measure of how big the ship is.
   d. **Column 4** gives the number of explosive units that hit the ship to put it out of action.
   e. **In analyzing this data, you will be looking for a correlation between full load displacement (the *x* – variable) and the number of hits it takes to put the ship out of action (the *y* – variable).**

   f. The data is divided into three parts. The first part lists ships put out of action by torpedoes, the second part by gunfire, and the third by bombs.

[15] Data comes from LT Thomas R. Beall, USN, "The Development of a Naval Battle Model and Its Validation Using Historical Data," (Monterey, CA, Unpublished Naval Postgraduate School Master's Thesis, 1990), 102 - 104.

SHIP	NATIONALITY	x variable - Displacement (Thousands of Tons)	y variable - Number of Explosive Unit Hits
DAMAGED BY TORPEDOES			
GNEISENAU	Germany	38.90	0.59
LITTORIO	Italy	45.24	0.59
VENETO	Italy	45.03	0.59
ARETHUSA	Great Britain	7.40	0.87
MANCHESTER	Great Britain	11.65	0.87
PHOEBE	Great Britain	6.85	0.87
GLASGOW	Great Britain	11.35	0.87
DENVER	U.S.	14.13	0.87
INDEPENDENCE	U.S.	14.75	0.87
INTREPID	U.S.	34.88	0.87
HOUSTON	U.S.	14.13	0.88
HATSUKAZI	Japan	1.90	0.97
KISUMI	Japan	2.37	0.97
LUTZOW	Germany	16.10	1.10
NURNBERG	Germany	8.40	1.10
SCHARNHORST	Germany	38.90	1.10
CLEOPATRA	Great Britain	6.85	1.20
CAPETOWN	Great Britain	5.18	1.30
COVENTRY	Great Britain	5.24	1.30
FIJI	Great Britain	10.45	1.30
HAMBLETON	Great Britain	2.40	1.30
KEARNEY	U.S.	2.40	1.30
KELLY	Great Britain	2.35	1.30
LIVERPOOL	Great Britain	11.65	1.30
AGANO	Japan	8.53	1.31
KUMANO	Japan	11.00	1.31
TAMA	Japan	5.83	1.31
SELFRIDGE	U.S.	2.60	1.32
MYOKO	Japan	13.67	1.46
CHICAGO	U.S.	11.42	1.60
JUNEAU	U.S.	8.34	1.60
NEW ORLEANS	U.S.	12.46	1.60
PENSECOLA	U.S.	11.51	1.60
PORTLAND	U.S.	12.76	1.60
ST. LOUIS	U.S.	12.21	1.60
KUMANO	Japan	11.00	1.87

SHIP	NATIONALITY	x variable - Displacement (Thousands of Tons)	y variable - Number of Explosive Unit Hits
DAMAGED BY GUNFIRE			
SCHARNHORST	Germany	38.90	0.21
SOUTH DAKOTA	U.S.	44.52	0.26
HIEI	Japan	31.79	0.12
WARSPITE	Great Britain	31.50	0.20
EXETER	Great Britain	10.49	0.08
AOBA	U.S.	10.65	0.11
BOISE	U.S.	12.21	0.05
SAN FRANCISCO	U.S.	12.46	0.24
ONSLOW	Great Britain	2.27	0.03
RALPH TALBOT	U.S.	2.25	0.04
AARON WARD	U.S.	2.40	0.01
ACASTA	Great Britain	1.30	0.01
BROOKE	Great Britain	1.25	0.03
ONSLOW	Great Britain	1.25	0.01
DEFENDER	Great Britain	0.99	0.10
GWIN	Great Britain	2.40	0.02
NORFOLK	U.S.	14.60	0.08
EXETER	Great Britain	11.00	0.04
DAMAGED BY BOMBS			
ITALIA	Italy	45.24	1.11
ILLUSTRIOUS	Great Britain	29.24	0.86
SHOKAKU	Japan	32.11	1.00
ZUIKAKU	Japan	32.11	1.01
SHOKAKU	Japan	32.11	1.01
ZUIHO	Japan	14.20	0.47
KYUHO	Japan	16.70	0.47
AMAGI	Japan	22.80	1.00
MARBLEHEAD	U.S.	7.05	0.23
MOGAMI	Japan	1.32	0.47
MAYA	Japan	15.78	0.47
NAGANAMI	Japan	2.52	0.47
SHAW	U.S.	1.11	0.23
MAYRANT	Great Britain	2.25	0.32
MINEGUMO	Japan	2.37	0.47
MATSUYAKI	Japan	2.39	0.47
ISONAMI	Japan	2.39	0.47
NOWAKI	Japan	2.50	0.47

5. **Getting Started**. To complete this part of the final project, you must do the following:

    a. **First Task:** Enter all of the data into a spreadsheet completely and accurately. **Attach a print – out of this spreadsheet to this paper.**

    b. **Second Task:** Create five graphs of the data, insert **linear trend lines, trend line equations and the R^2 values** for:

        i. *Graph 1:* ‘Displacement’ vs. ‘Number of Explosive Unit Hits’ for all ships damaged by torpedoes. **Attach a print – out of this graph to this paper.**

        ii. *Graph 2:* ‘Displacement’ vs. ‘Number of Explosive Unit Hits’ for all ships damaged by gunfire. **Attach a print – out of this graph to this paper.**

        iii. *Graph 3:* ‘Displacement’ vs. ‘Number of Explosive Unit Hits’ for all ships damaged by bombs. **Attach a print – out of this graph to this paper.**

        iv. *Graph 4:* ‘Displacement’ vs. ‘Number of Explosive Unit Hits’ for all ships damaged by bombs and guns. **Attach a print – out of this graph to this paper.**

        v. *Graph 5:* ‘Displacement’ vs. ‘Number of Explosive Unit Hits’ for all ships damaged by torpedoes, bombs and guns. **Attach a print – out of this graph to this paper.**

c. **Third Task: Record the results in the table below:**

	Linear Equation	**R^2 Value**
Damaged by Torpedoes		
Damaged by Gunfire		
Damaged by Bombs		
Damaged by Gunfire & Bombs		
Damaged by Torpedoes, Gunfire, and Bombs		

The R^2 value is a measure of how good a linear equation model is – how well it will predict future outcomes. The closer to "1.00" the R^2 value is, the better the linear equation model is. An R^2 value of less than "0.5" is generally an indication that the linear model should not be used.

d. **Fourth Task: Answer this question:** Of the five linear equation models, which should not be used?

____

____

____

____

____

e. **Fifth Task:** Complete the final analysis when handed out by the instructor.

Name:____________________________________

Final Analysis

Part I: Have Your Analysis Results Been What You Expected?

In this analysis problem, we have been looking for a correlation between a ship's size (as measured by its displacement) and how many hits it will take to put it out of action. We have been testing the following hypothesis?

The greater the full load displacement of a ship (a measure of how big the ship is) the more shells, bombs, or torpedoes it takes to put her out of action.

Look at the five graphs, their linear equations, and their R^2 values. Does each, in fact show that as size (displacement) increases, more hits are necessary to put it out of action?

1. Ships damaged by torpedoes: Yes / No

2. Ships damaged by gunfire: Yes / No

3. Ships damaged by bombs: Yes / No

4. Ships damaged by bombs and guns: Yes / No

5. Ships damaged from all sources: Yes / No

If your answer was "No" to any of these, explain why ***in a short paragraph*** (3 - 4 sentences) below:

____

____

____

____

Part II: Which Models Should You Choose?

1. The following illustrations might help you answer the following questions:

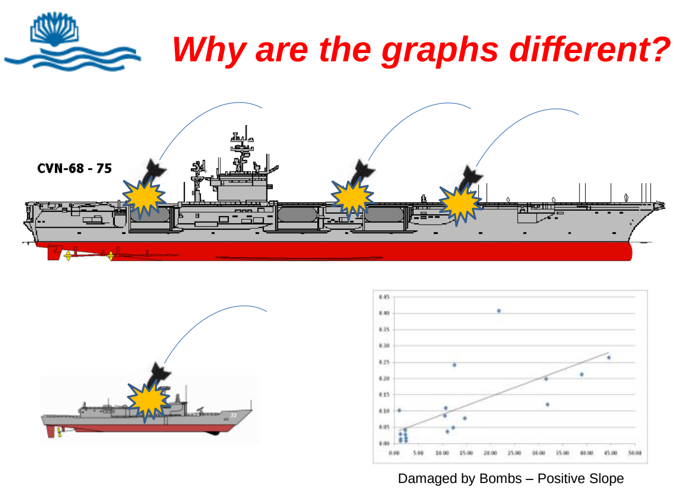


Figure 6: Damaged by Bombs

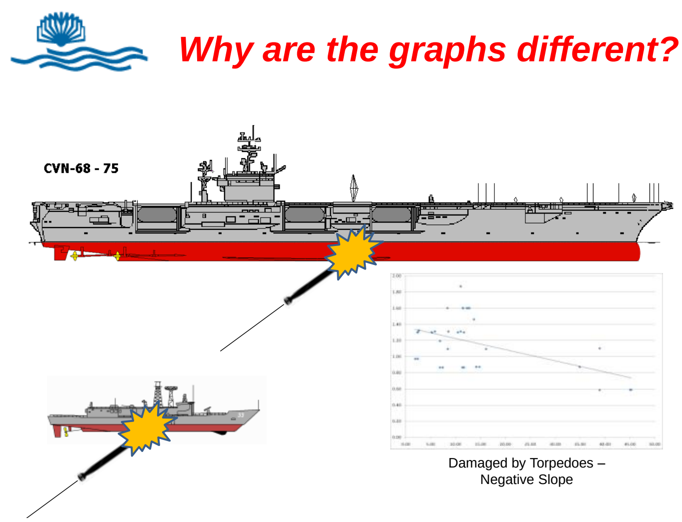


Figure 7: Damaged by Torpedoes

2. The model equation for "Damaged by Torpedoes" shows that as size (displacement) increases, the number of hits necessary to put a ship out of action goes down.

   In a short paragraph (2 – 3 sentences) below, discuss why this model may not be useful to predict how many hits are required to put a ship out of action.

3. The model equation for “Damaged by Torpedoes, Gunfire, and Bombs” shows that as size (displacement) increases, the number of hits necessary to put a ship remains about the same.

   In a short paragraph (2 – 3 sentences) below, discuss why this model may not be useful to predict how many hits are required to put a ship out of action.

4. The model equations for “Damaged by Gunfire”, “Damaged by Bombs”, and “Damaged by Gunfire and Bombs” do show clear relationships between size (displacement) and the number of hits necessary to put a ship out of action.

   In a short paragraph below (at least 3 – 4 sentences), discuss which of the three models, “Damaged by Gunfire”, “Damaged by Bombs”, or “Damaged by Gunfire and Bombs” would be useful to the builder (you may choose more than one). Why would it / they be useful to him?

Task II D – G – Ship Stability

Teacher Notes

Objectives:

1. Understand the importance of stability calculations in the design and construction of a ship.
2. Apply trigonometric functions to determine metacentric height and righting arm of a ship.
3. Graph and interpret stability curves using Microsoft EXCEL.
4. Develop self-confidence and resiliency by completing a challenging and unfamiliar task in which all steps may not be clearly identified.

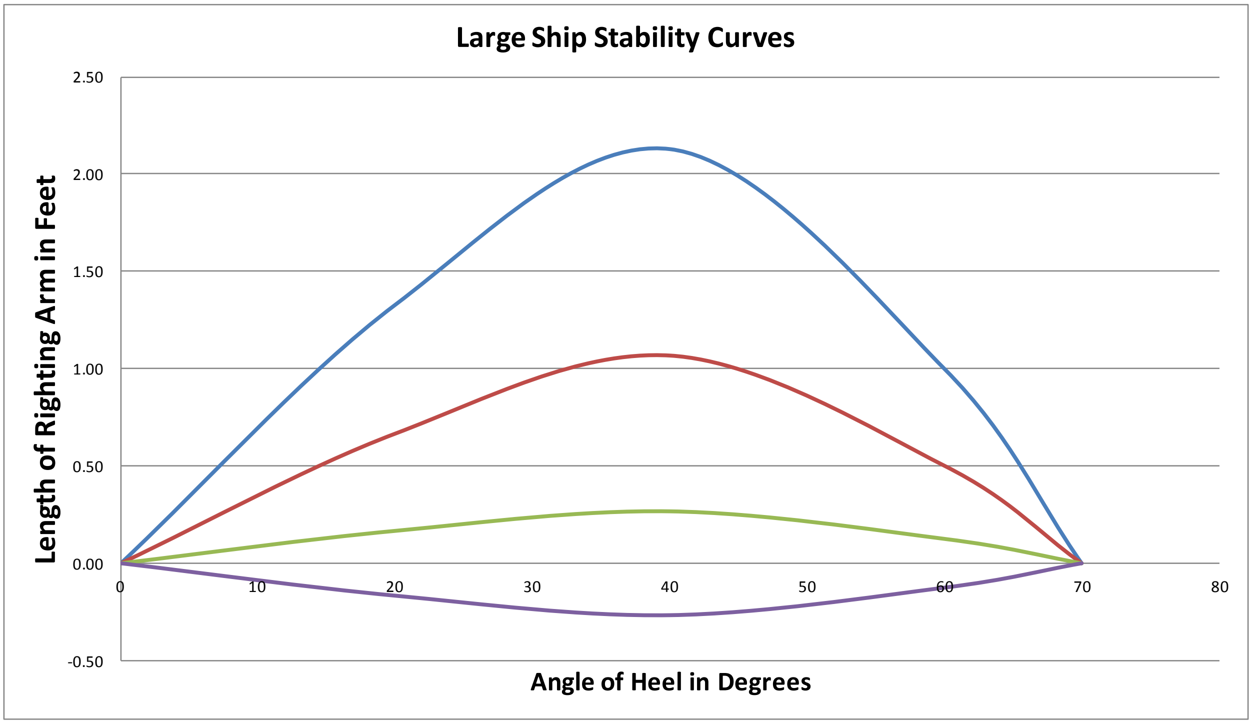
Teacher Support:

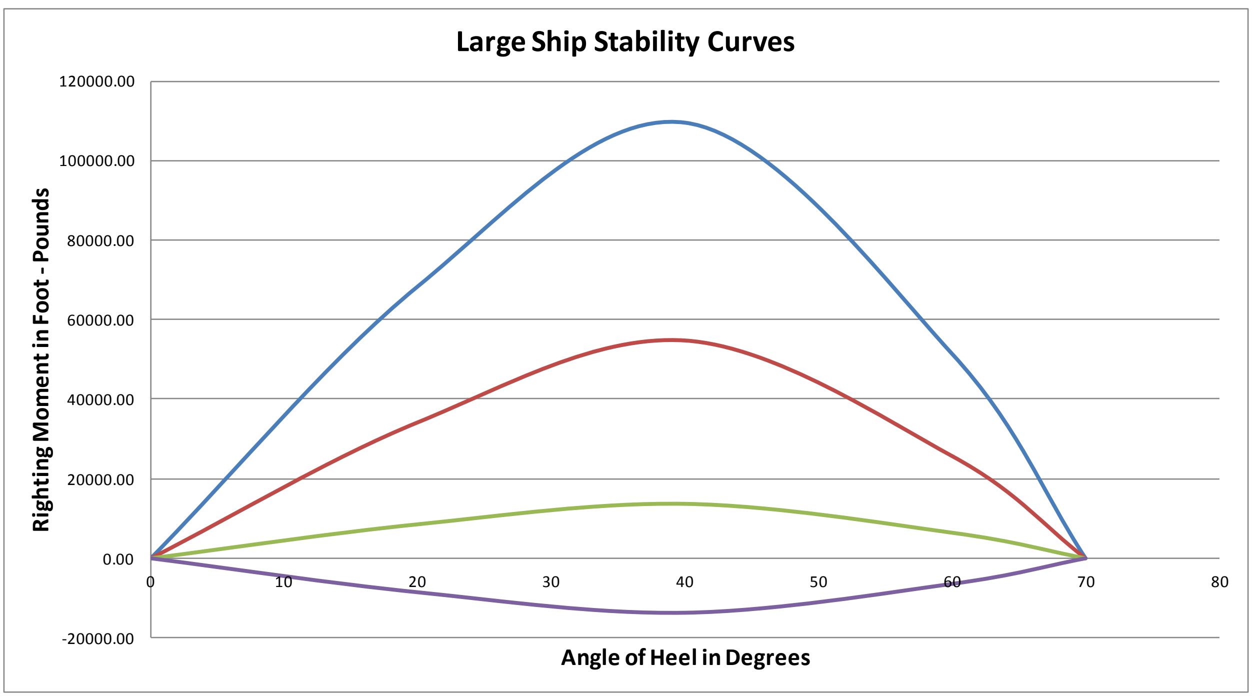
1. The formula asked for in paragraph 7 is: $\frac{GZ}{GM}$.

2. The following is the completed table for task 1:

θ	sin θ	cos θ	GM (in feet)	GZ (in feet)	ZM (in feet)
5°	0.087	0.996	7.18	0.625	7.151
10°	0.174	0.985	7.18	1.249	7.072
15°	0.259	0.966	7.18	1.860	6.936
20°	0.342	0.940	7.18	2.460	6.749
25°	0.423	0.906	7.18	3.037	6.505
30°	0.500	0.866	7.18	3.590	6.218

3. The following are the two graphs which are created from the table of data:





4. The answers to the task 3 questions are:

   a. At what angle of heel is the righting arm longest and the righting moment strongest?

      Approximately 40°.

   b. If the ship heels more than 40° will the ship be more or less likely to capsize (tip over on her side). Why?

      More likely because the righting moment increasingly diminishes as the angle of heel increases beyond 40°. In other words, the force to right the ship diminishes.

   c. The "negative stability" curve is negative, below the x – axis. In this case, what will the "moment" do to the ship?

      Capsize the ship.

Name: ____

Task II D – G – Ship Stability
This task is due no later than March 28th, 2014

1. **Background.** A ship floats because there are two forces acting on its hull in opposite directions.

   a. *Gravity* (G) is the force that is pulling the ship toward the center of the Earth. It is the same force that holds you to the Earth.
   b. *Buoyancy* (B) is the force that is caused by the ship pushing down on the surface of the water because of gravity. The water pushes back.

   G and B are vectors with direction and magnitude. When they act on a ship in opposite directions with equal magnitude (called opposing each other), the ship achieves *equilibrium*, the state in which it is floating on the water, not sinking and not moving up out of the water.

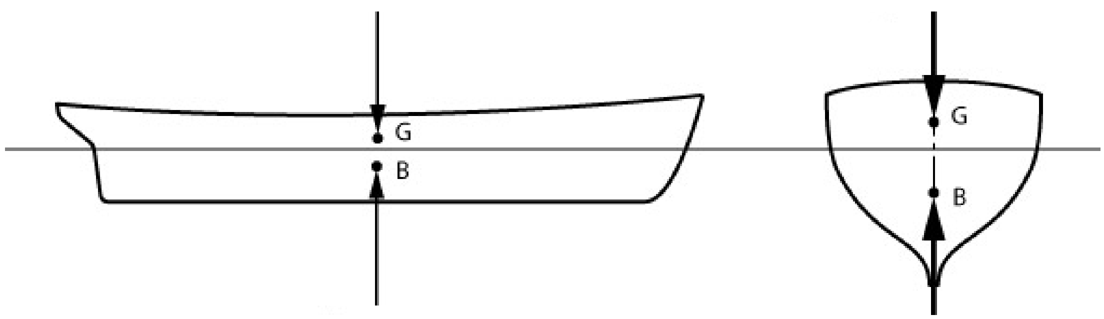


Figure 8: Vectors B and G opposing each other to keep a ship afloat.

2. If a ship did not rock and move with the flow of the sea, figure 8 would be a complete portrayal of how a ship stays afloat. The ship does move however and its these two vectors which bring it back into an upright condition. In figure 9, the ship is tipping (heeling) to the right. B is to the right of G so the two act to shift the ship back into an upright position.

   What do you think would happen if B was to the left of G as the ship heeled to the right?

   ____

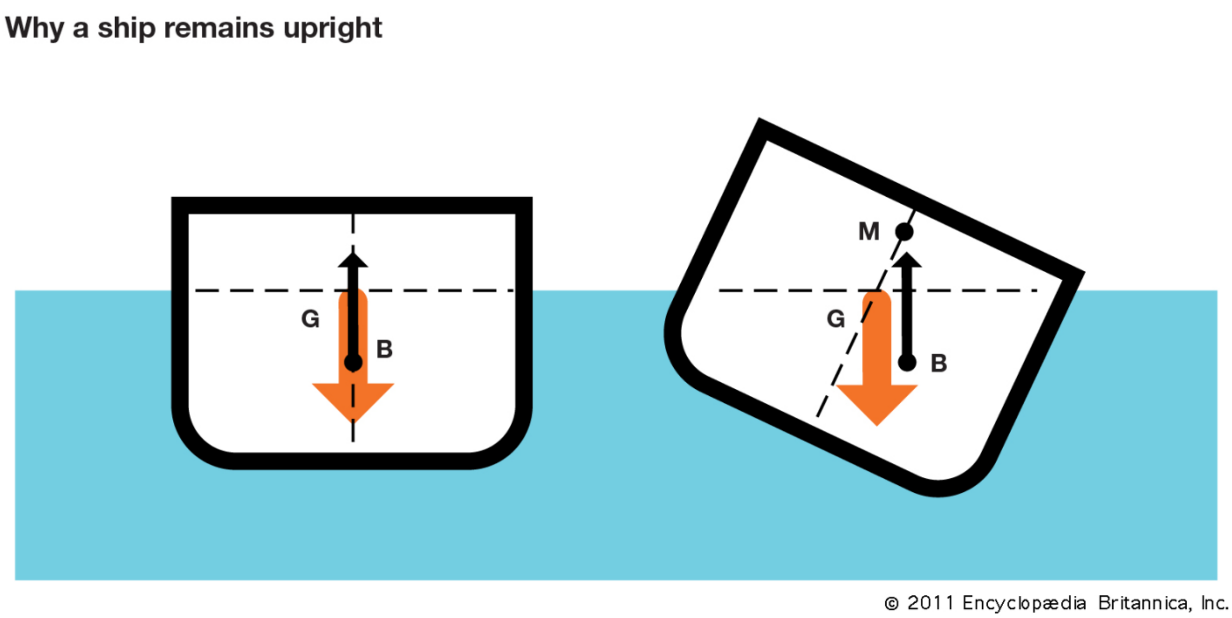


Figure 9: Why a ship remains upright.

3. When a ship heels, the G vector and the B vector do not directly oppose each other. Instead, they form an angle θ as you see in figure 10 below:

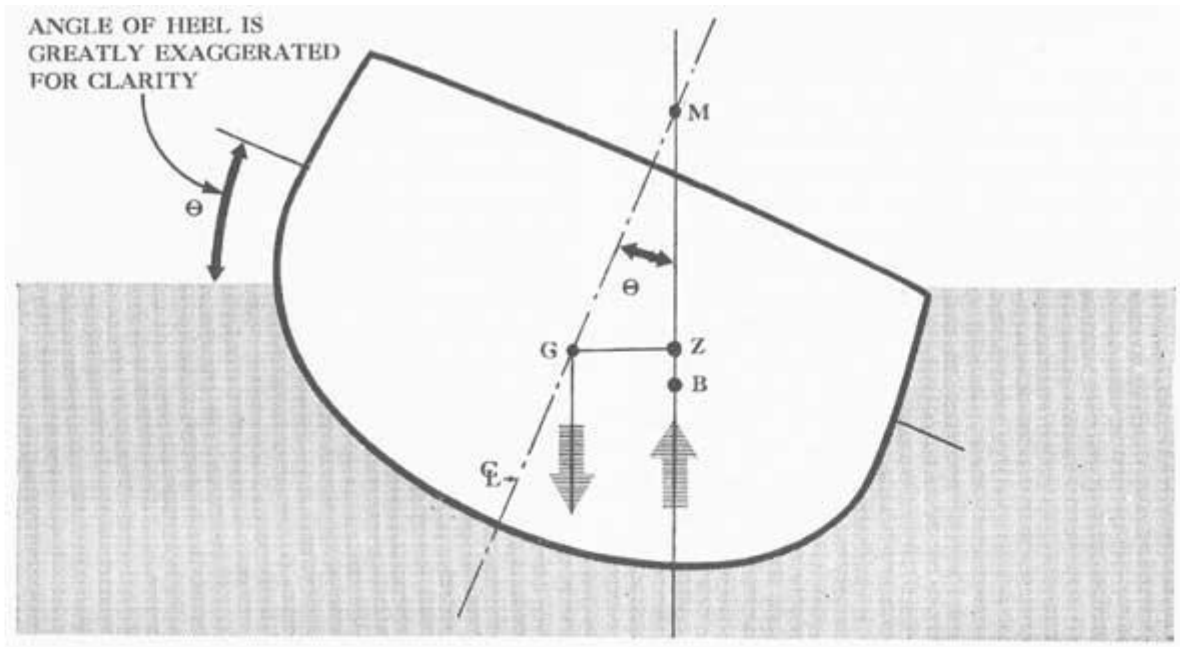


Figure 10: Ship healing to right, angle formed by G and B.

4. The apex of the angle formed by G and B is called the *metacentric height* (M).

Since M is the apex of the G and B vectors, it is a function of those vectors and, therefore, a measure of the ship's overall *stability* – its ability to remain afloat in an upright position.

5. The segment GZ in figure 10 is called the *righting arm*. The longer it is, the more force is exerted to bring the ship back to an upright position.

6. Recall from trigonometry the following relationships:

$$sin\ \theta = \frac{opposite}{hypotenuse},\quad cos\ \theta = \frac{adjacent}{hypotenuse}$$

7. In triangle ΔGMZ with ∠M = θ, therefore:

   $sin\ \theta =$ __________

8. For angles of heel of 30° or less, we can use the formulas in paragraph 7 to determine the length of the righting arm (GZ) and the metacentric height (GM).

   Task 1: Use the formulas to complete the following table:

θ	sin θ	cos θ	GM (in feet)	GZ (in feet)	ZM (in feet)
5°			7.18	0.625	7.151
10°			7.18		
15°				1.860	6.936
20°				2.460	
25°					6.505
30°				3.590	

9. The table above constitutes the basic stability table for U.S.S. *Massachusetts* (BB 59). Using this and other data gathered from a series of experience, naval architects (ship designers) can develop stability diagrams.

10. **Task 2:** The table below incorporates data from those experiments. Enter this data into Microsoft EXCEL and graph it. You need one graph to include all four "Length of Righting Arm" columns and one to include all four "Righting Moment" columns. A total of two graphs. Attach both to this handout when you turn it in.

Angle of Heel (in degrees)	Length of Righting Arm in Feet	Righting Moment (Normal Stability)	Length of Righting Arm in Feet (Reduced Stability)	Righting Moment (Reduced Stability)	Length of Righting Arm in Feet (Reduced Stability 2)	Righting Moment (Reduced Stability 2)	Length of Righting Arm in Feet (Negative Stability)	Righting Moment (Negative Stability)
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	1.33	68344.71	0.67	34172.36	0.17	8543.09	-0.17	-8543.09
40	2.13	109454.31	1.07	54727.16	0.27	13681.79	-0.27	-13681.79
60	1.00	51387.00	0.50	25693.50	0.13	6423.38	-0.13	-6423.38
70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

11. **Task 3:** Answer the following questions based on the data in the tables and the graphs.

a. At what angle of heel is the righting arm longest and the righting moment strongest?

____

b. If the ship heels more than 40° will the ship be more or less likely to capsize (tip over on her side). Why?

____

c. The "negative stability" curve is negative, below the x – axis. In this case, what will the "moment" do to the ship?

____

Name: ____

9th Grade Final Project: Mathematics and History

Part III: Geometry – Building the Ships

1. **Historical Background.** In the spring of 1942, the United States was on the defensive in the Pacific. The Japanese Empire had conquered vast territories in the Pacific Ocean including the Dutch East Indies (present day Indonesia), the Philippine Islands, Korea, Vietnam, and all of Eastern China (see figure 1). The U. S. Pacific Fleet was still recovering from the Pearl Harbor attack while the U. S. Asiatic Fleet and the British East Indies Fleet had been destroyed in battles with the Japanese.

   While Admiral Chester Nimitz and his Pacific Fleet of three aircraft carrier task forces was holding the American line at the Hawaiian Islands, the U. S. Navy was rapidly building new ships to replace those lost at Pearl Harbor on December 7th, 1941. One of these ships, the *South Dakota* - class battleship U.S.S. *Massachusetts* (BB 59) was completed in May 1942, just in time to join the Fleet to help the Americans begin offensive operations against the Japanese.

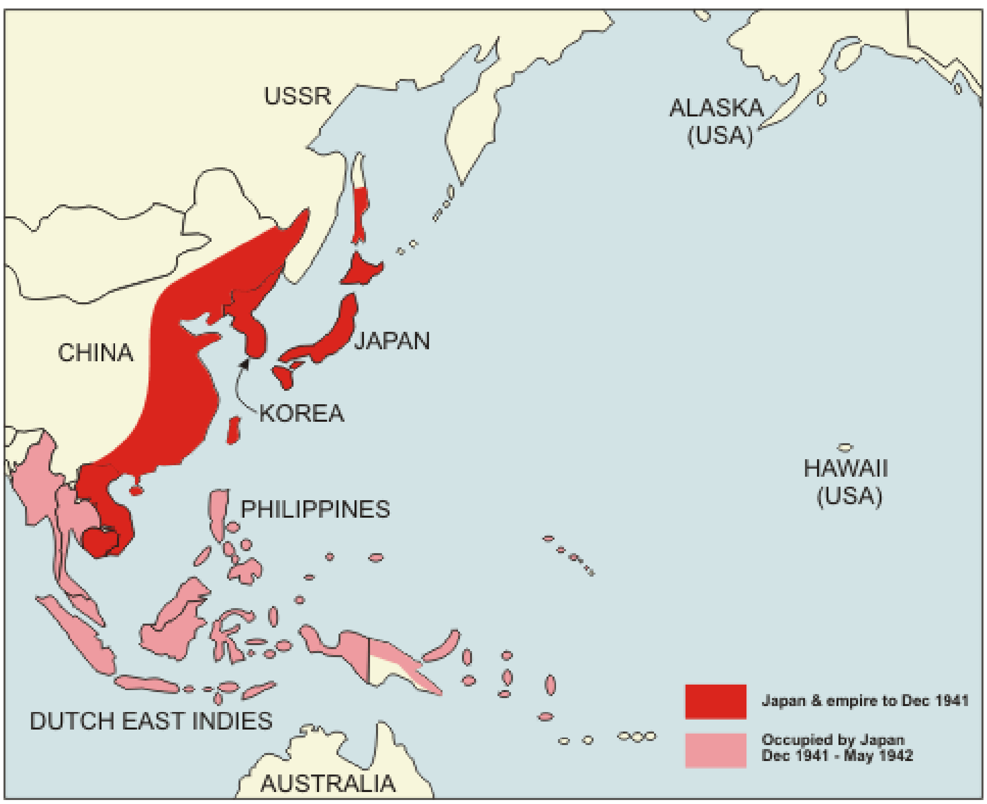


Figure 1: Japanese Empire May 1942

2. **U.S.S. *Massachusetts* (BB 59) Design Specifications.** *Massachusetts* was a battleship, a type of ship originally designed to fight with and against other battleships in great gun battles at sea. By 1942, naval aircraft flown from aircraft carriers and armed with bombs or torpedoes had replaced battleships as the primary offensive weapons at sea. Still, battleships like *Massachusetts* played important roles such as defending the fragile aircraft carriers from enemy attack and using their large caliber guns to bombard enemy positions on shorelines in preparation for attacks from the sea by U.S. Marines and U.S. Army troops.

   Specifications:

Displacement:		43,884 tons
Dimensions:	Length overall:	680 ft., 9.813 in.
	Maximum beam:	108 ft., 2.250 in.
	Maximum draft:	36 ft., 9.000 in.
Armament:	9 16 - inch / 45 caliber (Mark 6) guns	
	16 5 - inch / 38 caliber (Mark 12) guns	
	Assorted lower caliber guns	

3. **Field Trip to Battleship Cove.** In conjunction with this part of the project, and in order to see just what ships of the World War II period looked like, we will take a field trip to Battleship Cove in Fall River, MA. Information can be found at: http://www.battleshipcove.org/.

Part III: Geometry – Building the Ships

Task III A G/A: Drawing U.S.S. *Massachusetts*
Teacher Notes

Objectives:

1. Identify correct scale factor.
2. Use scale factor to produce a scaled drawing that is similar in dimensions to the actual item being drawn.
3. Develop self-confidence and resiliency by completing a challenging and unfamiliar task in which all steps may not be clearly identified.

Teacher Support:

1. The scale factor is determined by the following equation:

$$\frac{(3\ ft.)}{680.82\ ft.} = 0.0044$$

2. Each measure in feet from figure 4 is multiplied by this scale factor and then multiplied by 12 to convert the actual measure in feet to the scaled measure in inches. For example:

$$105\ ft. \times 0.0044 \times 12 \frac{in.}{ft.} = 5.54\ in.$$

3. The final scaled measures needed to prepare the drawing are listed in the following table:

Actual Dimension (feet)	Scale Factor	Drawing Dimension (feet)	Drawing Dimension (inches)
105.00	0.004406451	0.46	5.55
75.00	0.004406451	0.33	3.97
15.00	0.004406451	0.07	0.79
680.82	0.004406451	3.00	36.00
30.00	0.004406451	0.13	1.59
60.00	0.004406451	0.26	3.17
90.00	0.004406451	0.40	4.76
120.00	0.004406451	0.53	6.35
143.00	0.004406451	0.63	7.56

Name: ______________________________

Task III A – G/A

Drawing U.S.S. *Massachusetts*

This task is due no later than April 14th, 2014

1. **Task:** You will make a precise drawing of U.S.S. *Massachusetts* that is 36 inches (3 feet) in length.

   a. To meet the standard of this task (a grade of "3") you must reproduce the outboard profile shown in figure 4.

   b. To exceed the standard of this task (a grade of "4") you must reproduce the outboard profile ***and*** overhead view shown in figure 4.

   c. To exceed the standard of this task (a grade of "5") you must reproduce the views in figure 4 ***and*** the inboard profile of figure 5.

   d. To exceed the standard of this task (with a grade higher than "5") you must reproduce the views of figure 4, the inboard profile of figure 5, ***and*** the machinery schematic of figure 6.

3. **Getting Started.** Answer the following questions about your drawing and your model.

   a. If the actual ship is 680 ft. 9.813 in. in length and your model is 36 inches in length, use this equation to find the number you would multiply the actual length by to get the model length:

$$Actual\ ship\ length\ \times scale\ factor \times \frac{12\ inches}{1\ foot} = Model\ length\ in\ inches$$

   b. The number you just calculated is called the **scale factor**. It is the number by which you will multiply all of the ship's dimensions to get the model's dimensions.

   Scale factor: ______________________________

Using the scale factor and the dimensions in the drawing above, calculate all of your drawing's dimensions:

Actual Dimension (feet)	Scale Factor	Drawing Dimension (feet)	Drawing Dimension (inches)
105.00			
75.00			
15.00			
680.82			
30.00			
60.00			
90.00			
120.00			
143.00			

Table 1: Drawing Dimensions

c. To begin your drawing:

i. Using graph paper, tape together enough sheets to produce one large sheet that is at least 36 inches long and 9 inches tall.

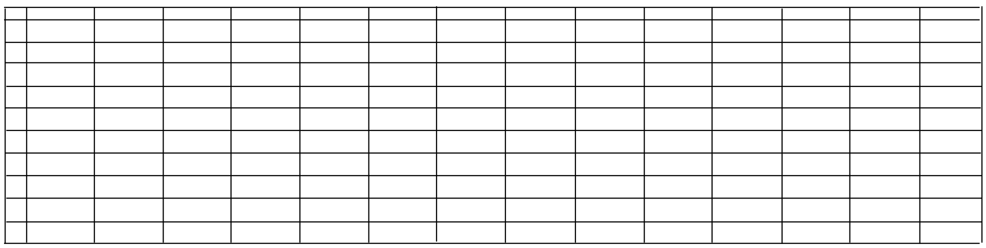
ii. Near the bottom of the large sheet, centered, draw a line 36 inches in length.

iii. Along this line, measure off segments that correspond to 100 feet in the drawing in figure 4 (multiply 100 by the scale factor and then by 12 inches to one foot). The last segment will be shorter.

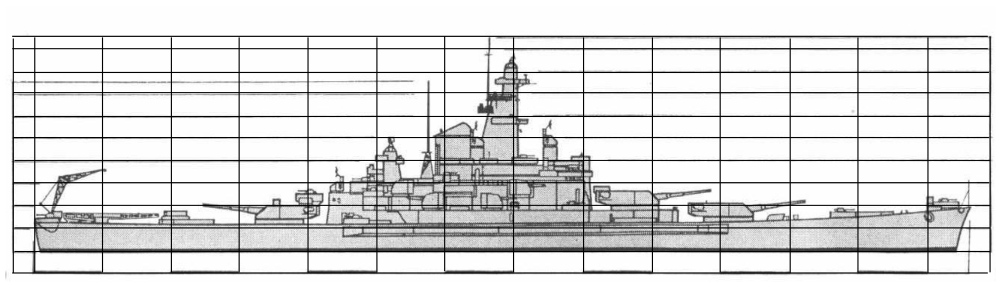
iv. At the left end of the 36 inch line, measure off segments that correspond to the measured segments on the left side of figure 3, using the "Drawing Dimension (inches)" numbers in Table 1.

v. Repeat using the measured segments on the right side of figure 3.

vi. You now have a reference grid in which to recreate the drawings in figures 4, 5, and 6. The grid might look something like figure 2 below, before and after the drawing is completed:



Before Drawing



After Drawing

Figure 2: Reference Grid

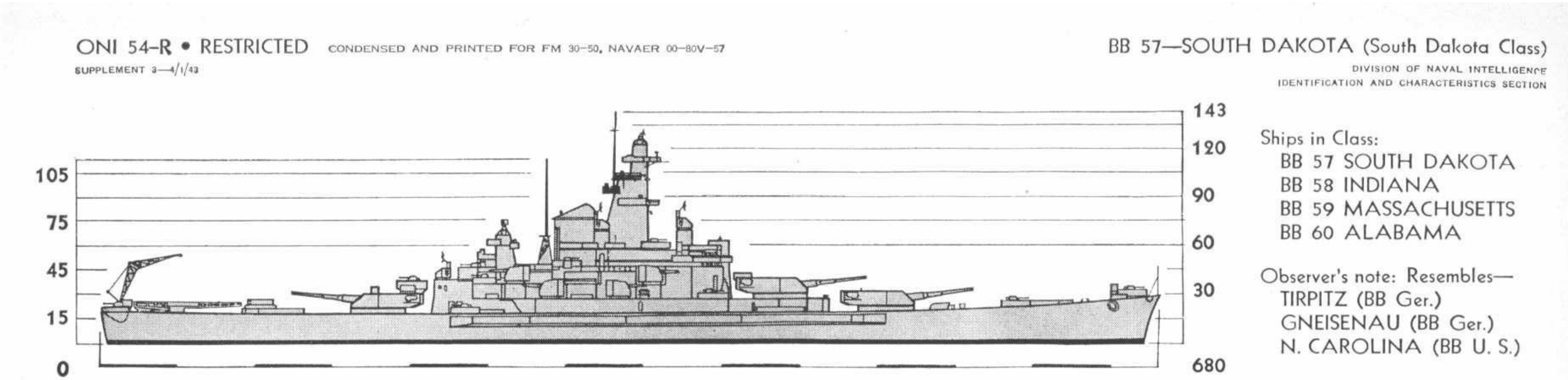


Figure 3: U.S.S. *Massachusetts* (BB-59) Dimensions

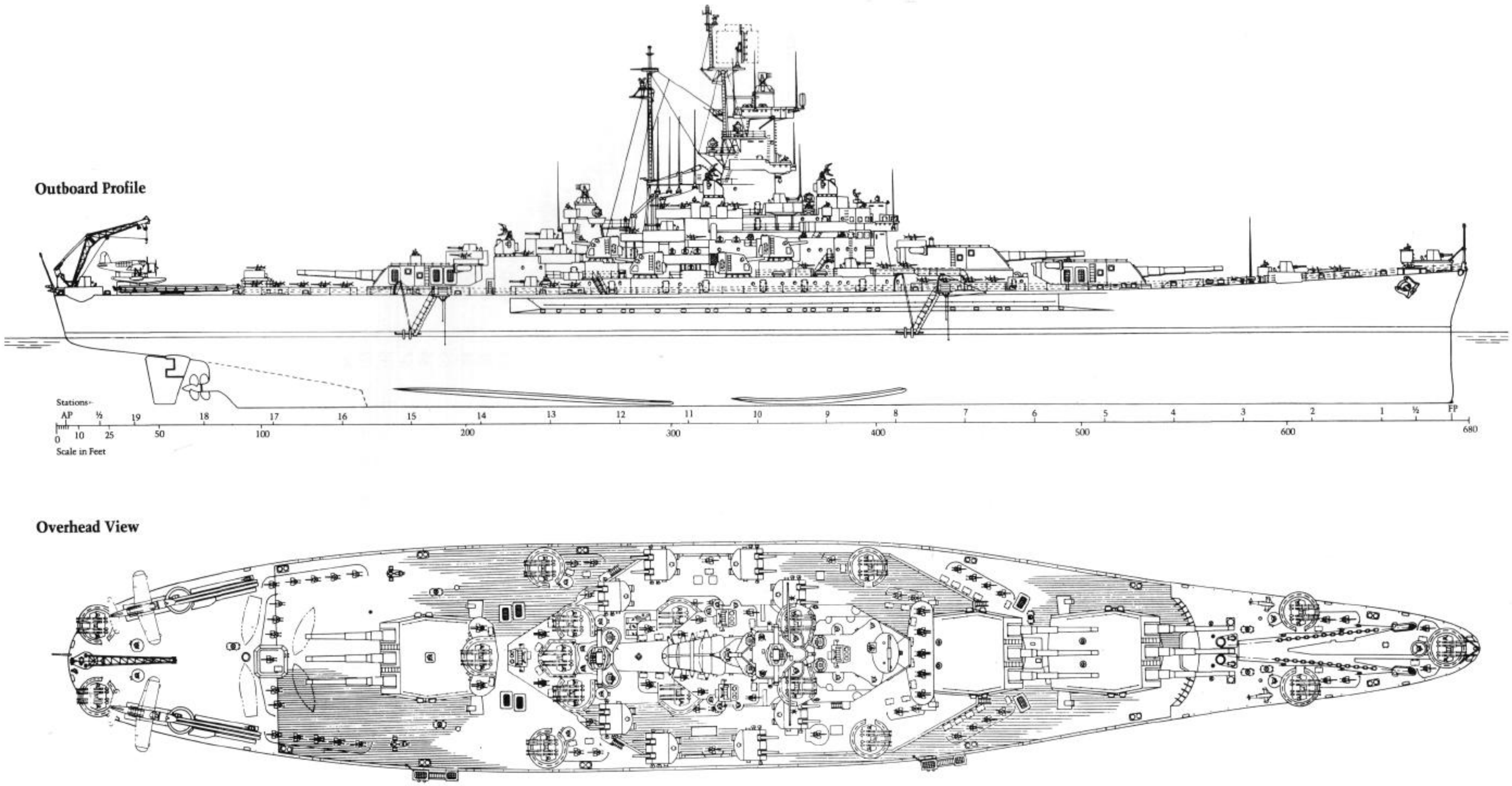


Figure 4: U.S.S. *Massachusetts* (BB – 59) Outboard and Overhead Views[16]

[16] Source: William H. Garzke, Jr. and Robert O. Dulin, Jr., *Battleships: United States Battleships, 1935 – 1992* (Annapolis: Naval Institute Press, Revised and Updated Edition, 1995), 104.

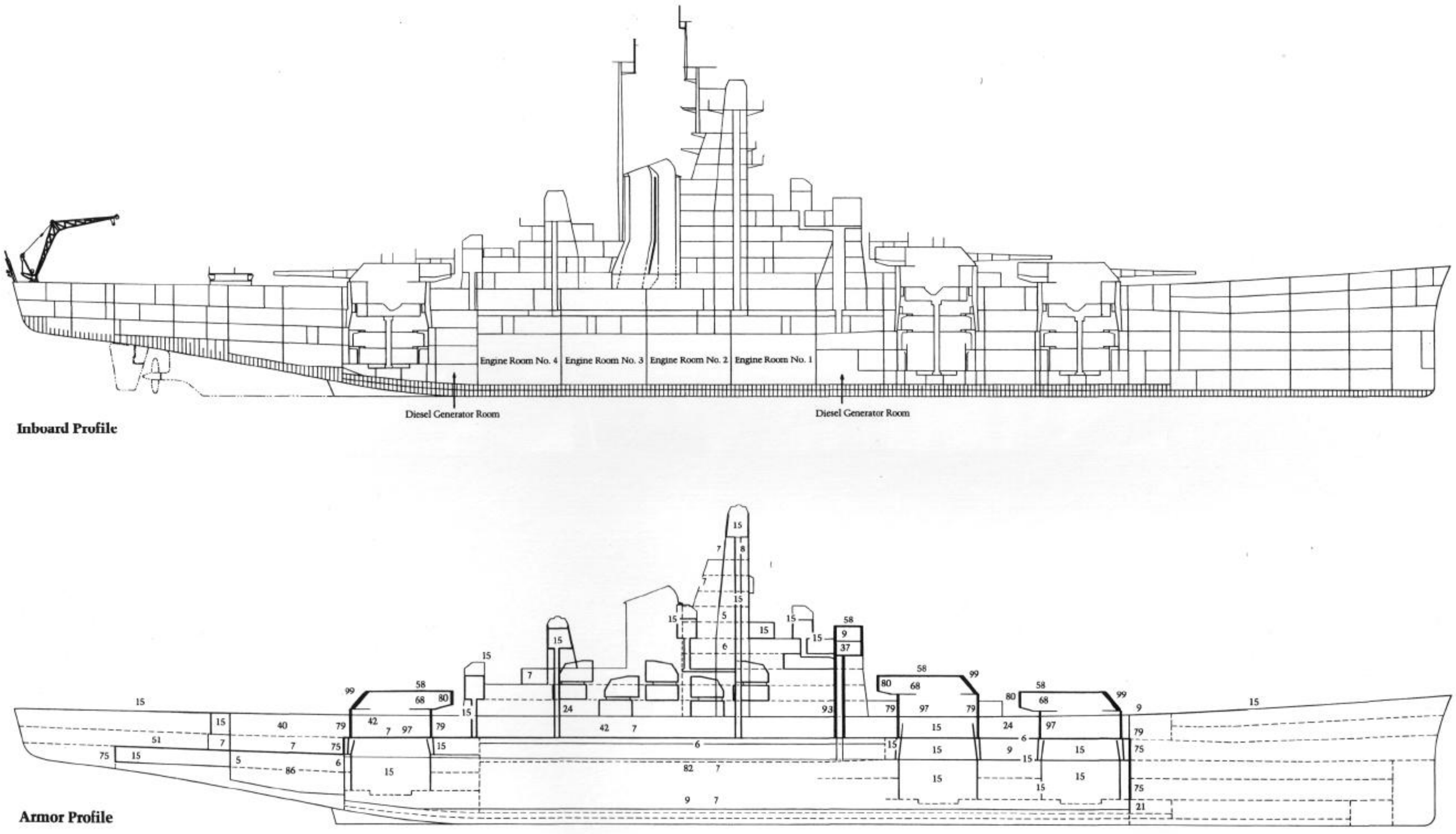


Figure 5: U.S.S. *Massachusetts* (BB – 59) Inboard and Armor Profiles[17]

[17] Source: William H. Garzke, Jr. and Robert O. Dulin, Jr., *Battleships: United States Battleships, 1935 – 1992* (Annapolis: Naval Institute Press, Revised and Updated Edition, 1995), 104.

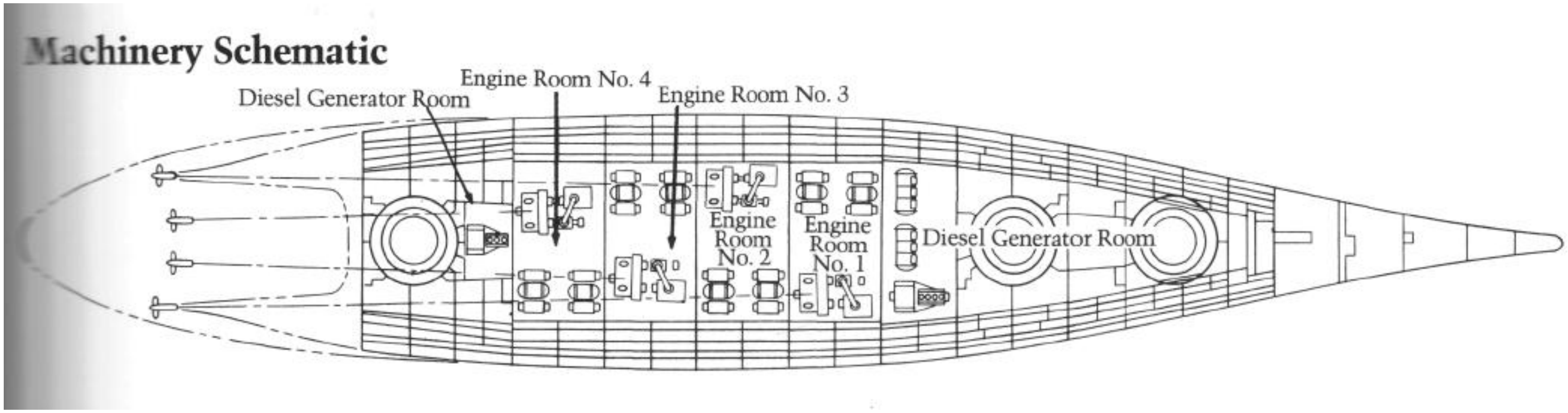


Figure 6: U.S.S. *Massachusetts* (BB – 59) Machinery Schematic[18]

[18] Source: William H. Garzke, Jr. and Robert O. Dulin, Jr., *Battleships: United States Battleships, 1935 – 1992* (Annapolis: Naval Institute Press, Revised and Updated Edition, 1995), 101.

Name: ___________________________________

Task III B – A / G
Understanding the Engineering Plant
This task is due no later than April 18th, 2014

1. **Task:**

    a. Reproduce the drawing of the Basic Ship Steam Cycle shown in Figure 7. The drawing should be done on a poster-board – sized sheet of paper. It should be labeled and color – coded just as in the original drawing below.

    b. From the document "U.S.S. *Massachusetts* Brief Description – Engineering Installation" find information on each component of the ship's steam engineering plant and place that information next to the component on your drawing. For example, for the boiler, you might put:

<u>Boiler</u>

1. **16500 gallons per hour into 600 psi. steam**
2. **850° F**
3. **Burns 1400 gallons of fuel oil per hour**

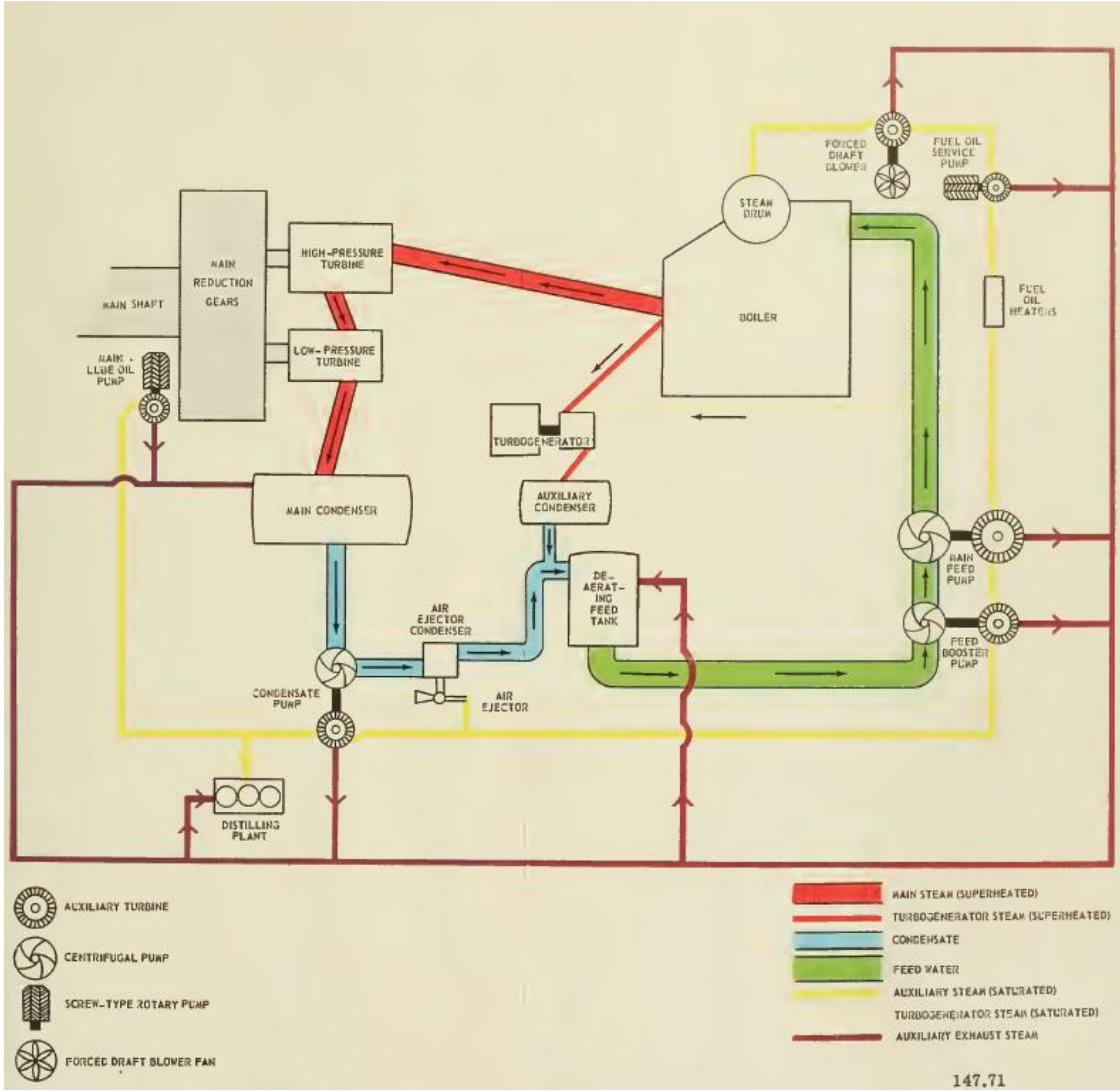


Figure 7: Basic Steam Cycle

Name: ___________________________________

Task III C – A
Constructing the Model

Teacher Notes

Objectives:

1. Reflect a detailed drawing accurately across an axis.
2. Create a three-dimensional shape from a two-dimensional drawing.
3. Develop self-confidence and resiliency by completing a challenging and unfamiliar task in which all steps may not be clearly identified.

Teacher Support: Students typically have the following problems with this task:

1. Achieving accuracy when reflecting the drawing. Many students try to draw it freehand. A better approach is to use a ruler and measure the distance of specific points on the completed half to the axis. These points can then be reflected precisely across the axis using those measurements. Once the points are reflected, it becomes a simple matter of connecting the dots to produce a fairly precise reflection.

2. Cutting the drawings to trace the frames. Many students try to cut all the edges first. Students should "cut and trace, cut and trace."

3. Creating the center portion. Once students have reflected the forward portion of the ship, the instructions clearly state that they are to cut out the entire drawing and trace it nine times. These form the center portion of the ship. *They do not need to trace the after portion nine times.*

4. Spacing the frames. The entire model is 36 inches long. Each frame should be evenly spaced using precise measurements. Students often overlook this step.

Name: ______________________________

Task III C – A
Constructing the Model
This task is due no later than May 2nd, 2014

Part 1: Preparing the Template

Directions: Each drawing is one half of the body plan of a naval architect's ship drawing. The first drawing depicts ½ of the ship as you would see it if you were standing directly in front of the ship looking at the bow. The second drawing depicts ½ of the ship as you would see it if you were standing directly behind the ship looking at the stern.

Your task today is to produce a precise mirror image of each drawing, connected to the first drawing so that you have a complete view of the plan of the forward part of the ship and the aft part of the ship.

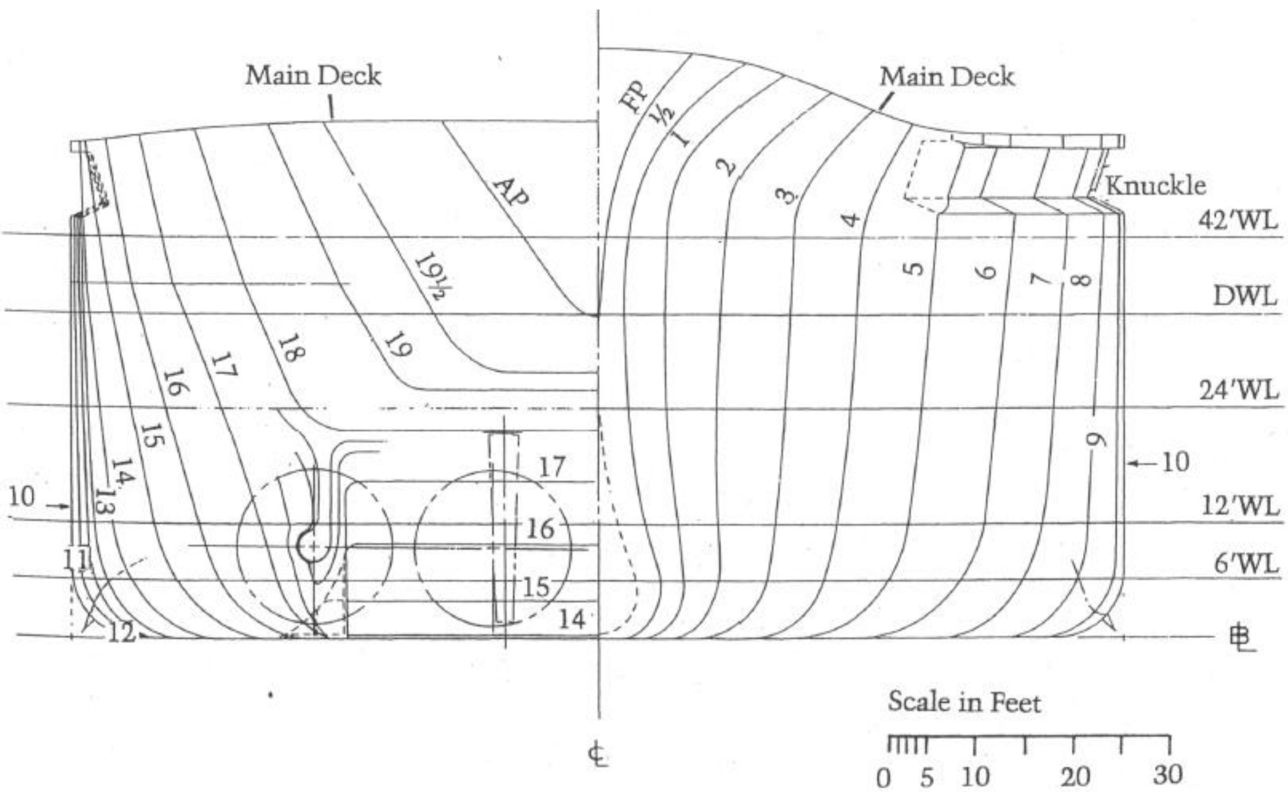


Figure 1: Complete Body Plan

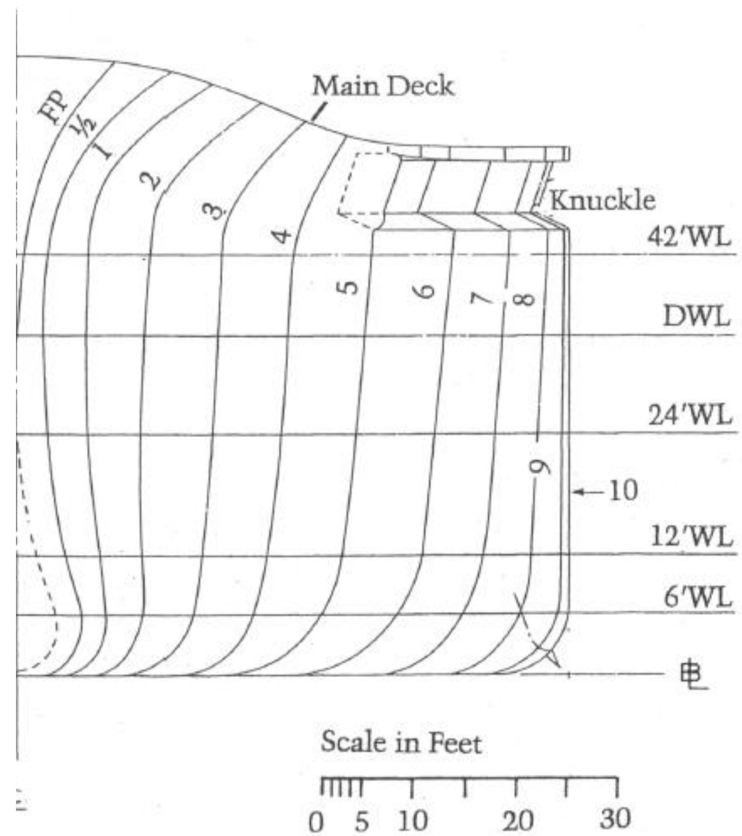


Figure 2: Body Plan – Forward Part of the Ship

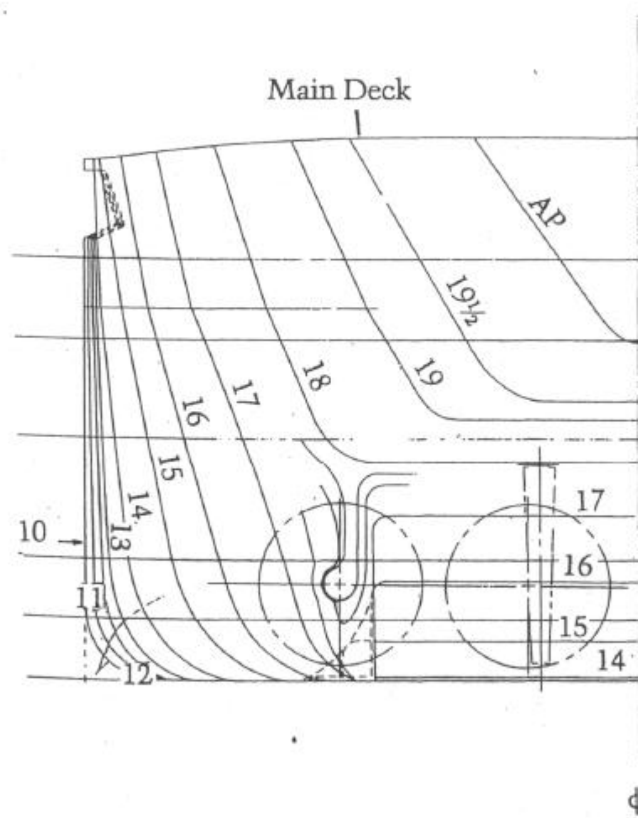


Figure 3: Body Plan – After Portion of the Ship

Name: ________________________________

Part 2: Constructing the Model

Getting Started: You will construct a builder's model ship from your drawings. When your drawings from Task III B Part 1 are approved by the instructor, follow these instructions very carefully.

1. For the drawing of the forward part of the ship:

   a. Cut out the entire drawing.

   b. Trace the outline on poster-board. Trace this outline (**and only this outline**) 9 times.

   c. Cut the outermost counter line on each side from the drawing and trace the remaining portion of the drawing onto poster-board.

   d. Repeat step 1.c, cutting the remaining outermost counter line on each side from the drawing and tracing the remainder. Repeat this until you have reached and traced the center-most contour lines.

   e. Cut out all traced outlines from the poster-board and set them aside.

2. For the drawing of the after part of the ship:

   a. Cut out the entire drawing.

   b. Trace the outline on poster-board. Trace this outline **only once**.

   c. Cut the outermost counter line on each side from the drawing and trace the remaining portion of the drawing onto poster-board.

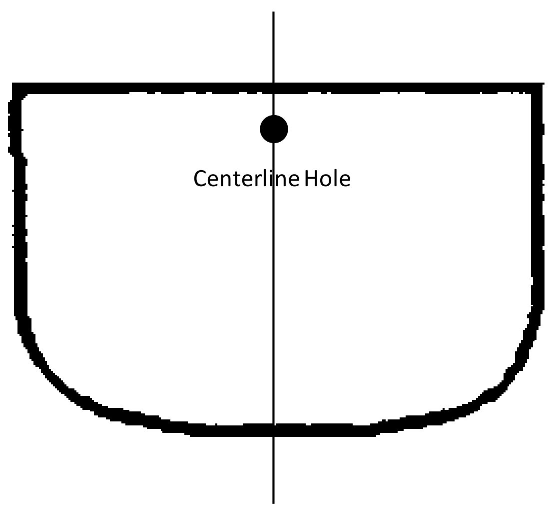
d. Repeat step 2.c, cutting the remaining outermost counter line on each side from the drawing and tracing the remainder. Repeat this until you have reached and traced the center-most contour lines.

e. Cut out all traced outlines from the poster-board and set them aside.

3. Take 4 of the kabob skewers, place them end to end (**do not glue them together yet**). With a ruler, measure 34 equidistant notches (one for each of your cut pieces).

4. Place all of the pieces of the forward part of the model (including the 9 identical pieces) on top of each other just as in the original picture.

   a. With a hammer and nail, drill a hole through all of the pieces.



   b. With a glue gun, glue each piece to a notch on one of your skewers. The pieces should be placed in order from smallest to largest with the identical pieces last.

5. Place all of the pieces of the after part of the model on top of each other just as in the original picture you drew.

   a. With a hammer and nail, drill a hole through all of the pieces.

b. With a glue gun, glue each piece to a notch on one of your skewers. The pieces should be placed in order from smallest to largest.

6. Glue the kabob skewers together such that they form the outline of the ship: forward part from smallest to largest, middle part with the 9 identical pieces, after part from largest to smallest.

7. Carefully trace the top of the model onto construction paper. Cut the trace out and glue it to the top of the model as a top deck.

8. To receive a grade of “4”, cover the entire hull such that your model will float. To receive a grade of “5”, build a superstructure onto your ship such that she resembles the actual ship as closely as possible.

Name: ______________________________

U.S.S. *Massachusetts* Layout and Design

1. **Ship Compartmentation.** Every room on a naval ship is called a compartment and is assigned an identifying letter - number symbol. This symbol is marked on a label that is found somewhere in the compartment.

   a. An example would be:

   C - 217 - A

   b. The first letter indicates the section of the ship the compartment is in. There are three sections:

      A - all compartments forward or ahead of the engine rooms.

      B - the engine rooms.

      C. - all compartments aft or behind the engine rooms.

      In the example above, the compartment is in the C section and is, therefore, aft of the engine rooms.

2. The number has three digits:

   a. The first number indicates the deck. "1" is the main deck. Decks above "1" have a "0" in front of them.

   b. In the example above, the compartment is on the second deck.

3. The next two numbers indicate the side of the ship the compartment is on. Odd numbers, like "17" above, are on the starboard side; even numbers are on the port side.

a. The final letter indicates the use of the compartment.

i. A is supply and storage
ii. C is control
iii. E is machinery
iv. F is fuel
v. L is living quarters
vi. M is ammunition
vii. T is trunks and passages
viii. V is voids
ix. W is water

In the example above the compartment is a supply and storage compartment.

Compartment	Section	Deck	Side	Use
C - 217 - A	C	2	Starboard	Supply
C - 201- L				
B - 204 - 1E				
A - 213 - L				
B - 0224 - L				
A - 0116 - C				
A - 0225 - E				
B - 0402 - C				
B - 326 - E				
A - 402 - F				
A - 425 - V				
C - 445 - W				
B - 501 - F				
B - 234 - V				
A - 601 - V				
A - 503 - M				
C - 0216 - C				
A - 0143 - E				
B - 409 - M				

Name: ____

U.S.S. *Massachusetts* (BB 59) Field Trip Reflection

This task is due no later than May 2nd, 2014

1. **Background.** 1800 men lived and worked aboard U.S.S. *Massachusetts* when she was in service in the Navy. They slept there, ate there, worked there, and relaxed there. As you walk around the ship, try to imagine what it would be like to do that for many months without the opportunity to go home, visit your family or friends, or even leave the ship.

2. **Tasks.**

   a. Find a compartment in *Massachusetts* (a room on a ship) where enlisted sailors (not officers) slept. Answer the following questions:

      i. What is the compartment number of this compartment? _______________

      ii. Estimate the number of square feet of this compartment _______________
      (Hint: think of the floor as a giant rectangle and find its area)

      iii. How many men could sleep in this compartment? ____________________

      iv. What would be difficult about sleeping in a place like this?

      ____

      ____

      ____

      ____

v. Think about a time when you have come to a place where everyone knew each other but no one knew you.

If you were a new sailor moving into this sleeping compartment and everyone else was a stranger, what kinds of things would you need to do to get along with your new shipmates with whom you will live for one year?

vi. Aboard *Massachusetts* there is no place for electronics such as phones or electronic games.

Imagine that you have to move aboard and live aboard for one year without going ashore and with no possibility that anyone can bring you anything.

You can only bring aboard one suitcase for all of your clothes and your personal items.

In addition to your clothes, what would you bring with you in the suitcase to last you for a year?

____

____

____

____

vii. Aboard the submarine, you can bring a suitcase, one half the size of the one you can bring aboard *Massachusetts*. What would you bring to live in the submarine for one year?

____

____

____

viii. You notice that there are no televisions aboard *Massachusetts* (they had only just been invented when the ship was built).

There is no place to watch TV or movies or to listen to recorded music.

How will you keep yourself entertained during your off hours for the year that you are onboard?

____

____

____

____

ix. You notice there are no vending machines nor is there a Walgreens outlet. The tiny ship's store only sells very limited items.

What will be the hardest thing to give up for the year you have to live in the ship?

____

____

____

Name: ______________________________

9th Grade Final Project: Mathematics and History

Part IV: War Game: Re – Fighting the Battle of Midway

1. **Historical Background.** While it was the Prussian Germans who developed the first modern war game, *Kriegspiel* in 1824, the modern naval war game was born in the State of Rhode Island when Lieutenant William McCarty Little, U.S.N., introduced a naval version of *Kriegspiel* at the Naval War College in Newport in 1886. The purpose of the war game was to simulate naval battle with enough realism that naval officers could practice the command decision – making they would use in an actual battle.

   In the days prior to electronic simulation, war games were played on gigantic game boards like the one seen below in Pringle Hall at the Naval War College. Each checkerboard square on the game floor represented some number of square miles. The men on their hands and knees used scaled measuring devices to measure direction and range of movement of various ships being employed in the game. The players sat around the play area, taking turns like in any board game. Umpires, such as the man in the grey suit standing with his hands behind his back, evaluated the outcome of each game move, awarding points and judging which player was the winner of the move.

   War gaming before electronics was a very complicated process involving much planning and detailed execution to ensure the game seemed real to the players. A successfully planned and executed war game would give the players insights into how to fight a real war. The Naval War College was very skilled at planning and executing such games, preparing generations of senior naval officers to fight real battles. Fleet Admiral Chester Nimitz, the U.S. naval commander in the Pacific Theater in World War II, said of his experience war gaming at the college, "The enemy of our games was always Japan, and the courses were so thorough that after the start of World War II, nothing that happened in the Pacific was strange or unexpected."[19] The war game planners of the Naval War College could be justly proud of their efforts helping prepare the

[19] http://www.pbs.org/wgbh/amex/macarthur/peopleevents/pandeAMEX90.html.

American naval commanders for their role in bringing about the great victory at the Battle of Midway.

Figure 6: A naval war game at Pringle Hall, U. S. Naval War College, Newport, RI.

We will refight the Battle of Midway by playing a war game that uses many of the techniques that Naval War College students such as Fleet Admiral Nimitz used in the 1920's and 1930's – the years before World War II. The outcome of our game may be different from that of the actual battle. It will be decided by the player(s) who most skillfully employ their naval forces in the simulated battle.

War Game Preparation and Training

Name: ________________________________

Task IV A – G: Maneuvering Board Exercise
This task is due May 9th, 2014

1. Ships of a carrier task force normally sailed together in a formation on a base course and base speed (the course and speed of the aircraft carrier). The aircraft carrier was normally in the center of the formation while cruisers and destroyers took positions around the aircraft carrier to protect it from attack by aircraft, other ships, or submarines.

    a. Battleships and cruisers normally took a point station around the aircraft carrier – each maintaining a fixed relative bearing and range from the carrier. For example, the cruiser U.S.S. *Astoria* (CA 34) might have been assigned a position 270° relative at 2000 yards from the aircraft carrier U.S.S. *Yorktown* (CV 5). She would therefore take a position on *Yorktown's* port (left) side at 2000 yards.

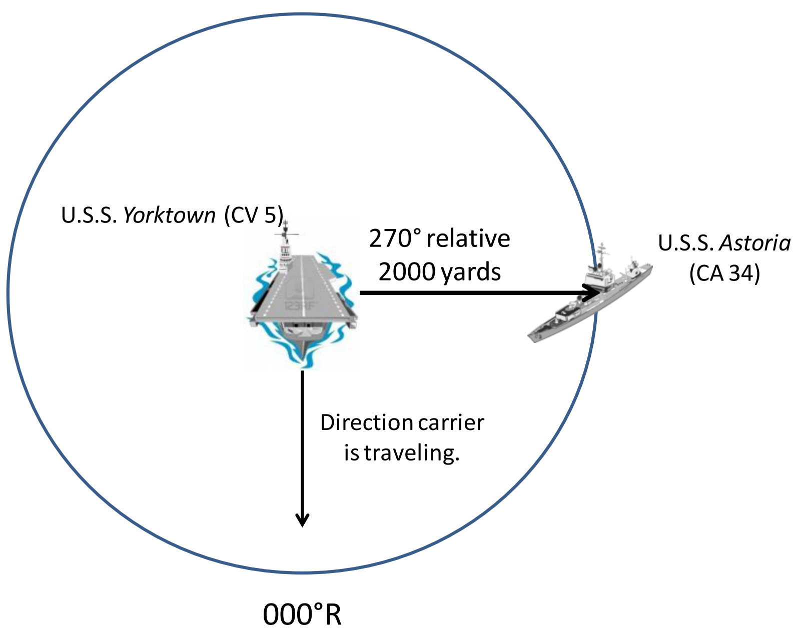
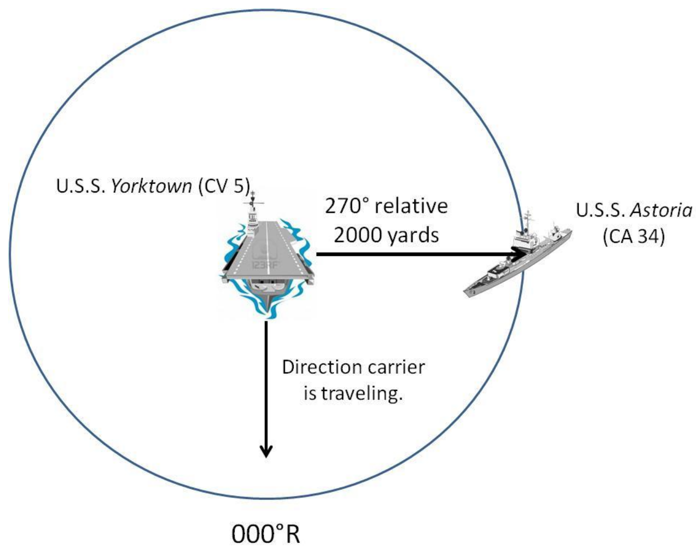


Figure 4: *Astoria* 270° relative, 2000 yards from *Yorktown*.

    b. Destroyers normally took a patrol sector between two bearings and two ranges from the aircraft carrier. This allowed them to patrol actively for enemy submarines that might try to sink the carrier. For

c. example, U.S.S. *Hammann* (DD 412) might be assigned a sector between 000° and 030° relative, 4000 yards to 6000 yards from *Yorktown.*



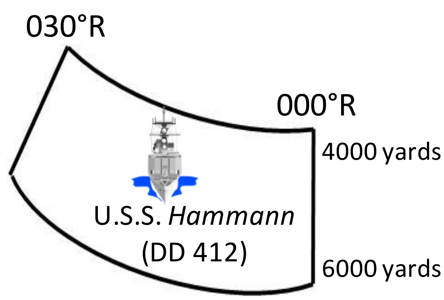


Figure 5:
Hammann* in sector station 000°R - 030°R, 4000 – 6000 yards from *Yorktown.

2. **Task.** You will decode the following message. It is encoded twice. Once you solve the encrypted number code, you will use EXTAC 1000 to break the code a second time.

   a. Break the code and attach the solution to this sheet.
   b. Plot the location of TF 17's ships (the *Yorktown* task group) on a maneuvering board sheet, and answer the questions below. Refer to the CINCPACFLT Operations Order (Appendix 3) for the names of TF 17 ships.

c. Answer the questions on the last sheet.

Getting Started.

1. The unencrypted message contains both letters and numbers.
2. As a first step, the letters were converted to their number equivalents, the numbers were left alone.
3. As a second step, an arithmetic operation was performed on all numbers. The operation was either multiplicative or power.
4. In the unencrypted message, the letter "a" appears most often, the number "0" appears most often.

361	9	324	25	25	196		121	81	144	225		676	676		625	225	324	121	400	225	529	196
0	49	1	1		0	16	0	36		1	361	400	225	324	81	1						
4	25	4	81		0	16	0	36		256	225	324	400	144	1	196	16					
4	64	9	4		0	36	1	0		169	225	324	324	81	361							
4	16	4	64		0	36	1	0		64	441	49	64	25	361							
4	0	4	16		0	36	1	0		1	196	16	25	324	361	225	196					
9	4	0	0		0	36	1	0		64	1	169	169	1	196	196						
0	0	0	16		0	36	1	0		324	441	361	361	25	144	144						
4		9	225	324	256	25	196		9	0	0											
4		361	256	25	25	16		4	25													

5. What is the area in square yards of:

Astoria's sector: ______________________________

Portland's sector: ______________________________

Hammann's sector: ______________________________

Hughes' sector: ______________________________

Morris' sector: ______________________________

Anderson's sector: ______________________________

Russell's sector: ______________________________

Name:____________________________________

Task IV B – G / A: Developing Communications Signals
This task is due May 9th, 2014

Using EXTAC 1000, encode the following basic signals:

MESSAGE	ENCODED SIGNAL
TAKE STATION 180° RELATIVE FROM THE GUIDE AT 2 NM	
WHEEL THE FORMATION TO THE RIGHT TO NEW COURSE 130° TRUE	
TAKE STATION 180° TRUE FROM THE GUIDE AT 5 NM	
TURN TOGETHER TO STARBOARD 40°	
ALTER THE DIRECTION OF THE SEARCH TO 190° T	
DECREASE SPEED BY 10 KNOTS	
BASE COURSE IS 000° T, BASE SPEED IS 15 KNOTS	
SHOW NO LIGHT	
REJOIN THE FORMATION WHEN PRESENT ORDERS HAVE BEEN CARRIED OUT	
PROCEED ON DUTY ASSIGNED	
MANEUVER INDEPENDENTLY TO AVOID ATTACK	
SUBMARINE SIGHTED BEARING 140° T AT 500 YARDS	
DEPTH OF SUBMARINE IS 100 FEET	

Name:_________________________________

Task IV C – G / A: Designing the Game Board
This task is due May 23rd, 2014

A. **Location and Layout.** The game board is located in the center of the 9th grade mathematics classroom (Room 327), inside the circle around which the desks are normally arranged. It will consist of a scaled map of the Hawaiian Islands, Midway Island and an area of the Central Pacific Ocean out to 600 nautical miles west of Midway Island.

1. North will be toward the north wall of the classroom (where the projector screen is).

2. The Hawaiian Islands will be depicted in the southeast corner of the map and will include the islands of Hawai'i, O'ahu, Maui, Kaua'i, Moloka'i, and Lana'i. The map drawings of the islands will be scaled to the map.

3. Midway Island will be depicted on the map northwest of the main Hawaiian Islands, also scaled to the map. The actual distance from Pearl Harbor on the island of O'ahu to Midway is 1300 nautical miles.

4. The map will be scaled to include the six main Hawaiian Islands, Midway Island and an area of a circle around Midway with a radius of 600 nautical miles.

5. The map's grid is already cut into the game board table. These will be subdivided as necessary to facilitate movement of game pieces across the map.

B. **Task 1: Creating the Map.** You will determine the scale of the game board and then produce scaled drawings of the Hawaiian Islands and Midway Island.

1. **Step 1:** Measure with a yardstick the distance from the southeast corner of the map to the northwest corner in inches. This is the map total distance in inches.

2. **Step 2:** The actual distance from the southeast corner of the mapped area to the northwest corner will be$1300\,nm + 600\,nm = 1900\,nm$. Convert 1900 nm to inches ($1\,nm = 2000\,yards$,$1\,yard = 36\,inches$). This is the actual total distance in inches.

3. **Step 3:** Determine the scale factor (SF) of the map using the map total distance in inches (M) and the actual total distance in inches (A). Write and solve the equation for the scale factor (SF).

4. **Step 4:** Complete the following table, converting actual measures to scaled measures or scaled measures to actual measures. For the Hawaiian Islands and Midway, use the maps below to estimate the actual east – west and north – south distances of each island. For the tile, find the length in inches, this is the scaled measure. We want to find the actual measure.

	Inches		x Scale Factor (SF)	in Inches		x Scale Factor (SF)	in Inches	
Distance from Pearl Harbor to Midway + 600 nm								
Length of a Tile								
	East - West	North - South		East - West	North - South		East - West	North - South
Island of Hawai'i								
Island of O'ahu								
Island of Maui								
Island of Kaua'i								
Island of Moloka'i								
Island of Lana'i								
Midway Island								

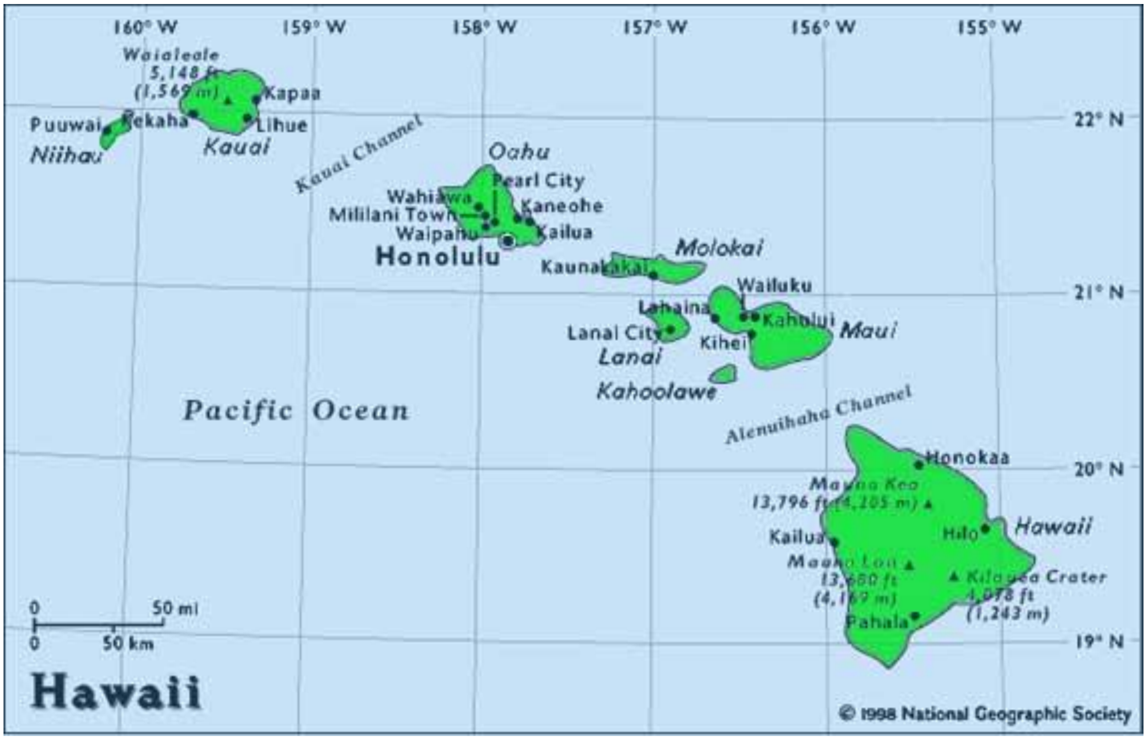


Figure 7: Hawaiian Islands

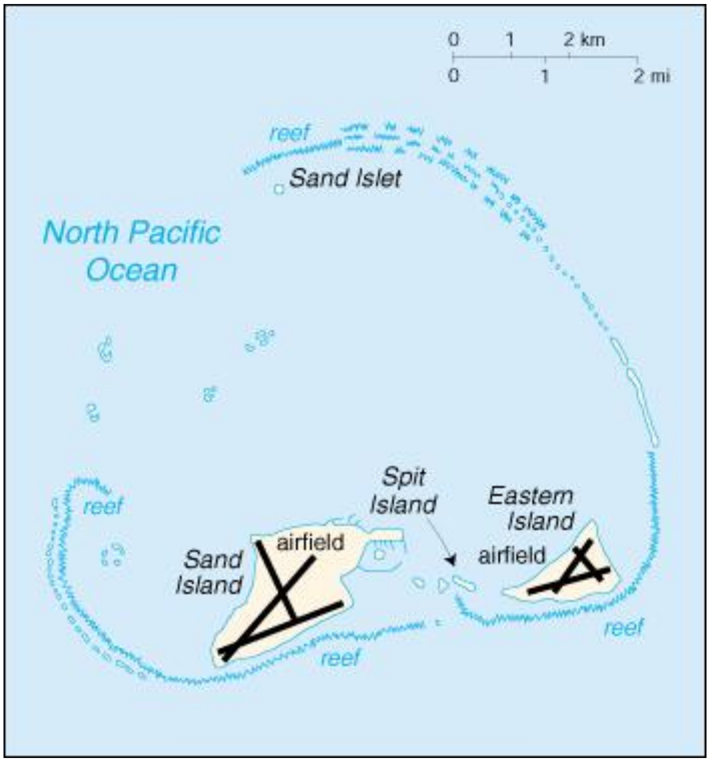


Figure 8: Midway Island

5. **Step 5:** Using the scaled measures from the table, sketch the Hawaiian Islands and Midway Island to the scale of the map. Attach the drawings to these sheets. The best sketch will be used to draw the game board map. The artist will receive a "5" on this portion of the project.

A. **Task 2.** The fleet tankers U.S.S. *Platte* and U.S.S. *Cimarron* have been ordered to sea to support the Midway Operation. They are to operate in the following area defined by latitudes and longitudes that are the solutions to these systems of equations.

1. Solve each system of equations using any two of the methods we have learned ***except the Microsoft EXCEL program*** to determine the latitude and longitude of each point (**show all work on separate sheets of paper**).

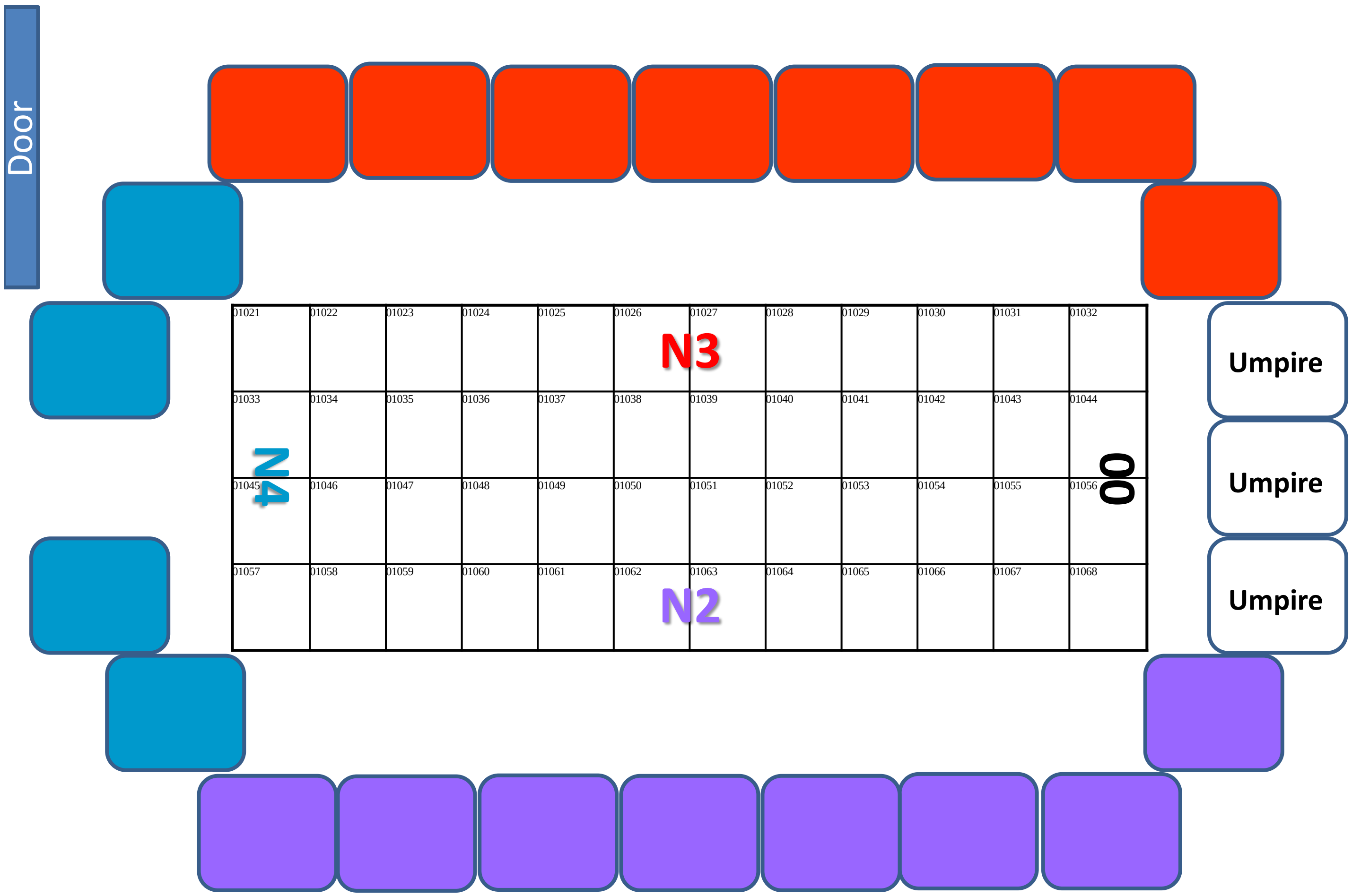
		x_i and y_i are solutions to the following systems of equations
Point A	x_1° N, y_1° W. x_1 = _____, y_1 = _____	$8x + y = 415$ $6x + y = 353$
Point B	x_2° N, y_2° W. x_2 = _____, y_2 = _____	$4x + -y = -55$ $2x + -y = -111$
Point C	x_3° N, y_3° W. x_3 = _____, y_3 = _____	$1x + 7y = 1167$ $-1x + 14y = 2256$
Point C	x_4° N, y_4° W. x_4 = _____, y_4 = _____	$-4x + 5y = 699$ $3x + 8y = 1391$

2. Determine the bearing and range of each point from Pearl Harbor, Hawaii. Then calculate the time in days, hours, and minutes it will take a ship traveling at 15 knots and 20 knots to reach each point from Pearl Harbor.

	Bearing	Range	Time at 15 kts.	Time at 20 kts.
Point A				
Point B				
Point C				
Point C				

Battle of Midway War Game Rules

Classroom Layout



Teams

Position	**Period 3**	**Period 5**	**Period 6**
Commander (00)			
COS (01)			
Intelligence (N2)			
Deputy (N2A)			
N2 Staff			
N2 Staff			
N2 Staff			
N2 Staff			
N2 Staff			
Operations (N3)			
Deputy (N3A)			
N3 Staff			
N3 Staff			
N3 Staff			
N3 Staff			
N3 Staff			
N3 Staff			
Logistics (N4)			
Deputy (N4A)			
N4 Staff			
N4 Staff			
N4 Staff			
N4 Staff			
N4 Staff			

War Game Sequence of Play

Time	Action	Responsibility
0:01	Display coded enemy intelligence disclosure on main screen.	Umpire
0:01 - 0:05	Brief team on outcome of previous move.	Umpire
0:05 - 0:15	N2 staff 1. Break coded enemy intelligence disclosure.	N2
0:05 - 0:15	N3 staff: 1. Begin preparation of Daily Briefing 2. Begin preparation of OPSUM Message 3. Schedule refueling requested by N4 to include location and time of rendezvous.	N3
0:05 - 0:15	N4 staff: 1. Determine fuel consumed last 24 hours - submit to N3 for inclusion in OPSUM 2. Identify any requests for refueling and submit request to N3.	N4
0:15 - 0:45	N2 Staff: 1. Begin to prepare encoded OPSUM message. 2. Submit intelligence input to N3 for inclusion into daily brief.	N2
0:15 - 0:45	N3 Staff: 1. Based upon intelligence received and previous day's turn outcome, develop air operations and surface operations plans for daily briefing to Admiral. 2. Advise N4 on movement intentions for next 24 hours. 3. Prepare daily OPSUM message and submit to N2 for encoding. 4. Begin preparation of daily briefing to Admiral.	N3
0:15 - 0:45	N4 Staff: 1. Determine latitude and longitude of task force. Submit to N3 for OPSUM. 2. Estimate fuel consumed next 24 hours for inclusion in OPSUM.	N4

0:45 - 0:55	Admiral's decision briefing. 1. Admiral approves recommendations of staff or modifies as desired. 2. Admiral advises umpire of decisions and basic strategy.	N2, N3, N4
0:55 - 0:60	1. Umpire, Commander, N-heads compile and submit daily grades. 2. N2: Finish encoding message for transmission. 3. N3 and N4 revise plans based upon Admiral's decisions.	N2, N3, N4

Procedures

1. There will be three teams:

   a. Period 3: Japanese Combined Fleet Striking Force
   b. Period 5: USN Task Force 16 / 17
   c. Period 6: USN Midway Defense Force

2. Each class period will be considered a team's turn.

3. Prior to the beginning of each turn, the umpire will ensure the game board is laid out reflecting that team's knowledge of the tactical situation.

4. At the beginning of each turn, the umpire will provide the team with the opponent's encoded OPSUM and will provide a summary of the outcome of the previous turn, to include:

   a. Outcome of any searches launched
   b. Outcome of any air strikes launched
   c. Points one or lost

5. Following the umpire brief, the Intelligence (N2) cell will work on decoding the opponent's OPSUM message. Once decoded, N2 will

immediately provide the decoded message to N3. Additionally, N2 will complete the following form and submit it to N3.

N2 Intelligence Analysis

____
Date / Time

Enemy force estimated position: ____________________________________
Grid Coordinates

Enemy force estimated course / speed: ______________________________

Enemy force estimated intentions:

Launch air strike against: ____________

Continue search for our forces: _______

Alter course / speed to: _____________

Additional notes: ____

____

6. Once N2 had decoded the opponent's OPSUM, N2 will begin to encode its own team's OPSUM, using information provided to N3. The OPSUM must be encoded, submitted to the umpire, and approved by the umpire no later than 5 minutes prior to the end of the turn period or the umpire will release an un-encoded version to the opposing team at the beginning of that team's turn.

7. Following the umpire brief, the Logistics (N4) cell will determine the task force's position (latitude and longitude) and compute fuel consumption for the previous day and estimated fuel consumption for the

coming day. N4 will submit this information in the following format to N3:

N4 Logistics Briefing

Umpire will not approve unless all work is attached.

Number	Name	Max. fuel capacity	Amount used last 24 hours	% used last 24 hours	% remaining onboard	Projected use next 24 hours
Carrier 1						
Carrier 2						
Carrier 3						
Carrier 4						
NAS Fuel Farm						

Tanker Services Required? Yes / No

8. Following the umpire brief, N3 will do the following:

   a. Develop the air daily search plan, ensuring that no aircraft is allowed to exceed its combat radius on a search operation.
   b. Develop the air strike plan (if enemy forces have been located) ensuring that no aircraft is allowed to exceed its combat radius on a strike operation without specific approval of the Commander.
   c. Compile and prepare the Daily Commander's Brief.
   d. Compile and prepare the Daily Operations Summary / Intentions Message. The Commander must initial this message, approving it before it is encoded and presented to the Umpire.

N3 / NAS CO Air Operations Plan
(Search)

Number of aircraft to be launched: ______________________________

Mission:

- Search

  Search axis (degrees true from home base):

  Aircraft 1: _______________

  Aircraft 2: _______________

  Aircraft 3: _______________

  Aircraft 4: _______________

  Aircraft 5: _______________

  Range: __ __________ nm

  Objective: ____

N3 / NAS CO Air Operations Plan
(Strike)

Umpire will not approve plan without work attached.

Number of aircraft to be launched: ______________________________

Mission:

Objective: ____

Location (bearing and range) of target: ______________________________

Is target within range of strike aircraft? Yes / No

Time of flight to target: ______________________________

Composition of strike force:

- Torpedo bombers: ______________________________
- Dive bombers: ______________________________
- Heavy bombers: ______________________________

N2 Intelligence Analysis

____
Date / Time

Enemy force estimated position: ________________________
Grid Coordinates

Enemy force estimated course / speed: ______________________

Enemy force estimated intentions:

Launch air strike against: __________

Continue search for our forces: ______

Alter course / speed to: __________

Additional notes: ____

____

N3 / NAS CO Air Operations Plan (Search)

Number of aircraft to be launched: ______________________

Mission:

- Search

Search axis (degrees true from home base):

Aircraft 1: ____________

Aircraft 2: ____________

Aircraft 3: ____________

Aircraft 4: ____________

Aircraft 5: ____________

Range: ___________ nm

Objective: __________________________________

N3 / NAS CO Air Operations Plan (Strike)

Umpire will not approve plan without work attached.

Number of aircraft to be launched: ______________________

Mission:

Objective: ________________________________

Location (bearing and range) of target: ______________________

Is target within range of strike aircraft? Yes / No

Time of flight to target: ______________________

Composition of strike force:

- Torpedo bombers: ______________________
- Dive bombers: ______________________
- Heavy bombers: ______________________

N4 Logistics Briefing

Umpire will not approve unless all work is attached.

Number	Name	Max. fuel capacity	Amount used last 24 hours	% used last 24 hours	% remaining onboard	Projected use next 24 hours
Carrier 1						
Carrier 2						
Carrier 3						
Carrier 4						
NAS Fuel Farm						

Tanker Services Required? Yes / No

Daily Commander's Briefing

e. N3 will revise plans, messages, and the briefing as necessary upon receipt of valid intelligence of enemy intentions or in response to changes directed by the Commander or Chief of Staff.

9. Upon receipt of the Daily Commander's Briefing, the Commander will approve or modify the staff's recommendations, advise the umpire of his / her decisions, and advise the umpire of the team's basic strategy.

Scoring.

The umpire, upon receipt of the above information will score as follows:

1. Strategy Selection. The umpire will assign points to each team once that team's commander has selected his / her team's strategy for the day.

First Turn and End of Turn if Neither Side Detected

Americans → Japanese ↓	**Reinforce Midway**	**Split Forces**	**Search for and Attack Japanese**
Attack Midway	0	-100	-200
Split Forces	+100	0	-100
Search for and Attack Americans	+200	+100	0

End of Turn - Americans Detected

Americans → Japanese ↓	Reinforce Midway	Split Forces	Search for and Attack Japanese
Attack Midway	0	-100	-200
Split Forces	+200	+100	0
Search for and Attack Americans	+300	+200	+100

End of Turn - Japanese Detected

Americans → Japanese ↓	Reinforce Midway	Split Forces	Search for and Attack Japanese
Attack Midway	0	-200	-300
Split Forces	+100	-100	-200
Search for and Attack Americans	+200	0	-100

2. Outcome of Play.

Location of Enemy Force	**25 points**
Detailed Identification of Enemy Force	**25 points**
Successful Strike of Enemy Force	**50 points**
Destruction of Enemy Aircraft Carrier or Airfield.	**25 points**
All Players Focused and Involved in Game.	**100 points**

Team Member Evaluation

1. Students will evaluate each other's performance throughout the game as follows:

   a. The Umpire will evaluate the Commander.
   b. The Commander will evaluate the Chief of Staff and the N-heads.
   c. The N-heads will evaluate members of their teams.

2. Evaluation Rubric.

Student Name: ____

Date: ________________________

	4	**3**	**2**	**1**
Completion of assigned tasks		Team member completed all assigned tasks on time		Team member did not complete assign tasks on time
Quality of Work	Team member completed work correctly and was displayed it in a highly professional manner	Team member completed work correctly and displayed it neatly	Team member failed to complete work **or** work was sloppy and poorly presented	Team member failed to complete work **and** work was sloppy and poorly presented.
Team Contribution	Team member was focused, professional, and helped members of the team to succeed	Team member was focused and professional	Team member was unfocused (i.e. failed to concentrate on completing assigned tasks) **or** was unprofessional (i.e. failed to maintain positive demeanor, used foul language, acted in a boisterous manner.	Team member was unfocused **and** unprofessional

Add up the points and divide by 3. Final grade is: ______________________

3. Final grade will be an average of the grades of all four days of play. A final grade of less than 3 will result in failure of the war game exercise.

Daily OPSUM Message / Intelligence Disclosure

1. Each team will prepare a daily operations summary and commander's intentions (OPSUM) message in the following format:

From: Task Force Commander

To: Fleet Commander

Subj: Daily OPSUM and Commander's Intentions for ______ June 1942

Task force position:

(provide latitude and longitude and grid coordinate from game board)

Fuel onboard:

Carrier	Gallons	Percent

(list your aircraft carriers, the number of gallons each has onboard, and the percent of total fuel load remaining onboard)

Estimated fuel onboard:

Carrier	Gallons	Percent

(list your aircraft carriers, the number of gallons each will have onboard tomorrow at this time, and the percent of total fuel load remaining onboard tomorrow at this time)

Operations last 24 hours:

(list the numbers and types of aircraft launched for search and the results of the search and list the numbers and types of aircraft launched for strikes and the results of the strikes)

Intentions next 24 hours:

(list the following:

1. *Numbers and types of aircraft to be launched for search*
2. *Area to be searched*

3. Numbers and types of aircraft to be launched for strike and the location and composition of the target to be struck
4. Where the task force will be (by grid number) at the beginning of the next turn)

2. The umpire must approve the OPSUM before it is encoded.

3. The OPSUM can be encoded in any number of ways using techniques we learned in class.

    a. Encoding using arithmetic operations is limited to addition, subtraction, multiplication, division, and raising to the 1st, 2nd, or 3rd powers.
    b. EXTAC 1000 may be used to encode the message.
    c. Difficulty may also be increased by removing word spaces and line returns in the message.

4. The umpire must approve the encoded message before it is accepted.

5. If the team fails to encode the message on time and properly, the umpire will release an un-encoded message to the opposing team(s).

6. Below is an example of an encoded message using Microsoft EXCEL.

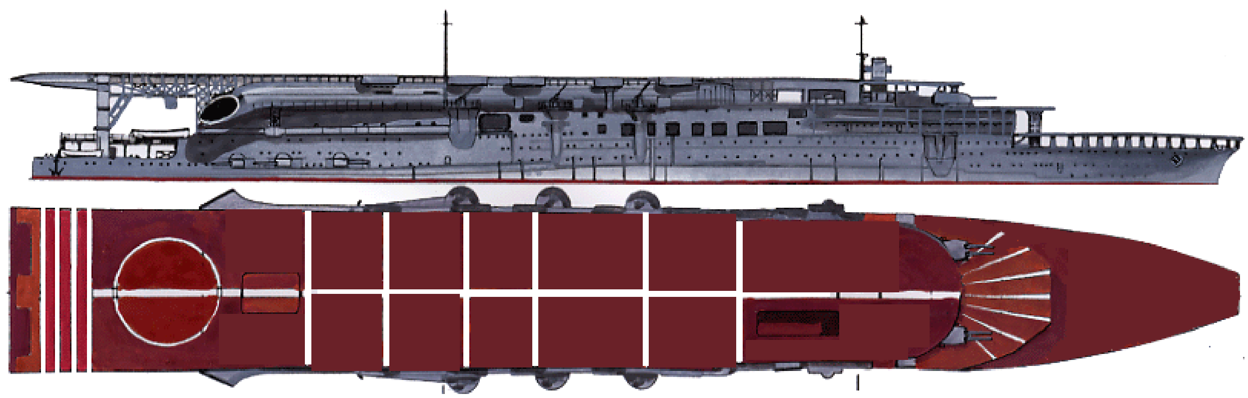
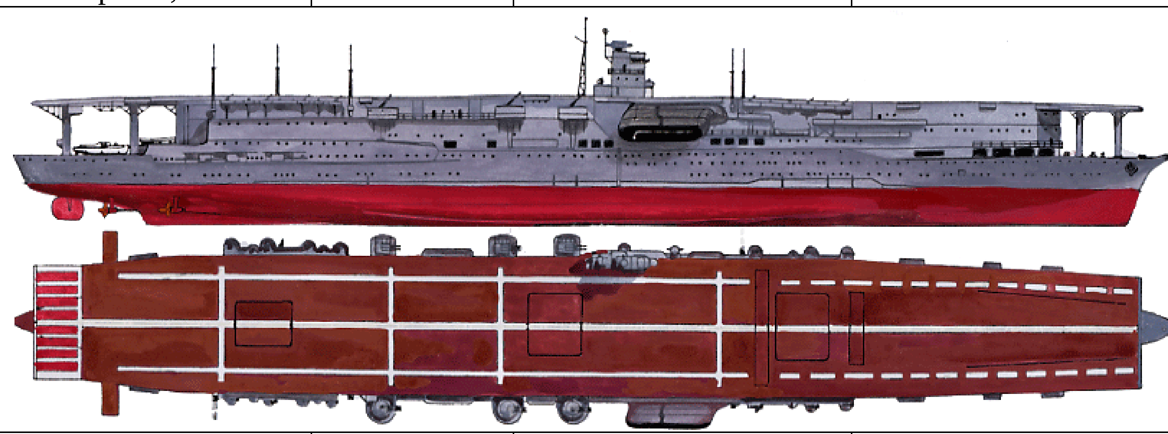
Message to be Coded																								
F	r	o	m	:	T	a	s	k		F	o	r	c	e		C	o	m	m	a	n	d	e	r
T	o	:		F	l	e	e	t		C	o	m	m	a	n	d	e	r						
S	u	b	j	:		D	a	i	l	y		O	P	S	U	M		a	n	d				
C	o	m	m	a	n	d	e	r	'	s		I	n	t	e	n	t	i	o	n	s			
f	o	r				J	u	n	e		1	9	4	2										
T	a	s	k		f	o	r	c	e		p	o	s	i	t	i	o	n	:					
28.2	N	177.35	W																					
F	u	e	l		o	n	b	o	a	r	d	:												
C	a	r	r	i	e	r				G	a	l	l	o	n	s		P	e	r	c	e	n	t
E	N	T	E	R	P	R	I	S	E	150	M	I	L	L	I	O	N		100					
H	O	R	N	E	T					150	M	I	L	L	I	O	N		100					
Y	O	R	K	T	O	W	N			150	M	I	L	L	I	O	N		100					
E	s	t	i	m	a	t	e	d		f	u	e	l		o	n	b	o	a	r	d	:		
C	a	r	r	i	e	r				G	a	l	l	o	n	s		P	e	r	c	e	n	t
E	N	T	E	R	P	R	I	S	E	145	M	I	L	L	I	O	N		97					
H	O	R	N	E	T					145	M	I	L	L	I	O	N		97					
Y	O	R	K	T	O	W	N			145	M	I	L	L	I	O	N		97					
I	n	t	e	n	t	i	o	n	s		n	e	x	t		2	4		h	o	u	r	s	:
C	o	n	t	i	n	u	e		e	n	r	o	u	t	e		P	o	i	n	t			
L	u	c	k																					

Encoded Message in Final Form with No Spaces																
1296	6250000	4879681	4100625	1296	160000	1185921	6765201	3418801	1296	4879681	6250000	1500625	1874161	81	4879681	4100625
160000	4100625	1185921	4477456	1679616	1874161	6250000	4879681	1296	1296	3748096	1874161	1874161	7311616	81	4879681	4100625
4100625	1185921	4477456	1679616	1874161	6250000	130321	7890481	1336336	3111696	1296	256	1185921	2825761	3748096	1.1E+07	50625
65536	130321	194481	28561	1185921	4477456	1679616	81	4879681	4100625	4100625	1185921	4477456	1679616	1874161	6250000	390625
6765201	6561	4477456	7311616	1874161	4477456	7311616	2825761	4879681	4477456	6765201	2085136	4879681	6250000	10000	7890481	4477456
1874161	50625	2401	20736	38416	160000	1185921	6765201	3418801	2085136	4879681	6250000	1500625	1874161	5308416	4879681	6765201
2825761	7311616	2825761	4879681	4477456	1296	632407	14	9.9E+08	23	1296	7890481	1874161	3748096	4879681	4477456	1336336
4879681	1185921	6250000	1679616	1296	81	1185921	6250000	6250000	2825761	1874161	6250000	2401	1185921	3748096	3748096	4879681
4477456	6765201	65536	1874161	6250000	1500625	1874161	4477456	7311616	625	38416	160000	625	104976	65536	104976	6561
130321	625	5.1E+08	28561	6561	20736	20736	6561	50625	38416	1E+08	4096	50625	104976	38416	625	160000
5.1E+08	28561	6561	20736	20736	6561	50625	38416	1E+08	390625	50625	104976	14641	160000	50625	279841	38416
5.1E+08	28561	6561	20736	20736	6561	50625	38416	1E+08	625	6765201	7311616	2825761	4100625	1185921	7311616	1874161
1679616	2085136	7890481	1874161	3748096	4879681	4477456	1336336	4879681	1185921	6250000	1679616	1296	81	1185921	6250000	6250000
2825761	1874161	6250000	2401	1185921	3748096	3748096	4879681	4477456	6765201	65536	1874161	6250000	1500625	1874161	4477456	7311616
625	38416	160000	625	104976	65536	104976	6561	130321	625	4.4E+08	28561	6561	20736	20736	6561	50625
38416	8.9E+07	4096	50625	104976	38416	625	160000	4.4E+08	28561	6561	20736	20736	6561	50625	38416	8.9E+07
390625	50625	104976	14641	160000	50625	279841	38416	4.4E+08	28561	6561	20736	20736	6561	50625	38416	8.9E+07
9834496	7311616	38416	20736	2560000	2825761	4477456	7311616	4879681	7890481	6250000	6765201	1296	6561	4477456	7311616	1874161
4477456	7311616	2825761	4879681	4477456	6765201	4477456	1874161	81	4879681	4477456	7311616	2825761	4477456	7890481	1874161	1874161
4477456	6250000	4879681	7890481	7311616	1874161	65536	4879681									

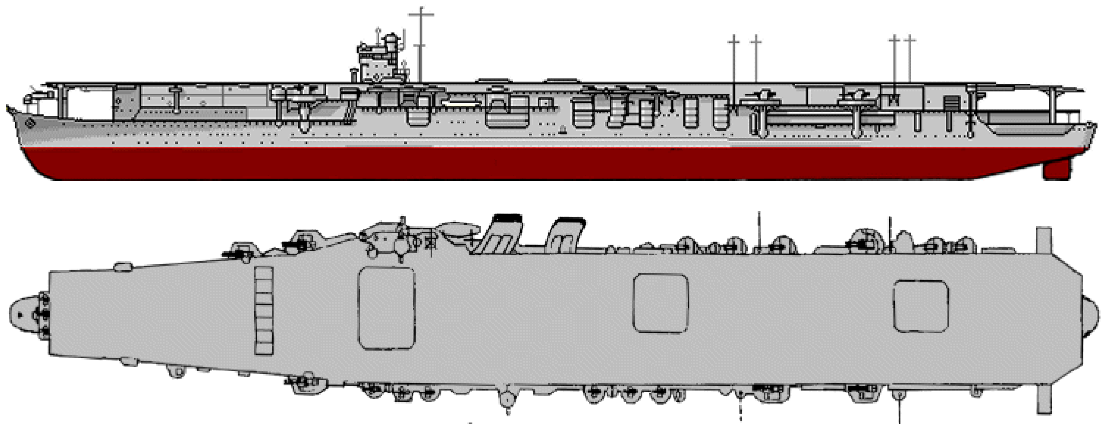
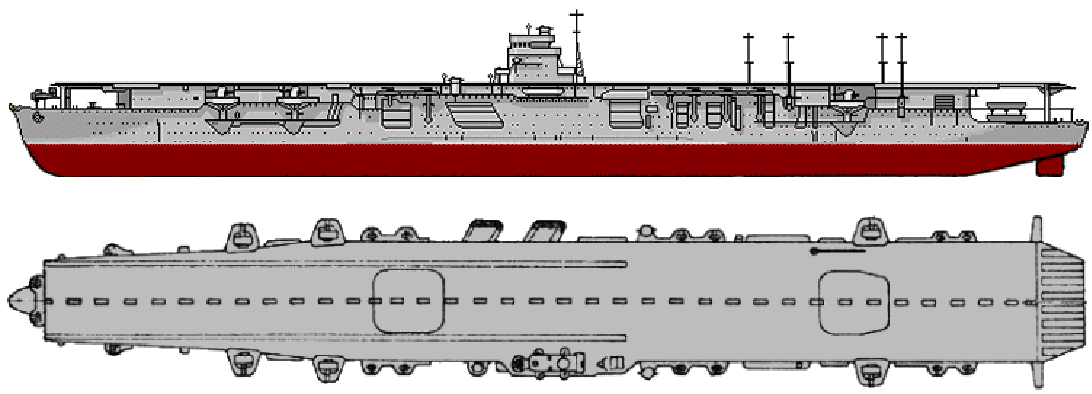
Ship and Aircraft Characteristics.

1. 機動部隊 (Japanese Mobile Striking Force).[20]

a. **Ships.** The speed and fuel consumption of the Japanese strike force will be based upon the characteristics of the four aircraft carriers of the *Kido Butai*. If operating together, the group can travel no faster than *Kaga's* maximum speed of 28 knots.

Ship	Max Speed	Max. Fuel Load	Consumption Rate
加賀 *(Kaga)* FL Disp. 42,541 tons	28 knots	1,666,666 gallons	at 15 knots: 166.67 gallons/nm
			
赤城 *(Akagi)* FL Disp. 41,300 tons	31.5 knots	1,562,500 gallons	at 16 knots: 156.25 gallons/nm
			
蒼流 *(Soryu)*	34.5 knots	1,092,593 gallons	at 18 knots: 142.45 gallons/nm

[20] Jonathan Parshall and Anthony Tully, *Shattered Sword: The Untold Story of the Battle of Midway* (Washington: Potomac Books, 2005), 462 – 486.

FL Disp. 19,800 tons			
			
飛龍 ***(Hiryu)*** FL Disp. 21,900 tons	34.5 knots	1,092,593 gallons	at 18 knots: 142.45 gallons/nm
			

b. **Carrier Aircraft.** P(hit) is the probability that the weapon will strike its target based upon performance in the actual battle.

Type	**Name**	**Cruising Range**	**Cruising Speed**	**Max. Speed**	**Armament**	**P(hit)**[21]
Fighter	A6M2 / 21 "Zero"	1010 nm	179.88 nm / hr	287.63 nm / hr	Machine guns / canon	
Torpedo Bomber	B5N2 "Kate"	528 nm	139.91 nm / hr	204.21 nm / hr	800 kg of bombs or torpedoes	2/6
Dive Bomber	D3A1 "Val"	795 nm	159.89 nm / hr	208.55 nm / hr	370 kg of bombs	2/6

[21] Beall, 53.

c. **Search Aircraft.**

Name	Cruising Range	Cruising Speed	Max. Speed
E13A "Jake"	1128 nm	119.92 nm / hr	203.34 nm / hr
E8N2 "Dave"	485 nm		161.63 nm / hr

2. U.S. Task Forces 16 and 17 and Midway Forces.

a. **Ships.**[22] The speed and fuel consumption of the U.S. task forces will be based upon the characteristics of the three aircraft carriers.

Ship	FL Disp.	Max Speed	Max. Fuel Load	Consumption Rate
***Yorktown* (CV 5)**	25,500 tons	32.5 knots	1,500,000 gallons	at 15 knots: 144.23 gallons/nm at 20 knots: 189.87 gallons/nm
***Enterprise* (CV 6)**	25,500 tons	32.5 knots	1,500,000 gallons	at 15 knots: 144.23 gallons/nm at 20 knots: 189.87 gallons/nm
***Hornet* (CV 8)**	25,500 tons	32.5 knots	1,500,000 gallons	at 15 knots: 144.23 gallons/nm at 20 knots: 189.87 gallons/nm

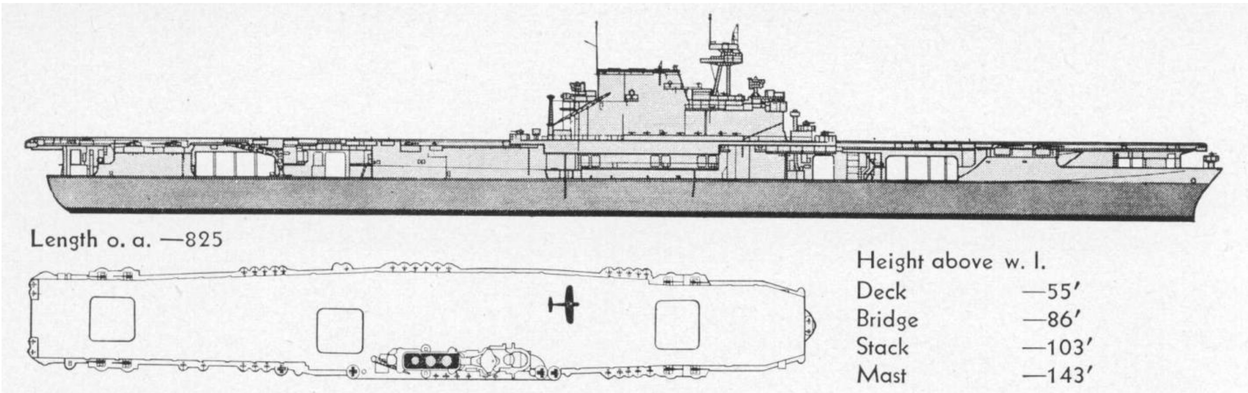


Figure 10 – *Yorktown* Class Aircraft Carrier

[22] "USS ENTERPRISE CV-6: The Most Decorated Ship Of The Second World War," accessed online May 28, 2012 at http://www.cv6.org/ship/big_e.htm.

b. **Carrier Aircraft.**[23] P(hit) is the probability that the weapon will strike its target based upon performance in the actual battle.

Type	**Name**	**Cruising Range**	**Cruising Speed**	**Max. Speed**	**Armament**	**P(hit)**[24]
Fighter	F4F-3 "Wildcat"	1690 nm (with drop tanks)		287.63 nm / hr	Machine guns	
Torpedo Bomber	TBD-1 "Devastator"	435 nm	111.23 nm / hr	179.00 nm / hr	454 kg of bombs or torpedoes	1/6
Dive Bomber	SBD-3 "Dauntless"	1130 nm	130.35 nm / hr	204.21 nm / hr	544 kg of bombs	2/6

c. **Long Range Aircraft.**[25]

Name	**Cruising Range**	**Cruising Speed**	**Max. Speed**	**P(hit)**
PBY-5 "Catalina"	3100 nm		170.32 nm / hr	
B-17E "Flying Fortress"	1737.95 nm	182.49 nm / hr	275.47 nm / hr	1/6

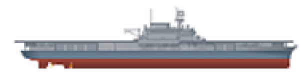
[23] "The Pacific War Online Encyclopedia," accessed online May 28, 2012 at http://pwencycl.kgbudge.com/Table_Of_Contents.htm.

[24] Beall, 53

[25] "The Pacific War Online Encyclopedia."

Fleet Aircraft Carriers

COMPARISON TABLE

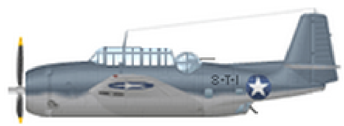
	Akagi, 赤城		**Kaga**, 加賀		**Sōryū**, 蒼龍		**Hiryū**, 飛龍		**Yorktown** class	
Displacement:	41,300 t (36,500 t Std)		42,541 t (38,200 t Std)		18,800 t (15,900 t Std)		20,250 t (17,300 t Std)		25,500 t (19,800 t Std)	
Max Length:	855 ft 3 in	260.7 m	812 ft 6 in	247.7 m	746 ft 5 in	227.5 m	745 ft 11 in	227.3 m	809 ft 6 in	246.7 m
Beam:	102 ft 9 in	31.3 m	106 ft 8 in	32.5 m	69 ft 11 in	21.3 m	73 ft 3 in	22.3 m	109 ft 6 in	33.4 m
Draght:	28 ft 7 in	8.7 m	31 ft 1 in	9.5 m	25 ft 0 in	7.6 m	25 ft 9 in	7.7 m	25 ft 11 in	7.9 m
Machinery:	19 boilers, 4 shafts		8 boilers, 4 shafts		8 boilers, 4 shafts		8 boilers, 4 shafts		9 boilers, 4 shafts	
Max Power:	133,000 hp	97 821 kW	127,400 hp	95 000 kW	152,000 hp	113 336 kW	153,000 hp	114 092 kW	120,000 hp	89 483 kW
Max Speed:	31.25 kts	57.88 km/h	28.5 kts	52.78 km/h	34.5 kts	63.89 km/h	34.5 kts	63.89 km/h	32.5 kts	60.2 km/h
Range:	8,200 nm	15 186 km	10,000 nm	18 520 km	7,750 nm	14 353 km	10,330 nm	19 131 km	12,500 nm	23 150 km
Flight Deck:	817 ft 6 in	249 m	815 ft 6 in	248.5 m	711 ft 6 in	216.8 m	711 ft 6 in	216.8 m	802 ft	244.5 m
Fighters:	27 × A6M2 Type 0		27 × A6M2 Type 0		27 × A6M2 Type 0		27 × A6M2 Type 0		27 × F4F-4 Wildcat	
Bombers:	18 × D3A1 Type 99		27 × D3A1 Type 99		18 × D3A1 Type 99		18 × D3A1 Type 99		36 × SBD-3 Dauntless	
Attack:	27 × B5N2 Type 97		27 × B5N2 Type 97		18 × B5N2 Type 97		18 × B5N2 Type 97		15 × TBD-1 Devastator	
Armor (belt):	6 in	154 mm	6 in	154 mm	1.8 in	46 mm	1.8 in	46 mm	4 in	104 mm
SP Guns:	6 × 1 × 8 in (203 mm)/50		10 × 1 × 8 in (203 mm)/50		—		—		—	
DP Guns:	6 × 2 × 4.7 in (120 mm)/45		8 × 2 × 5 in (127 mm)/40		6 × 2 × 5 in (127 mm)/40		6 × 2 × 5 in (127 mm)/40		8 × 1 × 5 in (127 mm)/38	
AA Guns:	14 × 2 × 0.98 in (25 mm)		11 × 2 × 0.98 in (25 mm)		14 × 2 × 0.98 in (25 mm)		9 × 3 × 0.98 in (25 mm) 3 × 2 × 0.98 in (25 mm)		4 × 4 × 1.1 in (28 mm) 24/32 × 1 × 0.79 in (20 mm)	

* *Akagi* and *Kaga* specifications as reconstructed at 1934-38.
** IJN carriers aircraft as in the largest known Air Groups ever carried (during the Pearl Harbor raid).
*** *Yorktown* class carriers light anti-aircraft armament: CV-5 - 24, CV-6 - 32, CV-8 - 30×1×0.79 in (20 mm)

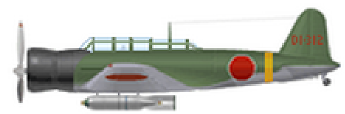
Carrier Torpedo Bombers

COMPARISON TABLE

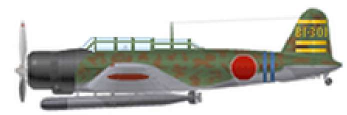
TBD-1



TBF-1



B5N1

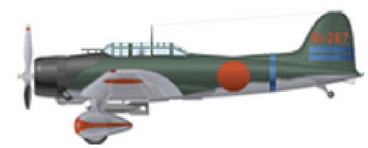


B5N2

	TBD-1		TBF-1		B5N1		B5N2	
Manufacturer:	Douglas Aircraft Company		Grumman Aircraft Engineering Corp		Nakajima Aircraft Industries Ltd.		Nakajima Aircraft Industries Ltd.	
Length:	35 ft	10.6 m	40 ft	12.1 m	33 ft 9 in	10.3 m	33 ft 9 in	10.3 m
Height:	15 ft 3 in	4.60 m	16 ft 5 in	5.00 m	12 ft 2 in	3.70 m	12 ft 2 in	3.70 m
Wingspan:	50 ft	15.2 m	54 ft 2 in	16.5 m	50 ft 11 in	15.52 m	50 ft 11 in	15.52 m
Wingarea:	422 sq ft	39.2 m^2	490 sq ft	45.5 m^2	406 sq ft	37.70 m^2	406 sq ft	37.70 m^2
Empty Weight:	5,600 lb	2539 kg	10,555 lb	4,786 kg	4,643 lb	2106 kg	5,024 lb	2279 kg
Gross Weight:	10,194 lb	4623 kg	16,412 lb	7443 kg	8,157 lb	3700 kg	8,380 lb	3800 kg
Fuel Capacity:	180 gal	682 L	335 gal	1268 L	306 gal	1160 L	306 gal	1160 L
Power Plant:	Pratt & Whitney R-1830-64		Wright R-2600-8 Cyclone 14		Nakajima Hikari 3		Nakajima NK1B Sakae 11	
Military Power:	900 hp	671 kW	1700 hp	1268 kW	840 hp	626 kW	1000 hp	746 kW
Power/Weight:	0.09 hp/lb	0.19 hp/kg	0.10 hp/lb	0.23 hp/kg	0.10 hp/lb	0.23 hp/kg	0.12 hp/lb	0.26 hp/kg
Max Speed:	178 kts	331 km/h	223 kts	413 km/h	199 kts	369 km/h	205 kts	378 km/h
Cruise Speed:	110 kts	206 km/h	132 kts	246 km/h	138 kts	256 km/h	140 kts	259 km/h
Rate of Climb:	720 ft/min	219 m/min	1,430 ft/min	435 m/min	1,256 ft/min	383 m/min	1,284 ft/min	390 m/min
Service Ceiling:	19,700 ft	6000 m	21,400 ft	6500 m	24,280 ft	7400 m	27,100 ft	8260 m
Combat Range:	380 nm	705 km	865 nm	1600 km	588 nm	1090 km	530 nm	980 km
Guns:	1× .30 (7.62 mm) forward 1(2)× .30 (7.62 mm) rearward		1× .30 (7.62 mm) forward 1× .50 (12.7 mm), 1×.30 rearward		1× .303 (7.7 mm) rearward		1× .303 (7.7 mm) rearward	
Torpedo:	1× Mark 13 - 1,927 lb (874 kg)		1× Mark 13 - 1,927 lb (874 kg)		1× Type 91 - 1,872 lb (849 kg)		1× Type 91 - 1,872 lb (849 kg)	
Bomb Load:	Up to 1,000 lb (454 kg)		Up to 2,600 lb (1180 kg)		Up to 1,775 lb (805 kg)		Up to 1,775 lb (805 kg)	

Carrier Dive Bombers

COMPARISON TABLE

	SB2U-3		SBD-2		SBD-3		D3A1	
Manufacturer:	Chance Vought Aircraft		Douglas Aircraft Company		Douglas Aircraft Company		Aichi Aircraft Company	
Length:	34 ft	10.36 m	32 ft 1 in	9.79 m	32 ft 8 in	9.96 m	33 ft 5 in	10.2 m
Height:	10 ft 3 in	3.12 m	13 ft 7 in	4.14 m	13 ft 7 in	4.14 m	12 ft 8 in	3.85 m
Wingspan:	42 ft	12.8 m	41 ft 6 in	12.66 m	41 ft 6 in	12.66 m	47 ft 2 in	14.4 m
Wingarea:	305 sq ft	28.34 m^2	325 sq ft	30.19 m^2	325 sq ft	30.19 m^2	375.6 sq ft	34.9 m^2
Empty Weight:	5,634 lb	2556 kg	5,652 lb	2564 kg	6,345 lb	2878 kg	5,309 lb	2410 kg
Gross Weight:	7,474 lb	3390 kg	10,360 lb	4699 kg	10,400 lb	4717 kg	8,047 lb	3650 kg
Fuel Capacity:	130 gal	492 L	310 gal	1173 L	260 gal	984 L	285 gal	1079 L
Power Plant:	Pratt & Whitney R-1535-02		Wright R-1820-32 Cyclone 9		Wright R-1820-52 Cyclone 9		Mitsubishi Kinsei 44	
Military Power:	825 hp	615 kW	1000 hp	746 kW	1000 hp	746 kW	1,070 hp	798 kW
Power/Weight:	0.11 hp/lb	0.24 hp/kg	0.09 hp/lb	0.21 hp/kg	0.10 hp/lb	0.23 hp/kg	0.13 hp/lb	0.29 hp/kg
Max Speed:	211 kts	390 km/h	222 kts	412 km/h	217 kts	402 km/h	210 kts	390 km/h
Cruise Speed:	130 kts	244 km/h	128 kts	237 km/h	135 kts	250 km/h	160 kts	296 km/h
Rate of Climb:	1,070 ft/min	326 m/min	1,080 ft/min	330 m/min	1,190 ft/min	363 m/min	1,540 ft/min	469 m/min
Service Ceiling:	23,600 ft	7190 m	26,000 ft	7925 m	27,100 ft	8260 m	30,500 ft	9300 m
Combat Range:	973 nm	1800 km	1,065 nm	1970 km	1,175 nm	2175 km	795 nm	1472 km
Guns:	1× .50 (12.7 mm) forward 1× .50 (12.7 mm) rearward		2× .50 (12.7 mm) forward 1× .30 (7.62 mm) rearward		2× .50 (12.7 mm) forward 2× .30 (7.62 mm) rearward		2× .303 (7.7 mm) forward 1× .303 (7.7 mm) rearward	
Bomb Load:*	1× Mk. 12 - 500 lb (227 kg)		1× Mk. 13 - 1,000 lb (454 kg) or 1× Mk. 12 - 500 lb (227 kg)		1× Mk. 13 - 1,000 lb (454 kg) or 1× Mk. 12 - 500 lb (227 kg) and 2× Mk. 11 - 100 lb (45.4 kg)		1× Type 98 #25 - 534 lb (242 kg) or 1× Type 99 #25 - 553 lb (251 kg)	

* Used during the Battle of Midway

Carrier Fighters

COMPARISON TABLE

	F2A-3		F4F-3		F4F-4		A6M2b	
Manufacturer:	Brewster Aeronautical Corporation		Grumman Aircraft Engineering Corporation				Mitsubishi Aircraft Company	
Length:	26 ft 4 in	8.0 m	28 ft 9 in	8.76 m	28 ft 9 in	8.76 m	29 ft 9 in	9.06 m
Height:	12 ft	3.65 m	11 ft 10 in	3.60 m	9 ft 2 in	2.80 m	10 ft	3.05 m
Wingspan:	35 ft	10.6 m	38 ft	11.6 m	38 ft	11.6 m	39 ft 4 in	12.0 m
Wingarea:	209 sq ft	19.4 m^2	260 sq ft	24 m^2	260 sq ft	24 m^2	241 sq ft	22.44 m2
Empty Weight:	4,732 lb	2146 kg	5,342 lb	2422 kg	5,758 lb	2611 kg	3,704 lb	1680 kg
Gross Weight:	6,321 lb	2867 kg	7,000 lb	3175 kg	7,406 lb	3358 kg	5,313 lb	2410 kg
Fuel Capacity:	160 gal	605 L	144 gal	545 L	144 gal	545 L	144 (231*) gal	545 (875*) L
Power Plant:	Wright R-1820-40 Cyclone 9		Pratt & Whitney R-1830-76		Pratt & Whitney R-1830-86		Nakajima NK1C Sakae 12	
Military Power:	1200 hp	895 kW	1200 hp	895 kW	1200 hp	895 kW	950 hp	709 kW
Power/Weight:	0.19 hp/lb	0.42 hp/kg	0.17 hp/lb	0.38 hp/kg	0.16 hp/lb	0.36 hp/kg	0.18 hp/lb	0.39 hp/kg
Max Speed:	278 kts	515 km/h	286 kts	531 km/h	278 kts	515 km/h	287 kts	534 km/h
Cruise Speed:	140 kts	259 km/h	128 kts	237 km/h	128 kts	237 km/h	180 kts	333 km/h
Rate of Climb:	2,290 ft/min	698 m/min	2,265 ft/min	690 m/min	1,950 ft/min	594 m/min	3,100 ft/min	942 m/min
Service Ceiling:	33,200 ft	10 120 m	37,500 ft	11 430 m	34,000 ft	10 363 m	33,000 ft	10 000 m
Combat Range:	840 nm	1550 km	735 nm	1360 km	670 nm	1240 km	1,675 nm *	3100 km *
Nose Guns:	2 × .50 (12.7 mm), 750 rds **		—		—		2 × .303 (7.7 mm), 680 rds	
Wing Guns:	2 × .50 (12.7 mm), 450 rds		4 × .50 (12.7 mm), 430 rds		6 × .50 (12.7 mm), 240 rds		2 × .787 (20 mm), 60 rds	
Salvo Weight:	3.66 lb/sec	1.66 kg/sec	5.41 lb/sec	2.45 kg/sec	8.12 lb/sec	3.68 kg/sec	5.06 lb/sec	2.29 kg/sec

* With external 87.2 gal (330 L) drop tank, ** Allowance of ammunition, rounds per gun.

Briefing Templates

Intelligence Analysis

____
Date / Time

Enemy force estimated position: ______________________________________
Grid Coordinates

Enemy force estimated course / speed: __________________________________

Enemy force estimated intentions:

Launch air strike against: _____________

Continue search for our forces: _______

Alter course / speed to: ______________

Additional notes: ____

____

N3 / NAS CO Air Operations Plan
(Search)

Number of aircraft to be launched: ______________________________________

Mission:

- Search

Search axis (degrees true from home base): Aircraft 1: ____________________

Aircraft 2: ____________________

Aircraft 3: ____________________

Aircraft 4: ____________________

Aircraft 5: ____________________

Range: __ _____________ nm

Objective: ____

N3 / NAS CO Air Operations Plan
(Strike)

Umpire will not approve plan without work attached.

Number of aircraft to be launched: ____________________________________

Mission:

Objective: ____

Location (bearing and range) of target: ____________________________________

Is target within range of strike aircraft? Yes / No

Time of flight to target: ____________________________________

Composition of strike force:

- Torpedo bombers: ____________________________________
- Dive bombers: ____________________________________
- Heavy bombers: ____________________________________

N4 Logistics Briefing

Umpire will not approve unless all work is attached.

Number	Name	Max. fuel capacity	Amount used last 24 hours	% used last 24 hours	% remaining onboard	Projected use next 24 hours
Carrier 1						
Carrier 2						
Carrier 3						
Carrier 4						
NAS Fuel Farm						

Tanker Services Required? Yes / No

Intelligence Analysis

____
Date / Time

Enemy force estimated position: ________________________
Grid Coordinates

Enemy force estimated course / speed: ______________________

Enemy force estimated intentions:

Launch air strike against: ________

Continue search for our forces: _____

Alter course / speed to: ________

Additional notes: ____

____

N3 / NAS CO Air Operations Plan (Search)

Number of aircraft to be launched: ____________________

Mission:

- Search

Search axis (degrees true from home base):

Aircraft 1: ____________

Aircraft 2: ____________

Aircraft 3: ____________

Aircraft 4: ____________

Aircraft 5: ____________

Range: __________ nm

Objective: ________________________________

N3 / NAS CO Air Operations Plan (Strike)

Umpire will not approve plan without work attached.

Number of aircraft to be launched: ____________________

Mission:

Objective: ________________________________

Location (bearing and range) of target: ____________________

Is target within range of strike aircraft? Yes / No

Time of flight to target: ____________________

Composition of strike force:

- Torpedo bombers: ____________________
- Dive bombers: ____________________
- Heavy bombers: ____________________

N4 Logistics Briefing

Umpire will not approve unless all work is attached.

Number	Name	Max. fuel capacity	Amount used last 24 hours	% used last 24 hours	% remaining onboard	Projected use next 24 hours
Carrier 1						
Carrier 2						
Carrier 3						
Carrier 4						
NAS Fuel Farm						

Tanker Services Required? Yes / No

Daily War Game Preparation Problems

Day 1:

A. U.S.S. *Yorktown* (CV – 5) burns the following amount of fuel per nautical mile:

144.23 gallons per nautical mile at 15 knots
189.7 gallon per nautical mile at 20 knots

Yorktown must travel 1300 nautical miles at 15 knots.

i. How much fuel will she burn?
ii. How long in hours will the voyage take?

B. In the following encoded message, the letter "a" or the letter "e" ***is the vowel that appears most often***. Decode the following message:

5 100 100 5 15 55   65 45 20 115 5 125

Day 2:

A. U.S.S. *Yorktown* (CV – 5) burns the following amount of fuel per nautical mile:

144.23 gallons per nautical mile at 15 knots
189.7 gallon per nautical mile at 20 knots

Yorktown must travel 450 nautical miles at 20 knots.

i. How much fuel will she burn?
ii. How long in hours will the voyage take?

B. In the following encoded message, the letter "a" or the letter "e" ***is the vowel that appears most often***. Decode the following message:

24 2 42 28 6 16   2 18 36 6 36 2 12 40

Day 3:

A. U.S.S. *Yorktown* (CV – 5) burns the following amount of fuel per nautical mile:

144.23 gallons per nautical mile at 15 knots
189.7 gallon per nautical mile at 20 knots

Yorktown must travel 130 nautical miles at 15 knots.

i. How much fuel will she burn?
ii. How long in hours will the voyage take?

B. In the following encoded message, the letter "a" or the letter "e" ***is the vowel that appears most often***. Decode the following message:

9 2.5 3 10.5 2.5 6 10 7.5 2 0.5 12.5

Day 4:

A. U.S.S. *Yorktown* (CV – 5) burns the following amount of fuel per nautical mile:

144.23 gallons per nautical mile at 15 knots
189.7 gallon per nautical mile at 20 knots

Yorktown must travel 573 nautical miles at 20 knots.

i. How much fuel will she burn?
ii. How long in hours will the voyage take?

B. In the following encoded message, the letter "a" or the letter "e" ***is the vowel that appears most often***. Decode the following message:

1 144 400 25 324 9 225 441 324 361 25

Day 5:

A. U.S.S. *Yorktown* (CV – 5) burns the following amount of fuel per nautical mile:

144.23 gallons per nautical mile at 15 knots
189.7 gallon per nautical mile at 20 knots

Yorktown must travel 573 nautical miles at 20 knots.

i. How much fuel will she burn?
ii. How long in hours will the voyage take?

B. In the following encoded message, ***no letter appears most often***. Decode the following message:

64 1 5832 1331 125 2744 6859 512 729 4096

Day 6:

A. Given the observed altitude of the Sun (O) and the declination of the Sun (D), Latitude (La) can be determined by the following formulae:

$La = 90° - O + D$ (Sun in the Northern Hemisphere)
$La = 90° - O - D$ (Sun in Southern Hemisphere)

Find La if:

$$O = 60°$$
$$D = 12°N$$

B. Decode the following message:

100 105 90 70 30 75 90 65 5 100 45 75 70

Day 7:

A. Given the observed altitude of the Sun (O) and the declination of the Sun (D), Latitude (La) can be determined by the following formulae:

$La = 90° - O + D$ (Sun in the Northern Hemisphere)
$La = 90° - O - D$ (Sun in Southern Hemisphere)

Find La if:

$$O = 55°$$
$$D = 15°N$$

B. Decode the following message:

4 324 25 1 121 9 225 196 400 1 9 400

Day 8:

A. Given the observed altitude of the Sun (O) and the declination of the Sun (D), Latitude (La) can be determined by the following formulae:

$La = 90° - O + D$ (Sun in the Northern Hemisphere)
$La = 90° - O - D$ (Sun in Southern Hemisphere)

Find La if:

$$O = 60°\,12'$$
$$D = 12°N$$

B. Decode the following message:

36 10 28 8 10 44 30 42 38 2 40 0 32 30 18 28 40 2

Day 9:

A. Given the observed altitude of the Sun (O) and the declination of the Sun (D), Latitude (La) can be determined by the following formulae:

$La = 90° - O + D$ (Sun in the Northern Hemisphere)
$La = 90° - O - D$ (Sun in Southern Hemisphere)

Find La if:

$$O = 55°\,12'$$
$$D = 12°\,17'N$$

B. Decode the following message:

6 0.5 7 2 0.5 4.5 9 1.5 9 0.5 3 10

Day 10:

A. Given the observed altitude of the Sun (O) and the declination of the Sun (D), Latitude (La) can be determined by the following formulae:

$La = 90° - O + D$ (Sun in the Northern Hemisphere)
$La = 90° - O - D$ (Sun in Southern Hemisphere)

Find La if:

$$O = 68°\,15'$$
$$D = 15°\,19'N$$

B. Decode the following message:

81 196 9 324 25 1 361 25 361 256 25 25 16

Day 11:

A. Recall that to compute ***longitude*** in the ***Western Hemisphere*** you follow these steps:

(1) $$Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) - 12{:}00 = h{:}mm$$

(2) $$h \times \frac{15^\circ}{hour} = degrees\ west\ of\ 0^\circ\ longitude$$

(3) $$mm \div \frac{4'}{1^\circ} = additional\ degrees\ west\ of 0^\circ\ longitude$$

(4) $$add\ (2)\ and\ (3) together\ to\ get\ ship's\ west\ longitude$$

Find the west longitude if $GMT\ of\ LAN = 22{:}28$.

Day 12:

A. Recall that to compute ***longitude*** in the ***Western Hemisphere*** you follow these steps:

(1) $$Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) - 12{:}00 = h{:}mm$$

(2) $$h \times \frac{15^\circ}{hour} = degrees\ west\ of\ 0^\circ\ longitude$$

(3) $$mm \div \frac{4'}{1^\circ} = additional\ degrees\ west\ of 0^\circ\ longitude$$

(4) $$add\ (2)\ and\ (3) together\ to\ get\ ship's\ west\ longitude$$

Find the west longitude if $GMT\ of\ LAN = 22{:}50$.

Day 13:

A. Recall that to compute ***longitude*** in the ***Western Hemisphere*** you follow these steps:

(1) $Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) - 12{:}00 = h{:}mm$

(2) $h \times \frac{15°}{hour} = degrees\ west\ of\ 0°\ longitude$

(3) $mm \div \frac{4'}{1°} = additional\ degrees\ west\ of 0°\ longitude$

(4) $add\ (2)\ and\ (3) together\ to\ get\ ship's\ west\ longitude$

Find the west longitude if $GMT\ of\ LAN = 23{:}12$.

Day 14:

A. Recall that to compute ***longitude*** in the ***Western Hemisphere*** you follow these steps:

(1) $Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) - 12{:}00 = h{:}mm$

(2) $h \times \frac{15°}{hour} = degrees\ west\ of\ 0°\ longitude$

(3) $mm \div \frac{4'}{1°} = additional\ degrees\ west\ of 0°\ longitude$

(4) $add\ (2)\ and\ (3) together\ to\ get\ ship's\ west\ longitude$

Find the west longitude if $GMT\ of\ LAN = 23{:}44$.

Name: ____________________________

Name: ____________________________

War Game Practice Drill Number One

Show all work on separate sheets of paper.

If you complete, you get the points. If you do not complete, you lose the points

1. Your task force is in grid 01039 heading west at 15 knots.

2. Your receive the following coded message identifying the first target. Break the code (**1 point)**.

1728	1	5832	343	125		125	2744	125	2197	15625	
216	1728	125	125	8000		343	5832	729	64		1034
64	729	6859	8000	1	2744	27	125		854	2744	2197

3. Determine which aircraft you can send against this target. Use the tables on the large white board and circle the aircraft you want to use (you can use more than one type so long as each selected type can make it to the target) (**1 point)**:

   a. SB2U-3 Dive Bomber

   b. TBD-1 Torpedo Bomber

   c. F4F-3 Fighter

4. Determine how long it will take your selected aircraft to reach the target (use the Cruise Speed). **Do this only for the aircraft you selected in "3"** (**1 point**).

Time to target in hours

a. SB2U-3 Dive Bomber ______________________

b. TBD-1 Torpedo Bomber ______________________

c. F4F-3 Fighter ______________________

5. **Bonus Points (5 points):** Only the SB2U-3 Dive Bomber and the TBD-1 Torpedo Bomber can strike the target. You can take up to six of each if each can reach the target.

   a. The SB2U-3 has a probability of 0.5 that it will hit the target. The TBD-1 has a probability of 0.3 that it will hit the target. If you take 3 of each type, what is the probability that all will hit the target?

   ____

Name: ___________________________

War Game Practice Drill Number Two
Show all work on separate sheets of paper.
If you complete, you get the points. If you do not complete, you lose the points

1. Your task force is in grid 01037 heading west at 15 knots.

2. Your receive a coded message identifying the first target. This message will be found in your team leader's school e-mail box.

3. Determine which aircraft you can send against this target. Use the tables on the large white board and circle the aircraft you want to use (you can use more than one type so long as each selected type can make it to the target). You need 2 dive bombers or 2 torpedo planes, or 1 of each to kill an aircraft carrier. Determine how many planes of each you want to take.

Number

   a. SB2U-3 Dive Bomber ___________________

   b. TBD-1 Torpedo Bomber ___________________

   c. F4F-3 Fighter ___________________

4. Determine how long it will take your selected aircraft to reach the target (use the Cruise Speed). **Do this only for the aircraft you selected in "3"**.

Time to target in hours

   a. SB2U-3 Dive Bomber ______________________

   b. TBD-1 Torpedo Bomber ______________________

   c. F4F-3 Fighter ______________________

5. **Bonus Points (5 points):** Only the SB2U-3 Dive Bomber and the TBD-1 Torpedo Bomber can strike the target. You can take up to six of each if each can reach the target.

   a. The SB2U-3 has a probability of 0.57 that it will hit the target. The TBD-1 has a probability of 0.29 that it will hit the target. If you take 4 of each type, what is the probability that all will hit the target?

      ____

List all team members here and grade them (1, 2, 3, 4) on their work today:

Name	Grade
______________________________	________
______________________________	________
______________________________	________
______________________________	________
______________________________	________
______________________________	________

01021	01022	01023	01024	01025	01026	01027	01028	01029	01030	01031	01032
01033	01034	01035	01036	01037	01038	01039	01040	01041	01042	01043	01044
01045	01046	01047	01048	01049	01050	01051	01052	01053	01054	01055	01056
01057	01058	01059	01060	01061	01062	01063	01064	01065	01066	01067	01068

Coded Message Nr. 1

28561	1	38416	390625		625	38416	625	28561	390625		
81	1	104976	104976	6561	625	104976	130321		160000	279841	50625
16	1	160000	160000	20736	625	130321	4096	6561	65536	130321	
130321	6561	331776		81	104976	194481	6561	130321	625	104976	130321
1	38416	256		1296	6561	1296	160000	625	625	38416	
256	625	130321	160000	104976	50625	625	104976	130321		1	160000
2401	104976	6561	256		1035						
81	130321	625		625	1	130321	160000		1	160000	
23		14641	38416	50625	160000	130321					

Coded Message Nr. 2

2197	1	2744	15625		125	2744	125	2197	15625		
27	1	5832	5832	729	125	5832	6859		8000	12167	3375
8	1	8000	8000	1728	125	6859	512	729	4096	6859	
6859	729	13824		27	5832	9261	729	6859	125	5832	6859
1	2744	64		216	729	216	8000	125	125	2744	
64	125	6859	8000	5832	3375	125	5832	6859		1	8000
343	5832	729	64		1035						
27	6859	125		125	1	6859	8000		1	8000	
23		1331	2744	3375	8000	6859					

Coded Message Nr. 3

39	3	42	75		15	42	15	39	75		
9	3	54	54	27	15	54	57		60	69	45
6	3	60	60	36	15	57	24	27	48	57	
57	27	72		9	54	63	27	57	15	54	57
3	42	12		18	27	18	60	15	15	42	
12	15	57	60	54	45	15	54	57		3	60
320		42	39		69	15	60				
9	57	15		15	3	57	60		3	60	
23		33	42	45	60	57					

Coded Message Nr. 4

3.25	0.25	3.5	6.25		1.25	3.5	1.25	3.25	6.25		
0.75	0.25	4.5	4.5	2.25	1.25	4.5	4.75		5	5.75	3.75
0.5	0.25	5	5	3	1.25	4.75	2	2.25	4	4.75	
4.75	2.25	6		0.75	4.5	5.25	2.25	4.75	1.25	4.5	4.75
0.25	3.5	1		1.5	2.25	1.5	5	1.25	1.25	3.5	
1	1.25	4.75	5	4.5	3.75	1.25	4.5	4.75		0.25	5
-3.25		3.5	3.25		5.75	1.25	5				
0.75	4.75	1.25		1.25	0.25	4.75	5		0.25	5	
-3.5		2.75	3.5	3.75	5	4.75					

Name: ______________________________

War Game Practice Drill Number Three

Show all work on separate sheets of paper.

If you complete, you get the points. If you do not complete, you lose the points

1. Your task force, with the carrier U.S.S. *Yorktown* is in grid 01049 heading west at 15 knots. Assume you have 80% fuel onboard.

2. Your receive a coded message from the Commander in Chief of the U. S. Pacific Fleet (CINCPACFLT). Directing you to move your ships to a new location to do something. Decode this message.

8000	1	8	8	-	4	-				
27	729	2744	27	4096	1	27	216	1728	8000	-
3375	4096	1728	1	2744		2	9	-	4	2
5832	6859	7	-	1						
8000	1	7	9	-	343	5832	729	64	"01051"	-
1	5	0	0	-	1728	3375	27	1	1728	-
4096	1728	1	8000	8000	1035					

3. Determine the course, speed you need to get to this new location on time. Determine how much fuel you have burned (gallons and percentage) when you get to the new location, and how much fuel you will need in gallons, to get to 100%.

Course: ________________________

Speed: ________________________

Time: ________________________

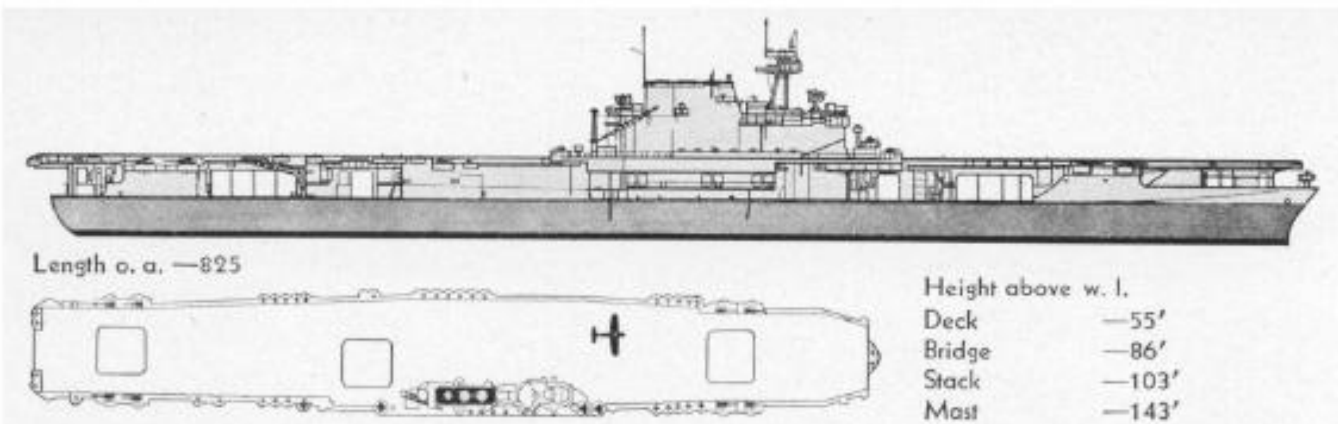
Fuel burned (gallons / percent) ________________________

Fuel needed (gallons) ________________________

List all team members here and grade them (1, 2, 3, 4) on their work today:

Name	Grade

Ship	FL Disp.	Max Speed	Max. Fuel Load	Consumption Rate
Yorktown (CV 5)	25,500 tons	32.5 knots	1,500,000 gallons	at 15 knots: 144.23 gallons/nm at 20 knots: 189.87 gallons/nm
Enterprise (CV 6)	25,500 tons	32.5 knots	1,500,000 gallons	at 15 knots: 144.23 gallons/nm at 20 knots: 189.87 gallons/nm
Hornet (CV 8)	25,500 tons	32.5 knots	1,500,000 gallons	at 15 knots: 144.23 gallons/nm at 20 knots: 189.87 gallons/nm



01021	01022	01023	01024	01025	01026	01027	01028	01029	01030	01031	01032
01033	01034	01035	01036	01037	01038	01039	01040	01041	01042	01043	01044
01045	01046	01047	01048	01049	01050	01051	01052	01053	01054	01055	01056
01057	01058	01059	01060	01061	01062	01063	01064	01065	01066	01067	01068

Name: ______________________________

War Game Practice Drill Number Four
Show all work on separate sheets of paper.
If you complete, you get the points. If you do not complete, you lose the points

1. You are commander of a Japanese force consisting of four aircraft carriers and many escort ships. You are located in grid 01046 on a course east at 18 knots.

2. You receive the following message:

敵は**480**
NMは東に見つけました。コース西。**25**ノットのスピード。二つの空母。多くの護衛船。爆撃と魚雷爆撃機で攻撃。

Translate the message and write the translation here. Team leaders will find this message in their e-mail.

____

____

____

____

____

3. At your current speed, determine how many hours it will take for your ships to reach the target (**This is a trick question. Hint:** Remember, the target is moving toward you at a certain speed.

____

4. Determine which aircraft you can send against this target. Use the tables on the large white board and circle the aircraft you want to use (you can use more than one type so long as each selected type can make it to the target). You need 2 dive bombers or 2 torpedo planes, or 1 of each to kill an aircraft carrier. Determine how many planes of each you want to take.

	Number
a. D3A1 Dive Bomber	___________________
b. B5N1 Torpedo Bomber	___________________
c. A6M2b Fighter	___________________

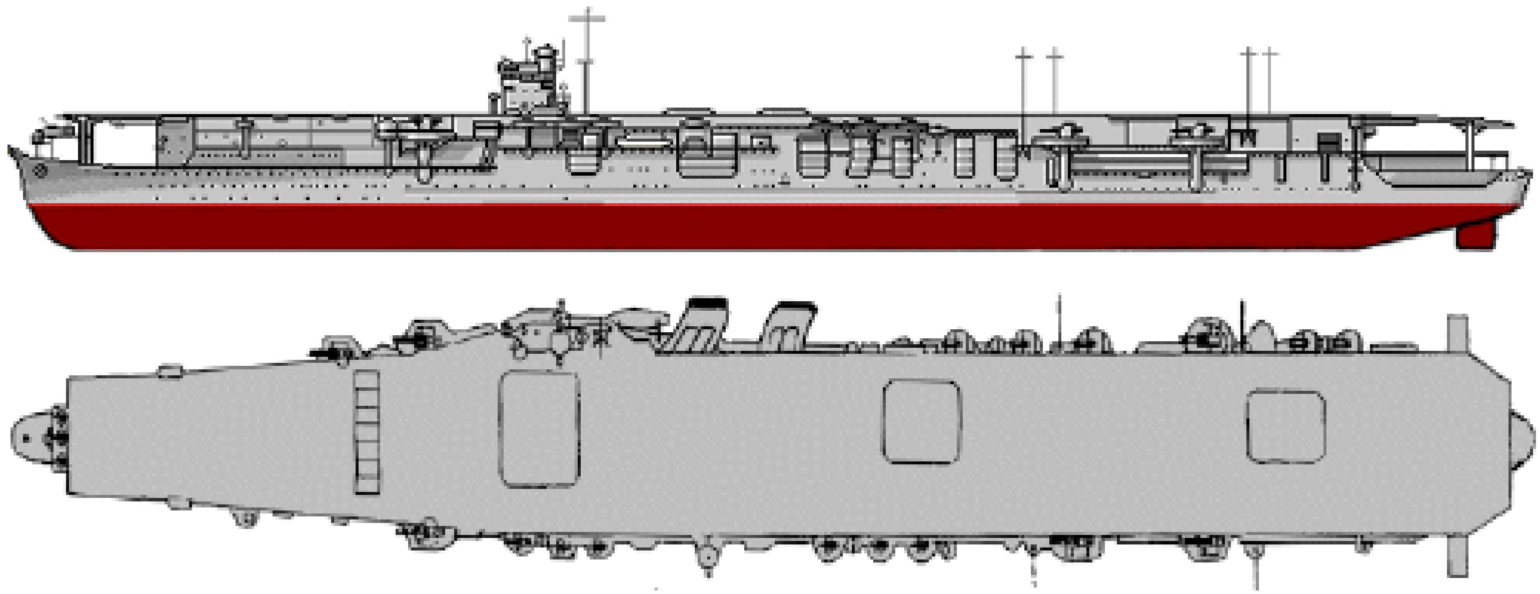
5. Determine how long it will take your selected aircraft to reach the target (use the Cruise Speed). **Do this only for the aircraft you selected in "3"**.

	Time to target in hours
a. D3A1 Dive Bomber	___________________
b. B5N1 Torpedo Bomber	___________________
c. A6M2b Fighter	___________________

6. List all team members here and grade them (1, 2, 3, 4) on their work today:

Name	Grade
____	________
____	________
____	________
____	________
____	________

蒼流 *(Soryu)* FL Disp. 19,800 tons	34.5 knots	1,092,593 gallons	at 18 knots: 142.45 gallons/nm



Game Board

01021	01022	01023	01024	01025	01026	01027	01028	01029	01030	01031	01032
01033	01034	01035	01036	01037	01038	01039	01040	01041	01042	01043	01044
01045	01046	01047	01048	01049	01050	01051	01052	01053	01054	01055	01056
01057	01058	01059	01060	01061	01062	01063	01064	01065	01066	01067	01068

Pre-Game Intelligence Disclosure 1

100	2916	2601	2401	4	576	1369	3025	2209		100	2601	2916	1521	1681		49	2601	2401	2401	1369	2500	1600	1681	2916
576	2601	4		100	2304	1681	1681	3136		49	2601	2401	2401	1369	2500	1600	1681	2916						
529	3249	1444	2116	4		64	1369	2025	2304	3721		361	400	529	625	289		1369	2500	1600				
49	2601	2401	2401	1369	2500	1600	1681	2916	441	3025		169	2500	3136	1681	2500	3136	2025	2601	2500	3025			
1764	2601	2916		2	3		289	1369	3721		1	9	4	2										
576	1369	3025	2209		1764	2601	2916	1521	1681		2704	2601	3025	2025	3136	2025	2601	2500	4					
121	2916	2025	1600		0	1	0	3	8															
100	3249	1681	2304		2601	2500	1444	2601	1369	2916	1600	4												
49	1369	2916	2916	2025	1681	2916				121	1369	2304	2304	2601	2500	3025		400	1681	2916	1521	1681	2500	3136
81	324	576	81	484	400	484	169	529	81	150	289	169	256	256	169	361	324		100					
144	361	484	324	81	576					150	289	169	256	256	169	361	324		100					
841	361	484	225	576	361	729	324			150	289	169	256	256	169	361	324		100					
81	3025	3136	2025	2401	1369	3136	1681	1600		1764	3249	1681	2304		2601	2500	1444	2601	1369	2916	1600	4		
49	1369	2916	2916	2025	1681	2916				121	1369	2304	2304	2601	2500	3025		400	1681	2916	1521	1681	2500	3136
81	324	576	81	484	400	484	169	529	81	145	289	169	256	256	169	361	324		97					
144	361	484	324	81	576					145	289	169	256	256	169	361	324		97					
841	361	484	225	576	361	729	324			145	289	169	256	256	169	361	324		97					
169	2500	3136	1681	2500	3136	2025	2601	2500	3025		2500	1681	3600	3136		2	4		1936	2601	3249	2916	3025	4
49	2601	2500	3136	2025	2500	3249	1681		1681	2500	2916	2601	3249	3136	1681		400	2601	2025	2500	3136			
256	3249	1521	2209																					

War Game Play and Outcome

Umpire Sheet - Day 1

1. Imperial Japanese Navy *Kido Butai*

    a. Location: Grid 01045
    b. Course: Northeast
    c. Speed: 18 knots
    d. Fuel Onboard (each carrier): 1,092,593 gallons
    e. Fuel Used Last 24 Hours 0
    f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm

2. Imperial Japanese Navy Midway Invasion Force

    a. Location: Grid 01035
    b. Course: East
    c. Speed: 18 knots

3. USN Task Force 16 (*Enterprise* and *Hornet*)

    a. Location: Grid 01025
    b. Course: West
    c. Speed: 15 knots
    d. Fuel Onboard (each carrier): 1,500,000 gallons
    e. Fuel Used Last 24 Hours 0
    f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm
    g.

4. USN Task Force 17 (*Yorktown*)

    a. Location: Grid 01038
    b. Course: Northwest
    c. Speed: 15 knots
    d. Fuel Onboard (each carrier): 1,500,000 gallons
    e. Fuel Used Last 24 Hours 0
    f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm

Intelligence Disclosures

To Japanese Forces:

-90	-108		54	84		18	6	108	108	150	54	84	42		90	126	120		120	48	30				
120	6	114	66		6	114	114	54	42	84	30	24		54	84		90	96	30	108	6	120	54	90	84
96	72	6	84		-84	-42	-114	-72	-84		150	90	126		138	54	72	72		12	30				
42	90	132	30	108	84	30	24		12	150		120	48	30		96	108	54	84	18	54	96	72	30	
90	36		18	6	72	18	126	72	6	120	30	24		108	54	114	66	-120		138	48	54	18	48	
150	90	126		114	48	6	72	72		54	84	120	30	108	96	108	30	120		120	90				
78	30	6	84		120	48	30		6	132	90	54	24	6	84	18	30		90	36					
30	144	96	90	114	126	108	30		90	36		150	90	126	108		36	90	108	18	30		120	90	
6	120	120	6	18	66		12	150		114	126	96	30	108	54	90	108		30	84	30	78	150		
36	90	108	18	30	114		138	54	120	48	90	126	120		42	90	90	24							
96	108	90	114	96	30	18	120		90	36		54	84	36	72	54	18	120	54	84	42	-120			
6	114	######	6	######	108	30	114	126	72	120		90	36		114	126	18	48							
30	144	96	90	114	126	108	30	-120		42	108	30	6	120	30	108		24	6	78	6	42	30		

1. IN CARRYING OUT THE TASK ASSIGNED IN OPERATION PLAN 29-42 YOU WILL BE GOVERNED BY THE PRINCIPLE OF CALCULATED RISK, WHICH YOU SHALL INTERPRET TO MEAN THE AVOIDANCE OF EXPOSURE OF YOUR FORCE TO ATTACK BY SUPERIOR ENEMY FORCES WITHOUT GOOD PROSPECT OF INFLICTING, AS A RESULT OF SUCH EXPOSURE, GREATER DAMAGE TO THE ENEMY.

To American Forces:

12167	1331	512		4096	1728	343	17576	64	21952		1728	4913	15625	64	10648	1728	5832	4913		729	5832	9261	216	512
17576	1728	3375	3375		64	9261	9261	1728	15625	512		5832	729	729		4096	1728	343	17576	64	21952			
1728	10648	3375	64	4913	343		5832	4913		19683	-4096	343	64	21952		64	4913	343						
1728	4096	4096	512	343	1728	64	12167	512	3375	21952		125	512	1000	1728	4913		3375	64	4913	343	1728	4913	1000
5832	6859	512	9261	64	12167	1728	5832	4913	10648	-4913		10648	512	216	13824	9261	1728	4913	1000		12167	1331	512	
64	1728	9261	729	1728	512	3375	343		64	4913	343		12167	1331	512		729	13824	512	3375				
343	512	6859	5832	12167		64	10648		1728	12167	10648		6859	9261	1728	4096	64	9261	21952					
5832	125	2197	512	216	12167	1728	15625	512	-3375															

THE MIDWAY INVASION FORCE WILL ARRIVE OFF MIDWAY ISLAND ON X-DAY AND IMMEDIATELY BEGIN LANDING OPERATIONS, SECURING THE AIRFIELD AND THE FUEL DEPOT AS ITS PRIMARY OBJECTIVE.

Umpire Sheet - Day 2

1. Imperial Japanese Navy *Kido Butai*

    a. Location: Grid 01036
    b. Course: Northeast
    c. Speed: 20 knots
    d. Fuel Onboard (each carrier): 1,024,217 gallons / 94%
    e. Fuel Used Last 24 Hours 68376 gallons
    f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm

2. Imperial Japanese Navy Midway Invasion Force

    a. Location: Grid 01037
    b. Course: East
    c. Speed: 18 knots

3. USN Task Force 16 (*Enterprise* and *Hornet*)

    a. Location: Grid 01023
    b. Course: West
    c. Speed: 15 knots
    d. Fuel Onboard (each carrier): 1,448,077.2 gallons / 93%
    e. Fuel Used Last 24 Hours 51922.8 gallons
    f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm
    g.

4. USN Task Force 17 (*Yorktown*)

    a. Location: Grid 01023
    b. Course: Northwest
    c. Speed: 20 knots
    d. Fuel Onboard (each carrier): 1,317,724.8 gallons / 88%
    e. Fuel Used Last 24 Hours 91137.6 gallons
    f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm

Situation

1. American forces remain undetected.

2. American forces detected a Japanese force, composition unknown, in grid 01045 heading east at approximately 20 knots 24 hours ago.

3. Point totals:

    a. Period 3 — 0 Points
    b. Period 5 — 100 Points
    c. Period 6 — 25 Points

Umpire Sheet - Day 3

1. Imperial Japanese Navy *Kido Butai*

   a. Location: Grid 01037
   b. Course: Various – remaining in 01037
   c. Speed: 20 knots
   d. Fuel Onboard (each carrier): 955,841 gallons / 87%
   e. Fuel Used Last 24 Hours 68,376 gallons
   f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm

2. Imperial Japanese Navy Midway Invasion Force

   a. Location: Grid 01037
   b. Course: Various – remaining in 01037
   c. Speed: 18 knots

3. USN Task Force 16 (*Enterprise* and *Hornet*)

   a. Location: Grid 01023
   b. Course: Various – remaining in 01023
   c. Speed: 15 knots
   d. Fuel Onboard (each carrier): 1,396,154.4 gallons / 97%
   e. Fuel Used Last 24 Hours 51922.8 gallons
   f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm

4. USN Task Force 17 (*Yorktown*)

   a. Location: Grid 01023
   b. Course: Northwest
   c. Speed: 20 knots
   d. Fuel Onboard (each carrier): 1,408,862.4 gallons / 94%
   e. Fuel Used Last 24 Hours 91137.6 gallons
   f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm

Situation

1. American and Japanese forces remain undetected.

2. Point totals:

   a. Period 3 — 0 Points
   b. Period 5 — 120 Points
   c. Period 6 — 105 Points

Umpire Sheet - Day 4

1. Imperial Japanese Navy *Kido Butai*

    a. Location: Grid 01037
    b. Course: Various – remaining in 01037
    c. Speed: 20 knots
    d. Fuel Onboard (each carrier): 887,465 gallons / 81%
    e. Fuel Used Last 24 Hours 68,376 gallons
    f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm

2. Imperial Japanese Navy Midway Invasion Force

    a. Location: Grid 01037
    b. Course: Various – remaining in 01037
    c. Speed: 18 knots

3. USN Task Force 16 (*Enterprise* and *Hornet*)

    a. Location: Grid 01023
    b. Course: Various – remaining in 01023
    c. Speed: 15 knots
    d. Fuel Onboard (each carrier): 1,344,231.6 gallons / 90%
    e. Fuel Used Last 24 Hours 51922.8 gallons
    f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm
    g.

4. USN Task Force 17 (*Yorktown*)

    a. Location: Grid 01023
    b. Course: Northwest
    c. Speed: 20 knots
    d. Fuel Onboard (each carrier): 1,317,724.8 gallons / 88%
    e. Fuel Used Last 24 Hours 91137.6 gallons
    f. Projected Use Next 24 Hours Speed * 24 * gallons burned / nm

Situation

1. Task Forces 16 and 17 detected by *Kido Butai*.

    a. 100 plane (52 torpedo bombers in five squadrons, 48 dive bombers in five squadrons) attacked American forces.
    b. 2 torpedo hits and 1 bomb hit.
    c. *Hornet* sunk.
    d. Points: $25 + 50 + 25 = 100$

2. ***Kido Butai*** detected by Task Force 16, not Task Force 17.

    a. Task Force 16: 72 dive bomber aircraft strike (7 squadrons).
    b. 3 hits.
    c. *Kaga* on fire and burning – destroyed.
    d. Task Force 16 points: $25 + 50 + 25 = 100$

3. Point totals:

    a. Period 3 100 Points
    b. Period 5 220 Points
    c. Period 6 105 Points

Intentions Next 24 Hours: *Kido Butai*

Intelligence Analysis

____
Date / Time

Enemy force estimated position: ____________________
Grid Coordinates

Enemy force estimated course / speed: ____________________

Enemy force estimated intentions:

Launch air strike against: __________

Continue search for our forces: ______

Alter course / speed to: __________

Additional notes: ____________________

____

N3 / NAS CO Air Operations Plan (Search)

Number of aircraft to be launched: ____________________

Mission:

- Search

Search axis (degrees true from home base):

Aircraft 1: __________

Aircraft 2: __________

Aircraft 3: __________

Aircraft 4: __________

Aircraft 5: __________

Range: __ __________ nm

Objective: ____________________

N3 / NAS CO Air Operations Plan (Strike)

Umpire will not approve plan without work attached.

Number of aircraft to be launched: ____________________

Mission:

Objective: ____________________

Location (bearing and range) of target: ____________________

Is target within range of strike aircraft? Yes / No

Time of flight to target: ____________________

Composition of strike force:

- Torpedo bombers: ____________________
- Dive bombers: ____________________
- Heavy bombers: ____________________

N4 Logistics Briefing

Umpire will not approve unless all work is attached.

Number	Name	Max. fuel capacity	Amount used last 24 hours	% used last 24 hours	% remaining onboard	Projected use next 24 hours
Carrier 1						
Carrier 2						
Carrier 3						
Carrier 4						
NAS Fuel Farm						

Tanker Services Required? Yes / No

Intentions Next 24 Hours: *Task Force 16*

Intelligence Analysis

____
Date / Time

Enemy force estimated position: ____________________
Grid Coordinates

Enemy force estimated course / speed: ____________________

Enemy force estimated intentions:

Launch air strike against: __________

Continue search for our forces: ______

Alter course / speed to: __________

Additional notes: ____

____

N3 / NAS CO Air Operations Plan (Search)

Number of aircraft to be launched: ____________________

Mission:

- Search

Search axis (degrees true from home base):

Aircraft 1: __________

Aircraft 2: __________

Aircraft 3: __________

Aircraft 4: __________

Aircraft 5: __________

Range: __________ nm

Objective: ______________________________

N3 / NAS CO Air Operations Plan (Strike)

Umpire will not approve plan without work attached.

Number of aircraft to be launched: ____________________

Mission:

Objective: ______________________________

Location (bearing and range) of target: ____________________

Is target within range of strike aircraft? Yes / No

Time of flight to target: ____________________

Composition of strike force:

- Torpedo bombers: ____________________
- Dive bombers: ____________________
- Heavy bombers: ____________________

N4 Logistics Briefing

Umpire will not approve unless all work is attached.

Number	Name	Max. fuel capacity	Amount used last 24 hours	% used last 24 hours	% remaining onboard	Projected use next 24 hours
Carrier 1						
Carrier 2						
Carrier 3						
Carrier 4						
NAS Fuel Farm						

Tanker Services Required? Yes / No

Intentions Next 24 Hours: *Task Force 17*

Intelligence Analysis

____

Date / Time

Enemy force estimated position: _______________
Grid Coordinates

Enemy force estimated course / speed: _______________

Enemy force estimated intentions:

Launch air strike against: ________

Continue search for our forces: ________

Alter course / speed to: ________

Additional notes: _______________

____

N3 / NAS CO Air Operations Plan (Search)

Number of aircraft to be launched: _______________

Mission:

- Search

Search axis (degrees true from home base):

Aircraft 1: ________

Aircraft 2: ________

Aircraft 3: ________

Aircraft 4: ________

Aircraft 5: ________

Range: ________ nm

Objective: _______________

N3 / NAS CO Air Operations Plan (Strike)

Umpire will not approve plan without work attached.

Number of aircraft to be launched: _______________

Mission:

Objective: _______________

Location (bearing and range) of target: _______________

Is target within range of strike aircraft? Yes / No

Time of flight to target: _______________

Composition of strike force:

- Torpedo bombers: _______________
- Dive bombers: _______________
- Heavy bombers: _______________

N4 Logistics Briefing

Umpire will not approve unless all work is attached.

Number	Name	Max. fuel capacity	Amount used last 24 hours	% used last 24 hours	% remaining onboard	Projected use next 24 hours
Carrier 1						
Carrier 2						
Carrier 3						
Carrier 4						
NAS Fuel Farm						

Tanker Services Required? Yes / No

Appendix 1 – Glossary

battleship	The largest and most heavily armed type of warship. A battleship's main armament was large caliber guns that could fire explosive shells at very long ranges. U.S.S. *Massachusetts* is a battleship.
bow	1) The forward part of a ship; as in: "Go stand watch on the bow." 2) The foremost outboard part of a ship; as in: "Notice how her bow cuts the water."
combatant	A country fighting in a war.
commander in chief	A senior military officer who commands a very large military organization such as an army or a fleet of ships.
correlation	a relationship that exists between two variables (normally the x variable and the y variable).
cryptanalysis	The analysis and deciphering of cryptographic writings or systems.
cryptanalyst	A decoder skilled in the analysis of codes.
keel	The central longitudinal structural member of a ship to which all the frames, stem and sternpost are fastened.
port	The left side of a vessel when seen by someone facing the bow.
reconnaissance	an exploratory military search to find an enemy.

starboard	The right side of a vessel when seen by someone facing the bow.
stem	The bow of a ship, referring to the timber between the forward end of the keel and the deck.
stern	The aftermost part of the hull of a ship, referring to the feature between the after end of the keel and the deck.
task force	A group of warships organized under one commander and assigned to perform a specific mission or task.

Appendix 2

U. S. S. MASSACHUSETTS

Brief Description

Engineering Installation

The four South Dakota class battleships represented the second group of 35,000-ton capital ships whose construction began shortly before the Second World War. Built with Fiscal Year 1939 appropriations, they were more compact and better protected than the preceding *North Carolina* class, but had the same main battery of nine 16"/45 guns in triple turrets. Their innovative hull design featured an internal armor belt, to protect the ships' vitals against 16" shells, and outboard propeller shafts that extended further aft than the inboard ones. They also had improved anti-torpedo side protection and more powerful engines, the latter being necessary to drive their shorter hulls at the designed 27-knot speed.

These ships were all completed in March-August 1942, providing a welcome reinforcement to the Navy's surface battle fleet at a critical stage of World War II. In 1942-43, they stood guard in the Atlantic against possible sorties by German battleships, took part in the invasion of North Africa and in operations around Guadalcanal. During the latter campaign, *South Dakota* was damaged in a gunnery engagement with a Japanese force that included the old battleship *Kirishima*. As the U.S. went on the offensive in the Central Pacific, they joined in escorting the fast carrier task forces, a job for which their heavy anti-aircraft gun batteries were well-suited. They also employed their main battery guns in shore bombardment, and were kept ready to form battle line in case their Japanese opposite numbers should appear.

All four *South Dakota* class battleships went into reserve after World War II and saw no further active service. When they were disposed of in the early 1960s, *Alabama* and *Massachusetts* became a memorials. The other two were sold for scrapping.

Photo # 19-N-28532 USS South Dakota under construction, April 1940

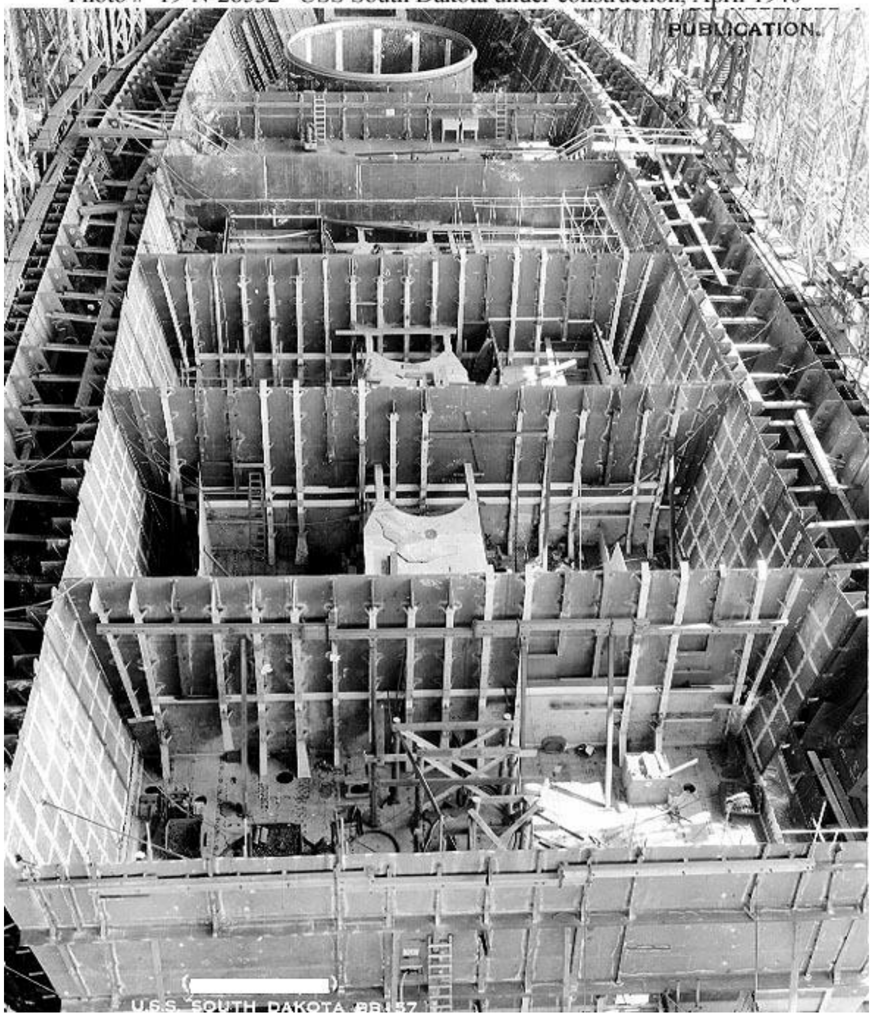


Photo # NH 93910 USS Indiana in drydock, March 1942. Stern view.

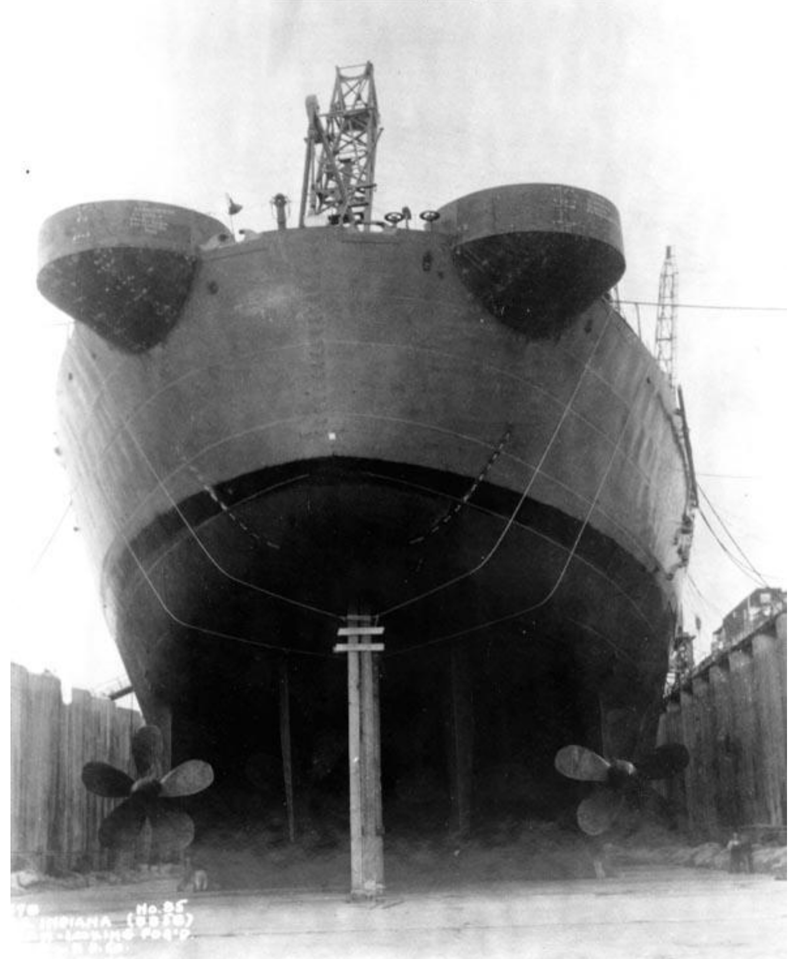


Photo # NH 93909 View below USS Indiana's hull, showing propellers and skegs, March 1942

Photo # NH 97255 USS Massachusetts off Port Wilson, Washington, July 1944

Photo # 80-G-156818 Drawing of battleships BB 58, 59 & 60 (Indiana, Massachusetts & Alabama), for camouflage planning, circa 1943 -- starboard side

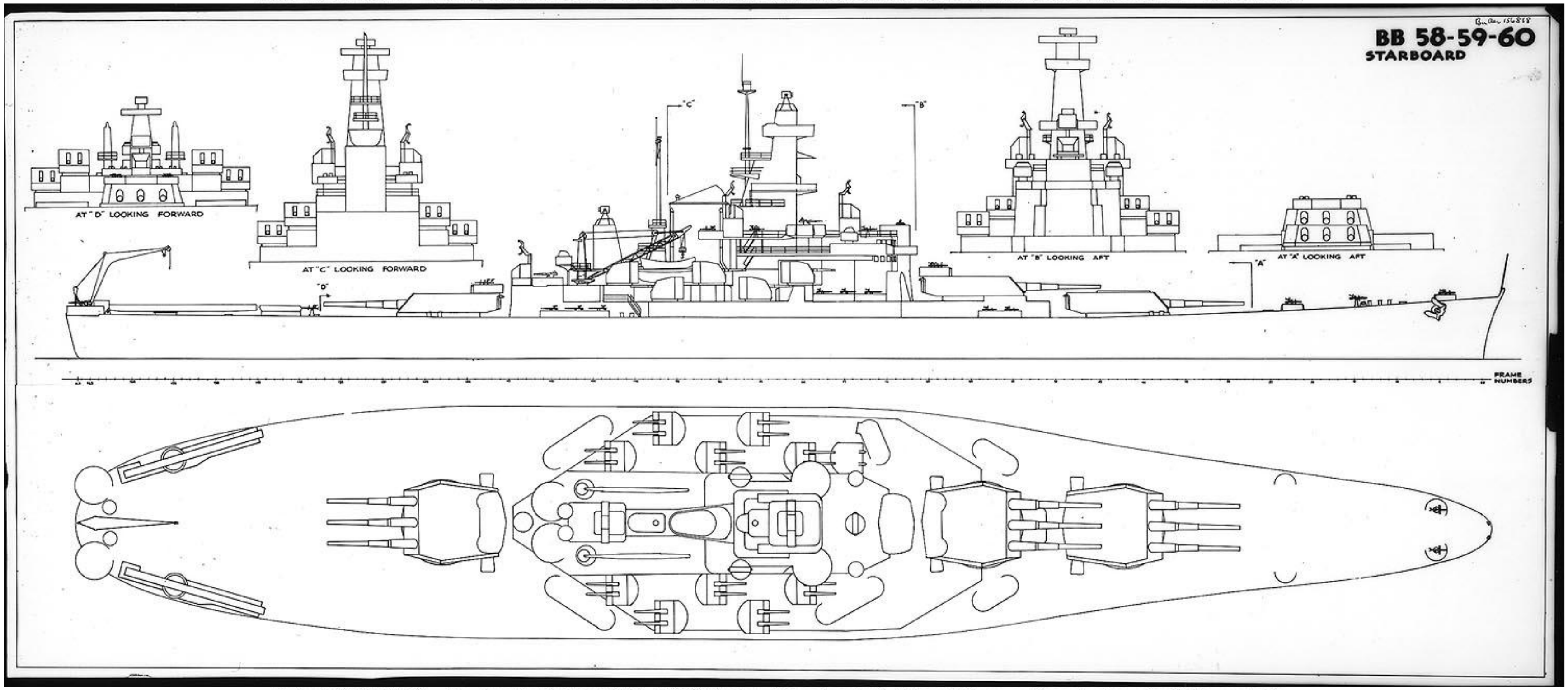
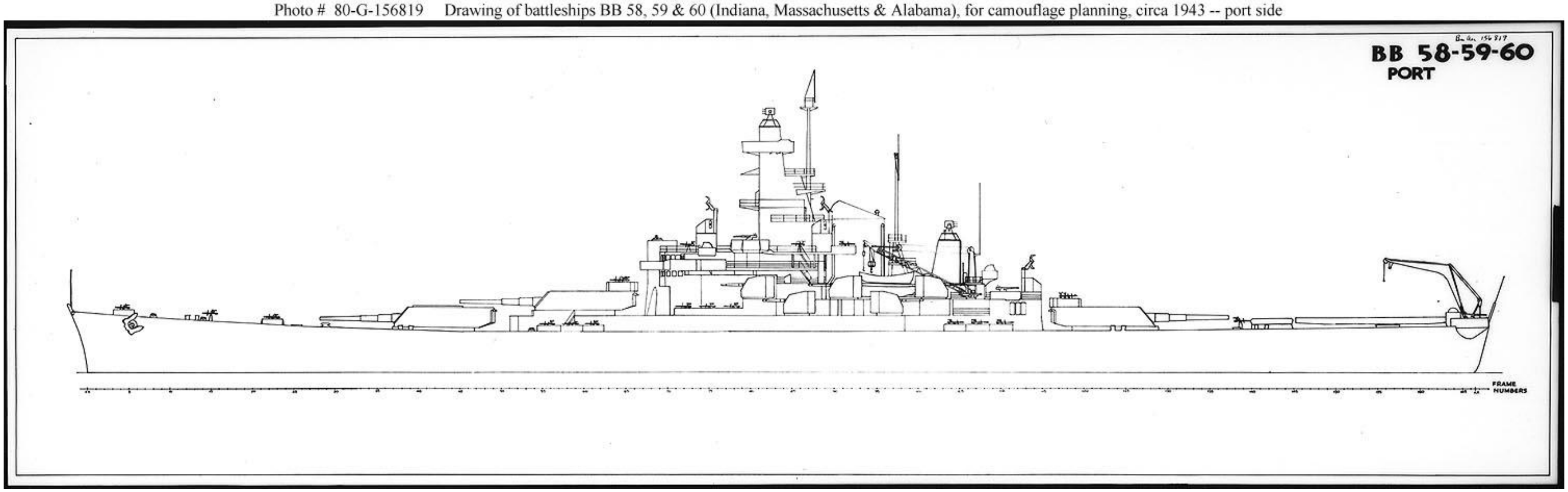


Photo # 80-G-156819 Drawing of battleships BB 58, 59 & 60 (Indiana, Massachusetts & Alabama), for camouflage planning, circa 1943 -- port side



A BRIEF DESCRIPTION OF THE ENGINEERING DEPARTMENT

PURPOSE. This brief is prepared with a view to enlightening those men, who join the ship without having much previous experience in engineering matters, on the scope of activities of the Engineer Department. It is hoped that by reading this paper carefully, much of the mystery regarding the raze of machinery and piping will vanish, and the new man will be able to proceed systematically to educate himself in his job below decks. By following the various steam, water, oil, electrical, and air cycles from start to finish, the novice can, step by step, trace the path of each and learn the purpose and method of operation of every unit along the path. To further aid self instruction, here are available the following means:

Operating Instructions at Each Unit.

Safety Instructions at Each Unit.

Manufacturers Instruction Books (in the LOG ROOM (Engineer Office)).

Blueprints (in Log Room Files).

Pamphlet Extracts of Chapters of the Manual of Engineering Instructions (in the Log Room).

Text Docks and Handbooks on Engineering Subjects (in the Log Room and Ship's Library).

Bureau of Navigation Courses (obtained from DIVISION OFFICER).

Operating Records and Various Stations (on which are printed much information).

No man should hesitate to ask the Log Room Yeoman for any of the above instructive matter. There are of course limited numbers of publications available, so you may have to wait your turn for a book.

Also, *do not hesitate* to ask officers or petty officers to explain things you don't understand.

It will be noted that numerous items throughout this paper are capitalized. This is done to emphasize an important unit or system which should be investigated separately in complete detail. When studying a unit, you should satisfy yourself on the following:

1. What is this unit called?
2. What is its purpose?
3. How does it fit into the cycle or machinery system of which it is a part?
4. How can I start it, bring it up to speed, stop it, cut it in or out of the line?
5. What are the most likely accidents or "casualties" that may occur?
6. What happens in the system if this unit stops?
7. Are there any thermometers, gages, tachometers, or other instruments and devices for the proper operation of this unit? If so, what should the readings be?
8. How is the unit lubricated? What kind of oil? How clean the oil?
9. How is the unit drained?

10. What is the procedure for preventing corrosion?
11. Are there any governors or overspeed devices? How do they work? What other safety devices are there?
12. What kind of glands are there? How are the shafts packed?
13. What readings of this unit are entered in the logs?
14. How are heat exchange surfaces such as boiler tubes, evaporator tubes kept clean?
15. Where are the steam, oil, water, and drain valves, or the switch box for starting this unit?

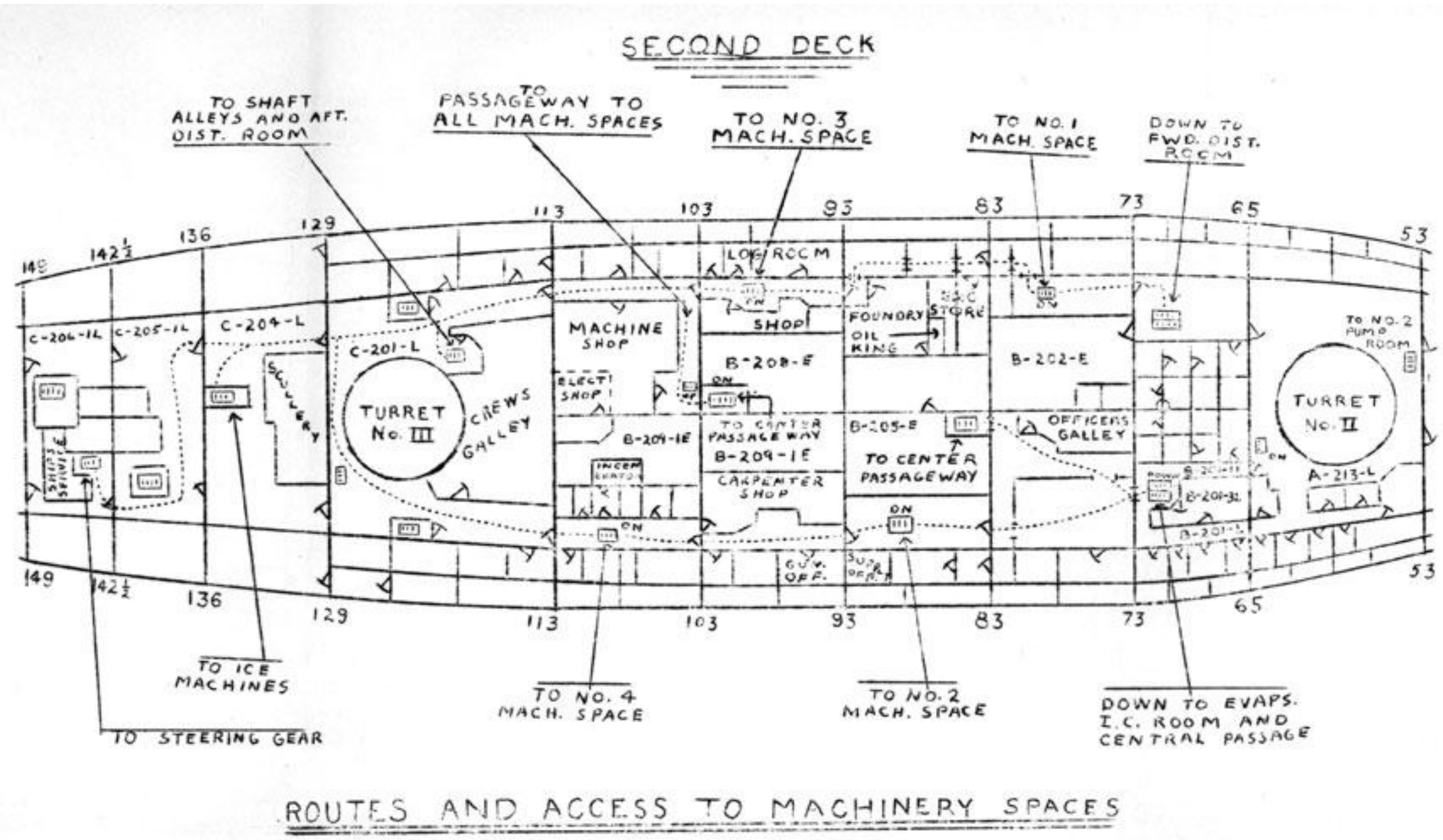
LOCATION OF MACHINERY SPACES

The principle units of the Engineering Department are installed in four large machinery spaces located below the second deck, extending from frame 73 to frame 113. Other units outside these spaces are located as follows: On the first platform deck, on the starboard side between frames 65 and 73 is found the Interior Communication room, while far aft on the same deck, between frames 136 and 142 ½ is located the Refrigerating Machinery. Still further aft, on the same level, between frames 142 ½ and 155 are the two separate Steering Engine roams. On the deck below, known as the second platform, between frames 65 and 68 is the Forward Electrical Distribution room, in which is #1 Distribution Board. Just aft, between frames 68 and 73 is the evaporator room, where most of the fresh water used on board is made. Between frames 113 and 117 ½ is the After Electrical Distribution room, in which is located #4 Distribution Board.

Directly under the After Distribution Room is located the after Emergency Diesel-Generator Room, in which, besides the generator, are diesel oil pumps and medium pressure air compressors. On the port side of the second deck, about amidships may be found the Engineer Office, the Machine Shop, Metalsmith Shop and the Fuel Oil Test Shop.

For watertight integrity reasons, the ship is so built that passage between spaces below the second deck is very difficult. In order to assist the newcomer aboard ship in finding his way to the many different engineering compartments mentioned above, a mimeographed diagram has been prepared which shows the hatches and routes to follow from the second dock to the different spaces. A copy of this diagram is attached.

Each man should take this guide and visit each room, going over the various possible routes many times, until he knows exactly how to get about the ship below decks. One must realize that many of the hatches shown will be closed during wartime cruising, therefore alternate routes must be known.


SECOND DECK
TO SHAFT ALLEYS AND AFT. DIST. ROOM
TO PASSAGEWAY TO ALL MACH. SPACES
TO NO. 3 MACH. SPACE
TO NO. 1 MACH. SPACE
DOWN TO FWD. DIST. ROOM
MACHINE SHOP
TURRET No. III
CREWS GALLEY
TURRET No. II
OFFICERS GALLEY
FOUNDRY
SHOP
CARPENTER SHOP
TO CENTER PASSAGEWAY
TO NO. 2 PUMP ROOM
TO ICE MACHINES
TO STEERING GEAR
TO NO. 4 MACH. SPACE
TO NO. 2 MACH. SPACE
DOWN TO EVAPS. I.C. ROOM AND CENTRAL PASSAGE
ROUTES AND ACCESS TO MACHINERY SPACES

DESCRIPTION MAIN PROPULSION INSTALLATION

The main power plant of the ship consists of four separate engine units, each developing 32,500 horsepower and each connected to its own propeller shaft. There are therefore four propellers. The propeller shafts are numbered from starboard to port, numbers one, two, three, and four respectively. Number one engine unit in #1 machinery space drives number one shaft. But in #2 machinery space, number four engine unit drives number four shaft. Likewise, in #3 machinery space, number two engine unit drives number two shaft, and in #3 machinery space, number three engine unit drives number three shaft, This may be confusing at first, but remember that the first two machinery units drive the outboard shafts, while the after machinery units drive the inboard shafts.

As each of the four engine units are similar, only one need be described. If you understand the plant in one machinery space, you'll be familiar with the other three as all four are identical.

First we'll start with the BOILERS. There are two in each machinery space. Each one can change 16,500 gallons of water an hour into steam at 600 lbs. per sq. in. pressure and 850 degrees F. Temperature. In order to do this it burns about 1400 gallons of fuel oil per hour.

Hundreds of thousands of gallons of fuel oil are carried in tanks spread the length of the ship along the sides and in the bottom. Those adjacent to the machinery spaces on the side of the ship are called service tanks; the others are called

storage tanks. Oil is pumped aboard ship through deck fittings and is distributed to all or any desired tanks through transfer piping by use of TRANSFER and BOOSTER PUMPS. One of these latter pumps is located in each machinery space, and one is in a pump room located in the HOLD between frames 46 ½ and 55. Important fuel oil manifolds are in the forward pump room between frames 28 & 31, in the C&R pump room between francs 46 ½ & 54, in the evaporator room, in each machinery space, and in the outer shaft alleys. The OIL KING is a petty officer who insures that the fuel oil is distributed properly and is available for the boilers.

For use in the boilers, fuel oil is drawn from one of the nearby SERVICE tanks by a FUEL OIL SERVICE PUMP. This pump discharges through STRAINERS, FUEL OIL HEATERS, OIL LETERS, and ATOMIZERS into the furnaces. In order to burn properly, the oil must be at the right PRESSURE and TEMPTERATURE. It must also have the right amount of AIR. Turbine-driven BLOWERS are therefore provided which force air into the furnaces under great pressure. The blowers are so regulated that just the right amount of air is sent into the furnace to burn completely all the oil sprayed by the atomizers. Water, which the boiler turns into steam, comes from the MAIN FEED . PUMP, through the ECONOMIZER located in the boiler uptake and then discharges through CHECK VALVES to the steam drum. Once inside the steam drum, the water flows down tubes in the "saturated" side of the boiler where it is changed into steam.

This steam is known as "saturated" steam because its temperature at 600 lbs. pressure is about 489 degrees temperature. If, at that pressure, it became cooler, it would begin to turn to water. This saturated steam is collected by a pipe in the top of the steam drum and sent through tubes in the "superheated" side of the

boiler. Here the steam is heated to 850 ° and is now called superheated steam, because, *at this pressure of 600 lbs*, it has to cool to 489 degrees before it begins to change to water. In other words, it is "superheated" far above the saturation point or the temperature at which it begins to condense. The reason why steam is heated to that temperature is that a great amount of energy is added to it which is used to drive the engines; it would be heated to higher temperatures if the metal of the pipes could stand up, but, at present, the steel will soften and "creep".

At high power, over ninety per cent of the steam made in the boiler goes out as superheated steam to drive the main engines and the electric generators. It leaves through STOP VALVES and enters the MAIN STEAM PIPE. The remaining ten per cent or loss of the steam made leaves the boiler as saturated steam through STOP VALVES and enters the AUXILLIARY STEAM LINE.

In each machinery space, some distance away from the boiler stop valve, there is a connection to the 600 lb. auxiliary steam line to a REDUCIG VALVE. This valve reduces the pressure from 600 lbs. to 150 lbs. and connects with a steam line known as the 150 lb. STEAM LINE. There are, then, three high pressure steam lines: the MAIN STEAM LINE, the AUXILLIARY STEAM LINE, and the 150 lb. STEAM LINE.

The main steam line goes directly to the MAIN TURBINES, passing through the THROTTLE TRIP VALVE, the STEAM STRAINER, to the TURBINE CONTROL THROTTLES. Ahead of the throttle trip valve is a connection for superheated steam to the MAIN GENERATORS.

The MAIN TURBINES are the main engines of the ship. By opening the-ahead or astern throttles, the ship is made to go ahead or astern. When a throttle valve is opened, steam is admitted to nozzles through which the steam blows at high velocity and strikes blades attached to wheels on the rotor. This action causes the rotor to turn. In the HIGH PRESSURE TURBINE, there are twelve sets of fixed nozzles blowing steam against twelve rotating wheels attached to the rotating shaft. These combinations of nozzles and wheels are known as stages. At high power, when steam leaves the last stage of the high pressure turbine, it has about 60 lbs. pressure remaining. This can still be used to drive the ship; so is piped over to the LOW PRESSURE TURBINE where it again expands through nozzles against blades on the rotor as in the high pressure turbine. When it leaves the last wheel of the low pressure turbine, the steam has about 1 lb. *absolute* pressure or about 14 lbs. less pressure then the air about us. It has also cooled by expansion to about 100 degrees F. temperature, and it is because of this low pressure and temperature that it is of no further use to us for producing power. In the low pressure turbine casing is installed the ASTERN TURBINE consisting of one pressure stage at each end of the low pressure rotor so arranged that steam will strike the blades and turn the rotor in the reverse direction.

But in order that the turbine can function at all, there are several features about its construction which must be understood. Each man must therefore investigate these features which follow, thoroughly, The turbines are supported by BEARINGS in which the shafts or journals as they are called rotate at high speed. Hence lubricating oil must be supplied under pressure to these bearings to keep then cool. Also the rotors must not move fore or aft so that the moving wheels hit the nozzles. TURBINE THRUST BEARIGS are provided for this purpose. In order to

know whether the BEARINGS or THRUSTS are wearing, MICROMETER GAGES are installed. As steam under high pressure is in one end of the turbine while at the other end it may be less than that of the outside air, it is necessary to have some way of providing steam from escaping along the shaft where, it comes through the casing, or of preventing the air from getting into the turbine. This is done by means of GLANDS which are sealed by the GLAND SEALING SYSTEM. This system operates automatically but its workings should be understood by engineers. The ahead throttle reach rod connects to a number of valves on the top of the high pressure turbine casing. Those valves control a number of nozzles. By opening them in turn more steam is gradually cut into the engine with the result that the speed of the ship increases. It will also be noted that there are valves installed at, various stages known as EXTRACTION VALVES. These exist for the purpose of taking steam out of the turbine to boost the pressure in the EXHAUST STEAM line which will be discussed later. There are other connections installed for DRAINAGE purposes.

Turbines must operate at very high speeds to work efficiently while propellers must turn relatively slowly. The rotors of the high pressure and low pressure turbines therefore are connected to the pinions (high speed gears) of the DOUBLE REDUCTION GEARS, where the high speeds of the turbine rotors are reduced so that the propeller turns at efficient speed. The reduction gears at full power reduce the speeds of the turbines from 6000 rpm. for the high pressure rotor and 4000 rpm. for the low pressure rotor to 185 rpm. for the propeller shaft. There are also a large number of bearings in the reduction gear which require lubrication while the gears themselves need plenty of oil. It is therefore of great importance that plenty of cool, clean oil be delivered to the gears, and that the

thermometers on the bearings be watched carefully for a rise in temperature which indicates trouble.

On the reduction gears is a JACKING GEAR for turning the turbines and gears without the use of steam.

When the propeller turns over, the shaft moves forward, pushing the ship. This thrust is absorbed by the THRUST BEARINGS located forward of the gears, and it is here that the ship is actually pushed through the water. Many thousands of pounds of pressure as applied to this thrust at high speeds; so how it is withstood by this device should be studied.

The SHAFTS extend from the bull gear flange to the propellers in sections known as line shafting, stern tube shafting, and propeller shafting. SPRING BEARINGS support the weight of the shaft and require good lubrication

Where the stern tube goes through the side of the ship is a STERN TUBE bearing. Water is prevented from entering the ship by a STERN TUBE GLAND. How to keep these spring and stern tube bearings cool should be investigated.

The THROTTLEMAN by means of his GAGE BOARD controls the operation of his engine and the speed of the propeller. On the gage board are instruments giving him all the information he needs for operating according to the wishes of the officer-of-the-deck on the bridge. Each man should study these instruments and satisfy himself that he knows the purpose of each.

It was mentioned before that when the steam leaves the last stage of the low pressure turbine, its pressure and temperature have decreased to the extent that

the steam can no longer be of use to develop power. It must nevertheless be recovered for use again in the boilers, where it will be heated and will then repeat its cycle through the engines. The unit where this steam is recovered is called the CONDENSER, located directly beneath the low pressure turbine. Here steam from ½ lb. to 1 ½ lbs. per. sq. in. *absolute* pressure exhausts from the low pressure ahead turbine, or, when going astern, from the astern turbine. But before it can be pumped back to the boilers, it must be turned to water. Therefore, thousands of tubes through which sea water circulates are placed in the condenser, and the steam in striking these tubes, cools, and condenses to water. This "condensate" water is fresh, very pure, and is called FEED WATER.

It is treated with BOILER COMPOUND to give it the right chemical characteristics for use in the boiler, otherwise the boiler would quickly become inoperative. Salt water leaking from the condenser tubes soon ruins the feed, and must be detected as soon as the leak occurs.

A pound of steam at 1 lb. absolute pressure occupies over 300 cu. ft. When this pound of steam condenses to water, the water formed is just one pint, which occupies very little space. This tremendous contraction of the steam on condensing leaves a lot of space containing nothing except a little air and water vapor and is the chief reason why the pressure in the condenser is so much lower than that of the outside air. If the steam space in the condenser where, completely devoid of any steam, air, or vapor, there would be no pressure at all inside the shell and the absolute pressure would be zero, or we would have what is called a complete VACUUM. If the top of a tube one square inch in diameter filled with mercury were connected to the condenser at this time, and the other

end were open to the air, the mercury would rise in the tube the number of inches equal to the reading of the barometer, which measures the outside air pressure at that moment. But if there is in the condenser some steam vapor, air, or other gas, there will be some pressure exerted by these gases which would press down on the top of the mercury in the tube and the mercury column would not be as high as that in the barometer. The difference in the height of the columns will then be a measure of the pressure in the condenser or the degree of vacuum in the condenser.

It is desired that as high a vacuum as possible be maintained in the condenser so that the steam expands as much as possible, otherwise a "back pressure" against the last stages of the turbines will be created which will reduce the power of the engines. Check up on the means of measuring vacuum provided on this ship.

The sea water flows through the condenser tubes is scooped in when the ship is moving through the water. When the Ship is stopped or backing, a MAIN CIRCULATING PUMP is used.

The condensed steam, now feed water, must continuously be removed from the condenser, This is accomplished by the CONDENSATE PUMP which draws the water from a well in the bottom of the condenser and discharges it, through the AIR EJECTOR CONDENSERS, the VENT CONDENSER on the DEARATING TANK, into the DEAERATERING TANK. A by-pass on the line from the condensate pump permits some condensate water to pass through the GLAND LEAK-OFF CONDENSER and then on to the DEAERATING TANK. Study the different thermo-control valves and by-passes on this section of condensate piping.

It was aforementioned that air in the condenser reduces the vacuum. Hence, as it is impossible to make an engine plant completely proof against leakage of air into the units under less than atmosphere pressure, some means must be provided to remove air which leaks into the system. The AIR EJECTOR performs this duty. Steam, expanding through nozzles, sucks air from the main condenser into the EJECTOR. Here it builds up in pressure until it is above the pressure of the outside air and it then discharges into engine room.

The steam from the nozzles condenses in EJECTOR condensers and the resulting feed water is returned to the system. Check carefully how this water returns, especially that from the first stage condenser.

The DEAERATING FEED TANK acts as a device for removing air from the feed water. If any air is contained in the feed, it carries oxygen to the boilers, where, under the influence of the high temperatures, this oxygen reacts with the steel of the drums and causes heavy pitting. Air is removed by causing the incoming feed water to mix with steam from the AUXILIARY EXHAUST STEAM LINE. Both water and steam are sprayed through nozzles with the result that the steam heats the water to the boiling point and all air is boiled out of it. The vapor formed by the boiling feed rises with the released air and the original steam from the exhaust line until they reach the vent condenser and pass between tubes containing relatively cool condensate water. The water vapor and steam then condense and drip to the bottom of the tank, while the air passes out to the atmosphere in the engine room.

Another function of the deaerating feud tank is to act as a reservoir of feed water. If too much water enters the tank, an overflow valve operates to discharge the excess water to a FEED BOTTOM as the boiler water tanks are known, if the water level gets too low, water can be sucked into the main condenser from a feed bottom. Find this connection and see how it is done. Also learn where the feed bottoms are.

It is necessary that some mechanism control the amount of steam that goes into the deaerating tank otherwise the pressure inside the tanks would vary as the amount of condensate water delivered to the tank varies and the deaerating process would not function. To insure a constant pressure, counterbalancing weights and linkages actuated by springs operate to control the opening of the exhaust steam valves inside the tank so that if the pressure decreases in the tank the valve is opened; if it goes too high, the valve is closed. It is possible to set the desired tank pressure by setting the spring pressure on the controls outside the tank.

From the deaerating feed tank, the feed water is drawn by the FEED BOOSTER PUMP which discharges it to the MAIN FEED PUMP. The main feed pump increases the pressure of the feed water to about 750 lbs. per sq. in. so that it can be sent into the boiler steam drum against the pressure of the boiler steam. The water has now returned to the boiler to be again made into high pressure, high temperature steam and the main cycle begins again.

MAIN GENERATOR (STEAM END)

We said in the, beginning that some of the superheated steam goes into the MAIN GENERATORS. In following this steam from the boilers along its cycle through the generator turbines and back to the boilers, we find that the generator plants are miniature main plants. The principle difference being that the generator turbines have electric generators connected to their shafts instead of propellers.

However, after the feed leaves the DYNAMO CONDENSATE PUMP it goes Into the main deaerating feed tank instead of a separate deaerating tank, for use in port, when the main feed pumps are not in use, in AUXILIARY FEED BOOSTER PUMP is provided which discharges to an EMERGENCY and PORT USE FEED PUMP.

MAIN LUBE OIL SYSTEM

The LUBRICATING OIL SYSTEM for the main machinery is very important, Whenever any piece of machinery operates, the proper type of lube oil is required, and great care must be used to insure that plenty of oil, clean, free from dirt, water, hard particles, and acid is supplied otherwise the unit served will soon be ruined, but the greatest of care must be used in maintaining the oil for the main turbines, gear, and thrusts. This latter OIL known as symbol ?190T is stored in the system in the sumps under the reduction gears. From here it is drawn through STRAINERS by the LUBRICATING OIL PUMPS and discharged through LUBE OIL COOLERS to the various bearings, and thrusts. The pressure at each bearing is regulated by NEEDLE VALVES. Thermometers at each bearing give an early indication that the bearing is becoming hot. Some of the oil goes to small pumps in the main turbine GOVERNORS which operate the steam trip valve

should the turbine overspeed. The oil, after passing through the bearings and thrusts etc., drains back to the pumps. Should it be necessary to purify the oil, it can be pumped to SETTLING TANYB and heated. Heating causes water in the oil to separate. Also, PURIFIERS are installed which act to separate the oil and water mechanically.

Water, especially salt water, has a very damaging effect on steel and must be kept out of all lubricating oil. The main units are usually well cared for, but many of the lesser units are sometimes neglected in this respect which results in their failure. *Take care of auxiliary machinery oil*. Spare oil is stored in STORAGE TANKS in the engine room. In order to determine what oils should be used in the different machinery units, reference should be made to the chapter on lubrication in the MEI (Manual of Engineering Instructions) and the manufacturers instruction book.

During the discussion of the main propulsion steam cycle, many machinery units were mentioned such as blowers, fuel oil pumps, feed pumps, air ejectors, etc. Those units themselves all require some power in order to operate. Most of them are driven by small turbines which do not exhaust to condensers as in the case of the big main drive and generator turbines because of weight and space limitations; hence it is impossible to expand the steam in them as much as it is in the big engines. Therefore, superheated steam is not used in the auxiliaries nor is the steam expanded to a vacuum. Instead 600 lb. *saturated* steam, exhausting to the AUXLIARY EXHAUST LINE at about 15 lbs. pressure, is used to run the auxiliary turbines and the RECIPROCATING PUMPS (the EMERGENCY FEED and the BILGE PUMPS). This 600 lb. steam is used also to heat the fuel oil in the heaters, whence

it drains as water through the HIGH PRESSURE DRAIN LINES. Make a list of all the connections to the auxiliary steam line.

Steam in the 150 lb. AXILIARY STEAM LINE is used chiefly for heating purpose and for operating the WHISTLES and SIREN. There are connections to this system for ship heating, galley use, obtaining shore steam, smothering fires in the bilges, steaming out tanks, heating fuel oil tank heating coils and raising the pressure of the AUXILIARY EXJAUST LINE to 15 lbs. if an insufficient amount of exhaust steam is obtained from auxiliary machinery or extraction from the main turbines or generator turbines. Make a list of all connections you can find on the 150 lb. line.

MAIN AUXLIARY PLANTS

So far in this paper, attention has been focused in the main on the machinery which drives the ship through the water, with some brief mention of the electric generating units. Yet in addition to the main engine plants there are needed in the ship several minor machinery plants to support the operation of the main units, and also to make it possible for two thousand men to live aboard for weeks away from shore.

MAIN GENERATORS

The most important of these minor plants is the ELECTRICAL GENRATOR INSTALLATION, which consists of seven 1000 Kilowatt GENERTORS. Two main generators are located in each of the machinery spaces with the exception that in number four machinery space, there is one. An emergency diesel-generator is

located in the port side of the main evaporator room forward and another is located in the after diesel generator room under the after distribution room.

The generators produce alternating voltage of 450 volts in three phases at 60 cycles, by the action of a rotating direct current field excited by a small D.C. generator attached to the rotating shaft whose flux cuts the windings of the stationary armature. The generator turbine rotates the shaft on which the main generator field and the exciter armature are attached. This turbine steam cycle has already been discussed. For the operation of electrical machinery it is essential the voltage and the frequency be constant regardless of the load upon the machine. On these machines, the voltage is maintained at constant value by a voltage regulator which controls the amount of current flowing through the generator field, while the frequency is maintained by keeping the speed of the turbine constant. This speed control is accomplished by an oil operated governor on the turbine throttle. When a motor is started somewhere on the ship, amperes flow through its armature in order to cause the motor to do work. These amperes flow back through the main generator armature and add to the other amperes already flowing, increasing the "load". The effect of these additional amperes is to cause the main generator rotor to slow down, which acts to decrease the voltage produced and lower the frequency. However, when the voltage goes down, the voltage regulator increases the strength of the generator field until the voltage is normal again, while at the same time, the oil operated turbine governor opens the turbine throttle so that more steam is admitted and the turbine speeds up. This procedure will continue until the wires of the generator armature are carrying all the amperes they can without burning up.

If, for some reason, the generator loses the load and the governor fails to shut off the steam, an emergency trip is provided to cut the steam supply before the turbine flies apart from overspeeding. as the load increases, the current lags the voltage more and more so that less of the current is in phase with the voltage. As only current in phase with the voltage produces power, the kilowatts produced will not equal the product of the volts times amperes, but just a percentage, known as the "power factor". These machines operate at 80% power factor at full load.

NOTE:- If the reader has not had any previous knowledge of electricity he should study the meaning of the terms "VOLT", "AMPERE", "KILOWATT", "PHASE", "CYCLE", "FREQUENCY", "FLAG", "LEAD", "POWER FACTOR", "RESISTANCE;", "IMPEDANCE", "REACTANCE", "INDUCTANCE", "CAPACITY", "SYNCHRONIZE", "FIELD", and "LINES OF FORCE".

DISTRIBUTION

The power produced by the main generators is lead to distribution boards. There are four of these main boards, one in the forward distribution room, one in number two machinery space, one in number three machinery space, and one in the after distribution room. The switchboard for controlling the generators are also attached to these distribution boards. Each man, especially those desiring to become electricians, should study these boards and learn the purpose of every device on then. Each one has an important function. The power delivered to the distribution boards is sent to all parts of the ship through feeders to LOAD CENTERS by closing the proper switch on the board.

Also the boards may be interconnected so that number one generator can supply power to number four board etc. From, the load centers, power can be further distributed through mains and sub-mains. If the units supplied do not operate on 440 volts, transformers are used if AC voltage is required; or motor-generator sets if DC voltage is needed. Big MOTOR GENERATOR sets are installed to provide direct current for searchlights, degaussing, and battery charging. The emergency diesel powered generators supply voltage for a few vital circuits in the event that the main voltage fails. Should the main voltage drop to 350 volts, the diesels will start automatically by air pressure and supply the vital circuits. When the main voltage rises to 405, the emergency voltage is disconnected and the diesels must be stopped manually. These sets have their own distribution boards, which can be energized from the main boards; however, the main boards cannot be energized by the diesel generators. Learn what circuits are connected to the various boards, low the boards are connected to each other, and how the generators can be cut in on the line or taken off.

Throughout the ship are innumerable electrical devices, motors, lights, heaters, etc. They are fed however by two principle circuits, POWER or LIGHTING. The leads have special markings on them telling what kind they are, and the degree of their importance. Learn these markings.

Electric power, as aforementioned, is directed to electrically operated devices by cables.

Between the distribution boards and the LOAD CENTERS the cables are known as FEEDERS; from the load centers to the distribution PANELS they are also called

feeders; but from the panels to JUNCTION BOXES they are called MAINS and SUBMAINS. They then separate into BRANCHES and SUBBRANCHES to the lights or units served. Every unit or light is operated from a switch or control box. As stated previously, some power units operate on less then 440 volts, so the voltage is reduced by transformers to the proper voltage. Three phase power, however, is employed nevertheless. On the LIGHTING CIRCUITS, the voltage is first reduced to 115 volts and single phase power is distributed to the various circuits by using just one of the three phases on a circuit. Study the connections to a lighting transformer and see the phase connections. Also get a piece of cable and note how three phase power is carried in this cable by the three wires inside the insulation.

For purposes of communicating throughout the ship, four systems are provided. One is the SHIP'S SERVICE TELEPHONE system, a miniature shore telephone installation. Dial telephones are located in the principle parts of the ship, and are connected through an AUTOMATIC SWITCHBOARD in the telephone exchange an the port side of the first platform deck just forward of frame 73. The next general communication means is the GENERAL ANNOUNCING SYSTEM, by which, through loud speakers, information may be passed over the entire ship to specially selected stations. Then, in addition, stations which must be in communication with each other during special periods are connected by means of SOUND-POWERED telephones.

Call bells are provided for this system so that other stations may be contacted. Some of the sound powered systems are paralled by AUXILIARY SOUND POWERED systems. The sound powered circuits are given numbers and letters to

distinguish them such as 1JV or 2JY. Each system connects a special group of stations, although it is possible to connect two or more systems by "Cross-jacking". Auxiliary systems are lettered the same as the main systems, but are preceded by an X, as X1JV.

The INTERIOR COMMUNICATION room on the starboard side of the ship from the telephone exchange contains the switchboards and motor generator sets for controlling all the interior communication systems, which include besides the sound powered telephones, all the interior alarm circuits, signaling circuits, steering circuits, etc.

Some STORAGE BATTERIES are provided for use on the automatic switchboard, motor boat starting, and a few other uses. These batteries require continuous care. Visit the battery charging station and familiarize yourself with what is done there. The ship is provided with four 36-inch gunnery SEARCHLIGHTS, four 24-inch signal SEARCHLIGHTS, and four 12-inch signal SEARCHLIGHTS. The two larger type burn carbons fed by a special mechanism whose workings must be thoroughly understood in order that the lamps are burned. These big lights use D.C. current furnished from motor-generator sets.

DISTILLING PLANT

Another very important auxiliary installation is the EVAPORATING PLANT. The main units of this plant are located in the space on the second platform deck forward, between frames 67 and 73. Over eighty thousand gallons of distilled water can be made every day by these sets. Another smaller set, capable of waking twelve thousand gallons a day will be located in #4 machinery space, so

that, should the big sets be damaged, the ship could still make some boiler water and be able to steam.

Sea water, as every one knows, contains a considerable amount of salt and other solid matter which render it unfit for either drinking purposes or for boiler feed water. The EVAPORATORS remove those salts and solids from sea water for use on the ship. Fresh drinking water need by distilled only to a purity which permits several grains of salt per gallon, but boiler water for use in modern high pressure boilers must be water of exceptional purity - less than 0.1 grains of salt per gallon for, as thousands of gallons of water are boiled per hour under great pressure, the tubes in the boiler would soon be coated with heavy scale from deposits of salt while dangerous chlorine acids would at the same time be formed which eat away the steel. Hence, the principle purpose of the evaporators is to prepare pure boiler water which is called BOILER FEED WATER.

In the forward evaporator room are two identical sets of forty thousand gallons daily capacity each.

As these two sets are the same, the operation of only one will be described.

Sea water is pumped by the DISTILLER CIRCULATING PUMP through the cooling tubes of the CONDENSATE (newly made fresh water) COOLER, then through the DISTILLING CONDENSER and overboard. This sea water has picked up some heat in going through these units so about one tenth is drawn from the overboard pipe by an EVAPORATOR FEED PUMP and discharged to the FIRST EFFECT SHELL, first passing successively through the INNER HEATER, the COIL DRAIN HEATER, the DISTILLER AIR EJECTOR CONDENSER, the second effect and the first effect VAPOR

HEATERS, and a FEED REGULATOR. In passing through these various heat exchangers, the evaporator feed water becomes progressively hotter. When it enters the first effect shell it is quite warm but not yet hot enough to boil. To accomplish boiling the evaporator feed and so causing the formation of *pure fresh water vapor*, steam from the auxiliary exhaust line is passed through coils in the shell where it condenses to water, drains through a DRAIN REGULATOR and is pumped by a TUBE NEST DRAIN PUMP to a feed bottom or deaerating feed tank as this drain water is *condensed boiler steam*, already treated with boiler compound. The exhaust steam, by condensing in the first effect coils, gives up its heat to the salt water in the shell and causes part of it to boil. The salt, however, remains in the remaining water making it saltier. The first effect vapor leaves the shell, passes through the first effect vapor heater where it heats the incoming evaporator feed and goes into the SECOND EFFECT *coils*, where it boils some of the saltwater in that shell.

It condenses in the second effect coils, and drains through a DRAIN REGULATOR to the bottom, of the THIRD EFFECT COILS. *This* condensed vapor is *pure fresh water*. The salt water in the second and third effect shells is the feed remaining from previous effects, which is pumped from one effect shell to the other by the second and third effect FEED BOOSTER PUMPS. From the third affect shell this feed water, which has now become one half again as salty as it was originally, is pumped overboard by a BRINE PUMP. The vapor from the second effect, as in the case of the vapor from the first effect, passes through a vapor feed heater to the third effect coils where it condenses, draining through a drain regulator, a COIL DRAIN HEATER, a regulator, to a FLASH CHAMBER. The vapor from the third effect flows through an INNER HEATER to the DISTILLING CONDENSER, where it

condenses and flows to the FLASH CHAMBER. In this flash chamber, the condensate whose temperature is such that it would "flash" into vapor at the pressure existing in the chamber, does so partially. This flashing of part of the condensate into vapor absorbs heat from the remaining condensate, and thereby cools it. The vapor formed returns to the distilling condenser. The condensate remaining in the chamber, still quite warm, is drawn off by the CONDENSATE PUMP, passed through a CONDENSATE COOLER to a MEASURING TANK, where the quantity made is recorded and tested for purity. It is then pumped by a FRESH WATER PUMP through a meter to a fresh water tank. As water boils at a lower temperature when the pressure on its surface is lower, a partial vacuum is maintained in the shells by a DISTILLER AIR EJECTOR which draws air from the distiller condenser.

This action coupled pith the condensing of the third effect vapor in the distilling condenser creates about 25 inches of vacuum in the distilling condenses with consequent partial vacuum in the succeeding shells, for they are connected by the vapor pipes. As result, it is possible to boil the evaporator feed water with steam whose temperature is much less than 212 degrees Fahrenheit. With regard to evaporator operation, it must be remembered that the incoming exhaust steam temperature must be between 200 and 230 degrees Fahrenheit, and that the pressure of the steam is not the governing factor. Hence steam of zero lbs. gage pressure or even less may be used. Also, it is the *condensation* of the steam in the *coils* which causes the feed to boil. Hence, do not permit the coils either to fill with water or permit the exhaust steam or the new vapor to blow clear through them. The feed in the shells must be neither at too high or too low a level, nor should the feed be allowed to become too salty. It is of paramount importance,

too, that the fresh water made is of great purity. Learn how it is tested, and also how the flow of steam and feed in the evaps are controlled.

REFRIGERATING PLANT

Aft, on the first platform dock, between frames 129 and 142 ½, are located the ship's refrigerated spaces where fresh meats, eggs, butter, and vegetables are stored. If the spaces are properly cooled, it is possible to carry fresh provisions for several weeks cruising.

The machines which cool these refrigerated rooms belong to the Engineering Department and are located at the after section of the ice box area. Three units are provided, any two of which will maintain the proper temperatures.

The units consist of an electrically driven compressors which compresses a gas known as FREON or F-12. This substance is a gas at room, temperatures, but by compressing it, and then removing the heat caused by it in a cooling unit known as the condenser, it turns to a liquid and will remain as a liquid so long as the pressure is maintained upon it. In the cooling system, however, it is passed though an expansion valve which allows just enough of the liquid Freon to enter the cooling coils as will permit a reduction of the pressure to the point at which Freon will boil. In boiling, heat is absorbed from the surrounding medium-in this case the air about the cooling coils in the ice boxes-and cooling is effected. Thus, liquid Freon enters the cooling coils, boils, absorbing heat, and leaves the coils as a gas on its way back to the compressor again to repeat its cycle. The expansion valves are controlled automatically by thermo valves, but nevertheless a careful check must always be maintained on the temperatures in the boxes or the food

will spoil. It is important that the Freon be kept free of water which would freeze in the expansion valves and prevent operation of the system. A drier or dehydrator is installed in the Freon lines to remove any water in the Freon.

Learn the usual box temperatures, Meat Box 15° F., Butter and Eggs 35° F., Fruit 40° F., how to detect Freon leaks, and how to replace Freon which leaks out.

STEERING GEAR

The ship is provided with two rudders, both of which are operated simultaneously by steering controls at the navigating bridge, at the secondary control station, the central station, or by the use of "Trick" wheels in each steering engine room.

At each helm, except in the steering engine rooms, are small SELSYN TRANSMITTER AC MOTORS whose stators, or stationary windings, are energized by 440 single phase voltage. The rotors of these motors are wound for three phase voltage and are connected to the rotors of similar SELSYN RECEIVER rotors in the steering engine room. In both transmitter and receiver rotors a voltage is induced by the alternating stator field, but if both rotors occupy the same relative position to their stators these voltages balance each other and no current would flow in the circuit connecting both rotors. Now when the steersman turns his wheel, he turns a shaft which is connected to the rotor of the motor at his helm and thereby causes the rotor to turn and change its position relative to its stator. By this movement an unbalanced voltage is induced which causes current to flow through the circuit to the rotor at the steering engine room. When this current flows through this latter rotor, it sets up a field which causes that rotor to turn

until it occupies the same position relative to its stator as the rotor at the helm does to its stator.

Thus the rotor in the steering gear room follows the one at the helm.

The rotor of the receiver motor in the steering gear room is connected by gears-end shafting; to the PILOT VALVE of a SERVO MECHANISM. Movement of this pilot valve permits oil under pressure from the SERVO PUMP to operate on either side of a piston, depending on which way the rotor moves the valve in response to movement of the helm. Movement of the piston, actuated by the aforementioned oil, moves a TILTING BOX on a WATERBURY SPEED GEAR. This Waterbury Gear is just a large oil pump operated by big 75 HP electric motor. Its operation is briefly as follows: this motor through a reduction gear, turns a shaft on which is a keyed cylinder. This cylinder has in it several little holes, into which project the pistons on the tilting block. These holes always contain oil. Then the tilting box is at right angles to the shaft, the pistons do not move relative to the cylinder and there is no pumping action in the holes. But if the tilting box is rotated so as not to be at right angles with the shaft, the pistons on one side of the box project further into the holes than the pistons on the opposite side as the cylinder rotates, and a pumping action results which puts pressure on the oil in the holes and piping to the rams. By tilting the box one way or the other, the pressure on the oil can be revered in direction.

The rudders are actually moved by CROSSHEAD operated by two oil filled RAMS.

If pressure is put on one ram and reduced simultaneously on the on the other ram, the crosshead will be turned, turning the rudder. This shifting of pressure in

the rams is accomplished by the tilting box as described above. On the crosshead gear shafts as a FOLLOW UP MECHANISM to return the tilting box to a vertical position as the rudder turns otherwise the rudder would keep on going to the "hard-over" position every time the tilting box is moved from the vertical.

In each steering engine room are two complete sets of steering motors and oil mechanisms for operating the rams. Shifts from one motors to the other can be quickly made, as can the shift of oil flow from one tilting box to the other. Furthermore, a separate motor is provided so that the rudder can be returned to the mid-ship position in case the main motors fail. Should the SELSYN SYSTEM fail, the "trick" wheel can be used to operate the pilot valve on the SERVO MECHANISM.

Each man should read the instructions regarding the shifting of controls from the different stations and know how to operate the switches necessary to line up the gear.

COMPRESSED AIR SYSTEMS

Three compressed air systems are provided for the ship, the HIGH PRESSURE, MEDIUM PRESSURE, and LOW PRESSURE. The high pressure system is supplied by tow motor driven H.P. COMPRESSORS, one in the forward emergency diesel generator room and on in the #4 Machinery Space.

These compressors will deliver thirty cubic feet of compressed air per hour at 3000 lbs. per sq. in. pressure, which air is used for charging air storage banks for

use by the turret guns and for starting certain diesel engines. The medium pressure system is supplied by four motor-driven M.P. COMPRESSORS, two being located in each emergency generator room. These compressors deliver 250 cubic feet of free air at 200 lbs. per sq. in. *per minute* used primarily for the rammers of the five-inch guns. The low pressure system is supplied by two rotor driven L.P. COMPRESSORS located one in the forward machinery space (#1) and one in the after machinery space (#4). These compressors deliver 100 cubic feet of free air per minute at 100 lbs. per sq. in. This air is used for miscellaneous ship work as cleaning motors, testing compartments for tightness, operating the forge, operating the pneumatic dispatch system, etc.

When compressing air, the principle features occurring are the high discharge temperatures and the precipitated water. Therefore, means must be provided for cooling the air as its pressure increases and for removing the water which is formed. This water is simply the water vapor which all free air condenses. Therefore, on compressed systems, look for these cooling provisions and the means for the manner in which water is removed.

The air after compression is usually stored in ACCUMULATORS

MISCELLLNEOUS SYSTEMS

In addition to the systems already described in brief, there are in the ship a number of installations necessary for its operation, but are of relatively miner engineering importance. They are listed below with a short description, mainly to bring their existence to your attention with a view that you will investigate them

carefully and learn of what each system consists and what peculiarities of operation may obtain.

THE HIGH PRESSURE STEAM DRAIN SYSTEM

This system collects the drains from high pressure valves on the main and auxiliary steam lines and a few other H.P. fittings and discharges through impulse traps to the deaerating feed tanks.

The FUEL OIL HEATER and FUEL OIL TANK HEATING COIL drain system collects the drains from these units and discharges them through inspection tanks to the deaerating feed tanks. Water seals are maintained in these tanks by means of needle valves.

THE FRESH WATER DRAIN COLLECTING SYSTEM.

This system collects fresh water which drains from various steam machinery units into open funnels and conducts this water to a DRAIN COLLECTING TANK in the bilges. From the drain collecting tanks, the water is drawn through a vacuum trap into either the dynamo or main condensers.

THE LOW PRESSURE AND WHISTLE AND SIREN DRAINS

The steam heating system drains are collected primarily by this system.

The drains from the whistle and siren connect to this system in the forward machinery space. Because of the extent of heating system drain piping throughout the vessel, the drain collecting tank for this system in the after machinery space (#4) is maintained under 15 inches of vacuum by a two stage AIR

EJECTOR in order to maintain the flow. Water collected by this system is pumped to the deaerating feed tanks by an L.P. STEAM DRAIN PUMP.

THE BILGE SUMP TANK DRAINS

This system collects contaminated water, waste oil, etc., from open funnels and discharges it to a BILGE SUMP TANK. From here this waste matter is pumped over the side by a BILGE PUMP.

THE FIRE AND FLUSHING SYSTEM

Five FIRE AND FLUSHING PULPS, four motor drive and one turbine drive, one located in each of the four machinery spaces and one in the forward emergency diesel generator room connect to the FIRE and to the FLUSHING MAIN. These pumps have a capacity of 1200 gallons per minute at 65 lbs. pressure and 750 gallons per minute at 150 lbs. pressure. They also discharge to JET PUMPS which operate at 150 lbs. pressure and increase the drainage capacity to 1200 gallons per minute per pump. Water for the machinery cooling service system can be obtained from this system pump. The FIRE AND pumps take suction only from the sea.

THE BILGE DRAINAGE SYSTEM

Five reciprocating BILGE PUMPS are provided, one in each machinery apace and one in the forward emergency diesel generator room.

These pumps have a capacity of 225 gallons per minute at 50 lbs. pressure. Suction can be taken from a bilge drain tank in the machinery spaces, from a

BILGE WELL in the forward emergency diesel generator room, and from contaminated fuel oil tanks. The MAIN CIRCULATING PUMPS can also be used for bilge drainage purposes. Discharge from the bilge pumps can be to the sea, to hose connections, and to contaminated fuel oil tanks.

THE MACHINERY COOLING WATER SERVICE SYSTEM

Six motor-driven AUXILIARY MACHINERY COOLING WATER SERVICE pumps, one in each of the four machinery spaces, and one in each of the emergency diesel generator rooms, connect to a COOLING WATER main which extends through the machinery spaces from the forward emergency generator room to the after emergency generator room. This system provides cooling water for auxiliary units as lube oil coolers, bearings, etc.

FUEL OIL TANK DRAINAGE SYSTEM

Three FUEL OIL TANK DRAIN PULTS of 50 gallons per minute capacity at 50 lbs. pressure are located, one in the pump room at frame 30, one in the C ∧ R pump room at frame 46, and one in the after Emergency diesel generator room, and one connected to a piping system which permits suction from all fuel tanks and discharge to the contaminated oil tanks.

DIESEL OIL TRANSFER AND SERVICE SYSTEM

Oil is stored in tanks directly beneath the emergency diesel generators.

In each diesel generator space is a motor driven DIESEL FUEL OIL SERVICE pump of 25 gallons per minute capacity at 50 lbs. pressure, a 150 gallon per hour PURIFIER,

a CLEAN OIL SERVICE TANK of 8 hours capacity and a GENERAL SHIPS USE TANK of 600 gallons capacity. The pumps take suction from the storage and service tanks either forward or aft and discharge to the clean oil tanks via purifier, the generator diesels, or to the SHIPS SERVICE line to boat filling connections, forges, etc.

THE STEAM HEATING SYSTEM

This system obtains its steam from the 150 lb. steam line in the forward (#1) and after (#4) machinery spaces through reducing valves which lower the pressure from 150 lbs. to 50 lbs. Two lines run from the machinery spaces, the CONSTANT pressure line and the INTERMITTENT steam line, and go practically to every part of the ship. The CONSTANT line supplies the galley, the SHIP SERVICE ACTIVITIES, principally the LAUNDRY, and also other stations where a continual flow of low pressure steam is needed. The drains from this system have been discussed.

AIR CONDITIONING PLANTS

There are thirteen AIR CONDITIONING UNITS scattered about the ship far the purpose of cooling essential battle stations and SICK BAYs. These units operate essentially like the refrigerating units, for cooling the air. The humidity is controlled by removing moisture from the air by means of lime trays, or adding moisture by use of steam.

SHOPS AND STOREROOMS

For upkeep purposes, a MACHINE SHOP, an ELECTRICAL WORKSHOP, a METALSMITH SHOP, and OIL TESTING SHOP, and a BATTERY MAINTENANCE

station have been provided. Visit those shops and learn the purpose of all the tools installed. A STOREROOM under the charge of the engineer force is also provided in which are kept tools and material necessary for daily use and minor repair.

ORGANIZATION OF THE ENGINEER DEPARTMENT

The Engineering Department of this ship is organized in three Divisions, the AUXILIARY (A), the PROPULSION (P), and the ELECTRICAL (E). Each Division is in the charge of a DIVISION OFFICER who has to assist him *junior division officers* and *repair officers*. The CHIEF ENGINEER is in charge of the Department and has as his assistant the SENIOR ASSISTANT ENGINEER.

The Engineer Office called the LOG ROOM is the headquarters of the Department. This office is in the charge of the LOG ROOM YEOMAN who has a number of assistants.

When you report aboard, and after you have been given a berth and locker, you will be assigned to a division. The division officer will assign you to a station which will be noted on the WATCH, QUARTER, and STATION BILL posted in a conspicuous place. Study his bill carefully and learn your job as required. Ask your division petty officers and your division officers if you are in doubt about anything.

NEVER FAIL TO CONSULT YOUR DIVISION OFFICER; IF YOU THINK IT NECESSARY, but TRY to help yourself.

SAFETY

In a subject so wide as safety, the listing of every situation that is likely to reproduce an accident is, of course, impossible, but an attempt has been made in the following outline to group the many kinds of accidents that occur aboard ship according to the nature of their causes. This outline can do little more than call the attention of everyone to these causes; it is then up to the individual to be continuously alert to see to it that neither through negligence nor ignorance is he the cause of damage to himself or others, or to material upon which the fighting ability of this vessel depends.

The attention of all hands is called to the following causes of accidents:

(a) *Collision.* This applies not only to collisions between ships and small boats, but collisions between persons, and persons and parts of the ship's structure. Watch your step. Constant care is necessary in going up and down hatches to avoid slipping or striking one's head on sharp projections.

(b) *Falling Weights.* Don't leave weights like buckets and tools lying around on overhead beams and elsewhere, where they may fall on persons or fragile material. To remain under suspended weights such as boats being hoisted aboard and materials being handled by booms and cranes is foolhardy and a direct tempting of Fate.

(c) *Falls*. Falls are a common on source of injury to people aboard ship.

Gear improperly secured or left adrift is a menace; and the next person who cores along trips over it, and receives severe cuts or bruises, or broken bones. Use

caution on slippery surfaces, particularly when carrying hot liquids or heavy weights.

(d) *High Temperatures*. Burns from hot gases and liquids are not only extremely painful but they frequently result in the laying up of a man for weeks. Labor is thereby made unavailable; the ship loses a man's services, and his shipmates must absorb the extra work load.

(e) *High Voltages.* Practically all the power and light circuits aboard the MASSACHUSETTS carry high alternating voltage. This voltage is vicious and many serious fatalities have resulted from carelessness in candling it. Should a person inadvertently strike one of the 440 circuits while it is energized, he would in all probability be fatally electrocuted. Men must never go aloft either on the mast or smokepipes without permission, as the high-frequency radio antennae constitute a continual hazard to personnel.

(f) *Pressure*. The danger of steam pressure is too well known for discussion here. However, water pressure too is dangerous; an unattended hose nozzle with water pressure on it can cause serious damage by swiping either persons or material. High air pressure used in charging gunnery air flasks and diesel starting units is very dangerous, and only experienced personnel will be permitted to operate valves on this system. A loose H.P. air lead is particularly deadly.

(g) *Moving Objects*. Care must always be exercised around machinery, rotating shafts, propellers etc., so that one's body is not brought into contact with them. Be careful that your clothes are not caught and serious injury sustained thereby,

(h) *Explosives*. The care and handling of explosives (both powder and *fuel*) are exhaustively covered by safety precautions. follow these precautions to the letter. Accidents from this source are almost invariably the result of carelessness.

(i) *Inflammables*. Wood, paper, or cloth fires are generally the result of a cigarette butt carelessly disposed of. Don't throw cigarette or cigar butts over the side; they may land in boats along-side. Ash receptacles are provided for smokers; use them. And don't smoke in bunks.

(j) *Weapons*. Handling of small arms, in particular the .45 automatic pistol, require eternal vigilance. Accidents resulting from small arms can only be the result of carelessness, because all personnel aboard the MASSACHUSETTS, who will be required to use them, will first be qualified in their use by the Gunnery Officer.

(k) *Asphyxiation*. Suffocation by gases has been one of the common causes of fatalities in our Navy. Men painting in enclosed compartments like fresh water tanks and cofferdams have been overcome by gases and in many instances killed. No work in such spaces will be undertaken unless under the direct supervision of an officer. Gases released by the use of CO2 fire extinguishers against fire in confined compartments or PYRENE extinguishers or an electric arc are other causes of asphyxiation, and must be always considered when CO2 or PYRENE fire extinguishers are used under these circumstances.

When Foamite is used on an electric arc or cable carrying voltage, there is danger of electrocution.

(l) *Drowning*. Each year many men are drowned in the Navy. It gales without saying that the best way to avoid drowning is to be a good swimmer. However, many drownings of both swimmers and non-swimmers can be prevented by prompt action on the part of the men on the spot; that is, the throwing of a life ring to the man overboard, by getting a boat out to the drowning man, and then after his rescue the application of resuscitation methods with which all men in the Navy should be acquainted.

Appendix 3 – CINCPACFLT Operations Order for Midway Operation[26]

Cincpac file
A16-3/(16)

UNITED STATES PACIFIC FLEET,
PEARL HARBOR, T. H.,
1800, 27 May 1942.

Serial 0114 W.

S E C R E T

REG. NO. 38

Operation Plan
No. 29-42

TASK ORGANIZATION

(a) Striking Forces - Senior striking force commander in MIDWAY Area.

Task Force SIXTEEN - Rear Admiral Spruance

ENTERPRISE, HORNET	2 CV
NORTHAMPTON, VINCENNES, PENSACOLA, MINNEAPOLIS, NEW ORLEANS	5 CA
ATLANTA	1 CL
Desron One, less HULL, McDONOUGH, FARRAGUT, DALE plus CONYNGHAM	1 DL, 5 DD
Desron Six, less CRAVEN, GRIDLEY, McCALL DUNLAP, FUNNING plus GWIN, MONSSEN	1 DL, 5 DD

Task Force SEVENTEEN - Rear Admiral Fletcher

YORKTOWN	1 CV
ASTORIA, PORTLAND	2 CA
Desron Two, less O'BRIEN, WALKE	6 DD

Task Force ELEVEN - Rear Admiral Fitch

SARATOGA	1 CV
CHESTER	1 CA
SAN DIEGO	1 CL
DALE, FANNING, AARON WARD, DUNLAP, CRAVEN, LAFFEY	6 DD

Oilers

PLATTE, CIMARRON	2 AO

(b) Submarine Force, Task Force SEVEN - Captain English

Midway Patrol

DOLPHIN, GATO, CATTLEFISH, GRENADIER, TAMBOR, TROUT, GRAYLING, NAUTILUS, GROUPER, GUDGEON, CACHALOT, FLYING FISH	12 SS

-1-

[26] Accessed online 21 Feb 13 at: http://midway1942.org/docs/usn_doc_00.shtml.

(b) (Continued)

Support Patrol

Submarines to be designated   about 4 SS

(c) Patrol Wings, Task Force NINE - Rear Admiral Bellinger

Patrol planes and tenders in readiness to be despatched to MIDWAY defense.

(d) Hawaiian Sea Frontier, Task Force FOUR - Rear Admiral Bagley

MIDWAY Local Defenses, Captain Simard

Patrol craft   4 YP, 11 PT
Sixth Marine Defense Battalion reenforced.
Marine Air Group plus aircraft reenforcements from Patrol Wings and Hawaiian Air Force.
Naval Air Station.

JOHNSTON ISLAND

Patrol planes made available for support for MIDWAY.

ISLANDS PATROLS

YPs with aviation gasoline, food and water stationed along island chain at following positions: YP 284 at LISIANSKI; YP 290 at LAYSAN; YP 345 at GARDNER'S PINNACLES; YP 350 at NECKER ISLAND.

(e) Hawaiian Department - Lieutenant General Emmons

Aircraft in readiness to be despatched to MIDWAY defense.

1. Information

(a) The enemy is expected to attempt the capture of MIDWAY in the near future. For this purpose it is

believed that the enemy will employ approximately the following: 2-4 fast BB; 4-5 CV; 8-9 CA; 16-24 DD; 8-12 SS; a landing force with seaplane tenders. The attack on MIDWAY may be preceded or followed by an attack on OAHU. A special intelligence annex is being supplied to Task Force Commanders only. The Commander-in-Chief, U.S. Pacific Fleet will keep Task Force Commanders informed of all pertinent information before and during the operations, including complete weather broadcast.

(b) It is estimated that enemy action against MIDWAY with comprise a full scale attack for its capture and its quick occupation and use against the Hawaiian Area. Operations beginning probably as soon as thirty May, are visualized as follows:

(1) Preliminary reconnaissance by submarines.

(2) Possibly diversionary bombing of positions including OAHU by patrol planes fueled by submarines.

(3) High speed approach by carriers.

(4) Preliminary attacks by carrier aircraft beginning at daylight or during moonlight and continuing for about two days or until defending air forces have been eliminated. It is thought that one or more carriers may take up close in daylight positions for this purpose. It is estimated a northwesterly bearing will be favored. The attack would be designed to be so incessant as to prevent refueling and rearming MIDWAY aircraft. This attempt may be continued by means of bombardment at night.

(5) Covering of attacking carriers against our surface forces by additional carrier groups, and fast battleships.

SECRET

Operation Plan
No. 29-42

(6) Landing attack, probably at night, featured by usual attempts at infiltration and extreme resolution on the part of the individual enemy soldier.

(7) Employment of incendiary bombs and possibly gas.

(8) If landing attack is successful, immediate occupation of the island with full base equipment, aircraft, KTBs, etc.

(9) Covering with concentrations of submarines designed to intercept our supporting surface forces both in the MIDWAY area and on some such line as about two hundred miles west of OAHU.

(10) It is probable that if our carriers are sighted early in the operations, they will become the primary object of the enemy carriers.

(c) Task Force ELEVEN will be ready to depart from West Coast ports on five June, and will be directed to arrive in the Hawaiian Area as soon as possible. The availability of Task Force SEVENTEEN as a unit is dependent of the condition of the YORKTOWN. If YORKTOWN is not available, instructions will be issued as to employment of remainder of force. GWIN and MUSTIN are now engaged in escorting between PEARL and MIDWAY and will join their Task Forces when released.

(d) Task Force EIGHT composed of 2 CA, 3 CL, 4 DD from the Fleet plus certain forces normally under the Commander Northwestern Sea Frontier and certain Army ALASKAN Air Forces are operating in the North Pacific Area against an expected attack in the ALEUTIANS.

(d) The defenses at MIDWAY have recently been reenforced to the totals indicated:

	Aircraft
Reenforcement	Total
16 VP; 7 VF; 18 VSB	16 VP; 27 VF; 36 VSB

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A16-3/(16)
Serial 0114 W.

Armament

Reenforcement	Total
4 - 60mm Mortar;	8 - 60mm Mortar
8 - 37mm AA;	8 - 37mm AA
12 - 3" AA;	24 - 3" AA
	4 - 3", 4 - 7", 6 - 5".

Troops

Reenforcement of 670 men bringing the total, including air personnel, to:

	Officers	Enlisted
6th Defense Battalion	52	1357
3d Defense Battalion	13	379
Raider Companies	9	269
Marine Air Squadron	45	470
Naval Air Squadron	22	372
Net Depot	12	
Cable Station	25	
Pan American Air	_	2
Total	141	2886

(f) Additional communication and air operations personnel are being made available to the Commanding Officer, MIDWAY.

(g) Some of the initial dispositions such as that of the submarines have been directed by separate directives but are being repeated herein in order to make this plan of operations complete

(h) This Plan deals primarily with the expected attack on MIDWAY - but the deployment herein made is also believed to promote the security of OAHU.

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No. 29-42

2. These forces will hold MIDWAY and will inflict maximum damage on the enemy by strong attrition tactics.

3. (a) Striking Forces.

(1) Inflict maximum damage on enemy by employing strong attrition tactics. Do not accept such decisive action as would be likely to incur heavy losses in our carriers and cruisers. A letter of instructions is being furnished separately to striking force commanders.

(2) Operate with Task Forces available initially to the northeast of MIDWAY commencing thirty May, in order to seize opportunity to obtain initial advantage against carriers which are employing their air groups against MIDWAY.

(3) Initially establish air search in the northwest sector from MIDWAY to eastward of bearing twenty degrees true from that place.

(4) Task Force SIXTEEN depart PEARL on twenty-eight May; other forces join Task Force SIXTEEN as directed by Commander-in-Chief, U.S. Pacific Fleet.

(5) Oilers depart in company with Task Force SIXTEEN and operate as directed by Senior Striking Force Commander in area of operations.

(b) Submarine Force.

(1) Inflict maximum damage to enemy. Priority of targets - carriers, battleships, transports, cruisers, auxiliaries.

MIDWAY Patrol

(2) As soon as available, submarines take stations as shown on diagram in Annex "A".

(3) When information is received proceed to attack objectives without regard for area assignment.

(4) Transmit such information as does not interfere with primary task of attack.

Support Patrol

(5) Take stations on patrol line 045°-225° with center at Latitude 30-00 N, Long. 169-30 W, distance thirty miles.

(6) Support striking forces which may be forced to retire over patrol line.

(c) Patrol Wings.

(1) Despatch patrol planes to MIDWAY and JOHNSTON as may be directed; to operate under Commanding Officers of Air Stations of those places.

(2) Station patrol tender at FRENCH FRIGATE SHOAL.

(d) Hawaiian Sea Frontier.

MIDWAY Local Defenses.

(1) Hold MIDWAY.

(2) Aircraft obtain and report early information of enemy advance by searches to maximum practicable radius from MIDWAY covering daily the greatest arc possible whit a number of planes available between true bearing from MIDWAY clockwise two hundred degrees dash twenty degrees. Inflict maximum damage on enemy, particularly carriers, battleships and transports.

(3) Take every precaution against being destroyed on the ground or water. Long range aircraft retire to OAHU when necessary to avoid such destruction. Patrol planes fuel from AVD at FRENCH FRIGATE SHOAL if necessary.

(4) Patrol craft patrol approaches; exploit favorable opportunities to attack carriers, battleships, transports and auxiliaries. Observe KURE and PEARL and HERMES REEF. Give prompt warning of approaching enemy forces.

(5) Keep Commander-in-Chief, U.S. Pacific Fleet and Commander Hawaiian Sea Frontier fully informed of of air searches and other air operations; also of weather encountered by search planes.

Johnston Island

Maintain daily searches with patrol planes in sector with median 295°.

Island Patrols

Supply gasoline and other assistance to aircraft as may be required.

(c) Hawaiian Department.

(1) Provide, as directed, a striking force of long range bombers and torpedo carrying aircraft to operate under Commanding Officer, MIDWAY.

(2) Hold special long range bombers in readiness at OAHU-KAUAI Area to strike enemy forces attacking MIDWAY.

(x)(1) Enemy submarines are not important objectives and their identification must be positive before they are attacked. Do not attack submarines in the areas shown in Annex "A".

(2) Recognition of own forces is vital. As one precaution against attack on our striking forces by own air forces, Commander-in-Chief, U.S. Pacific Fleet will inform Task Force Commanders of all air attacks ordered for shore-based aircraft.

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(3) Insure that contact reports are accurate, complete and prompt.

(4) This Operation Plan effective 1830 GCT twenty-eight May.

4. Gasoline for aircraft and limited fuel for CAs and smaller vessels available at MIDWAY. One AVD at FRENCH FRIGATE SHOAL. Fuel oil at PEARL and in attached oilers.

3. (a) Communications according to PAC SEVENTY except as follows:

Radio Frequency Plans

Task Forces ELEVEN, SIXTEEN, and SEVENTEEN use radio frequency plan four.
Oilers guard 4205 Kcs. series.
Task Forces SEVEN and Patrol Planes based MIDWAY and JOHNSTON guard 4265 kcs. from 0730 to 1830 GCT; 12795 kcs. 1830 to 0730 GCT, primary; 4385 kcs 0730 to 1830 GCT; and 13155 kcs. 1830 to 0730 GCT, secondary.
Army aircraft assigned defense MIDWAY use 4265 primary, 4385 secondary without shift to higher harmonics. Naval Air Station MIDWAY must guard this.
MIDWAY local defense as assigned by Commander.
All Units: In view of known Japanese practice of jamming radio frequencies calibrate transmitters and receivers for secondary frequencies. The use of NPM primary Fox is available for relay through NPM or Commander-in-Chief, U.S. Pacific Fleet, who guards 4205 kcs. series primary and 4295 ksc. series secondary.

Authentication

The Japanese are adopt at the practice of deception. Have authenticators ready for use when needed. Small craft and aircraft except patrol planes use two alternate letters from the expression:

"Farmer in the dell"

EXAMPLE: RE or EL or NH

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Use of CABLE to MIDWAY

Commander-in-Chief, U.S. Pacific Fleet, Radio NPM, MIDWAY are connected by cable. Use of plain language over the cable is authorized when speed is essential. Commandant, Fourteenth Naval District and Naval Air Station MIDWAY place U.S. Naval personnel at terminals to supervise operation, and at MIDWAY to communicate with Naval Air Station operational center.

Cryptographic Aids.

Naval Air Station MIDWAY, JOHNSTON, and PALMYRA hold following crypto-channels: 104, 105, 106, 107, 135, 135, 143, 144, 145, 171, 176, 180, and 184.

(b) Use chart 4000-Y extended westward to Longitude column 01.

(c) Commander-in-Chief, U.S. Pacific Fleet at Submarine Base, PEARL HARBOR, will coordinate operations of forces assigned in this Plan for the defense of MIDWAY.

C. W. NIMITZ,
Admiral,
Commander-in-Chief

Annexes:

A - Diagram of MIDWAY Submarine Positions.
B - Special Intelligence Annex.

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Cincpac file
A16-3/(16)
Serial 0114 W.

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Operation Plan
No. 29-42

DISTRIBUTION

Method	Address	No.	Reg. Nos.
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(a)	Comtaskfor 11 for distribution	11	40-50 incl.
(b)	PLATTE	1	51
(b)	CIMARRON	1	52
(b)	Comtaskfor 7	2	53,54
(b)	Comtaskfor 9 (plus 30 copies Annex "A")	2	55,56
(b)	Comtaskfor 4, (2 for delivery to MIDWAY Local Defenses)	4	57-60 incl.
(b)	ComGen. Haw.Dept (plus 30 copies Annex "A")	2	61,62
	File	3	63,64,65
	Reserve	20	66-85 incl.
(a)	CO JOHNSTON	1	86

(a) Clipper lock box air mail.
(b) Officer messenger.

NOTE: Annex "B" furnished Task Force Commanders only, i.e., with copies nos.7, 29, 40, 53, 55, 57 & 61.

P. C. CROSLEY,
Flag Secretary.

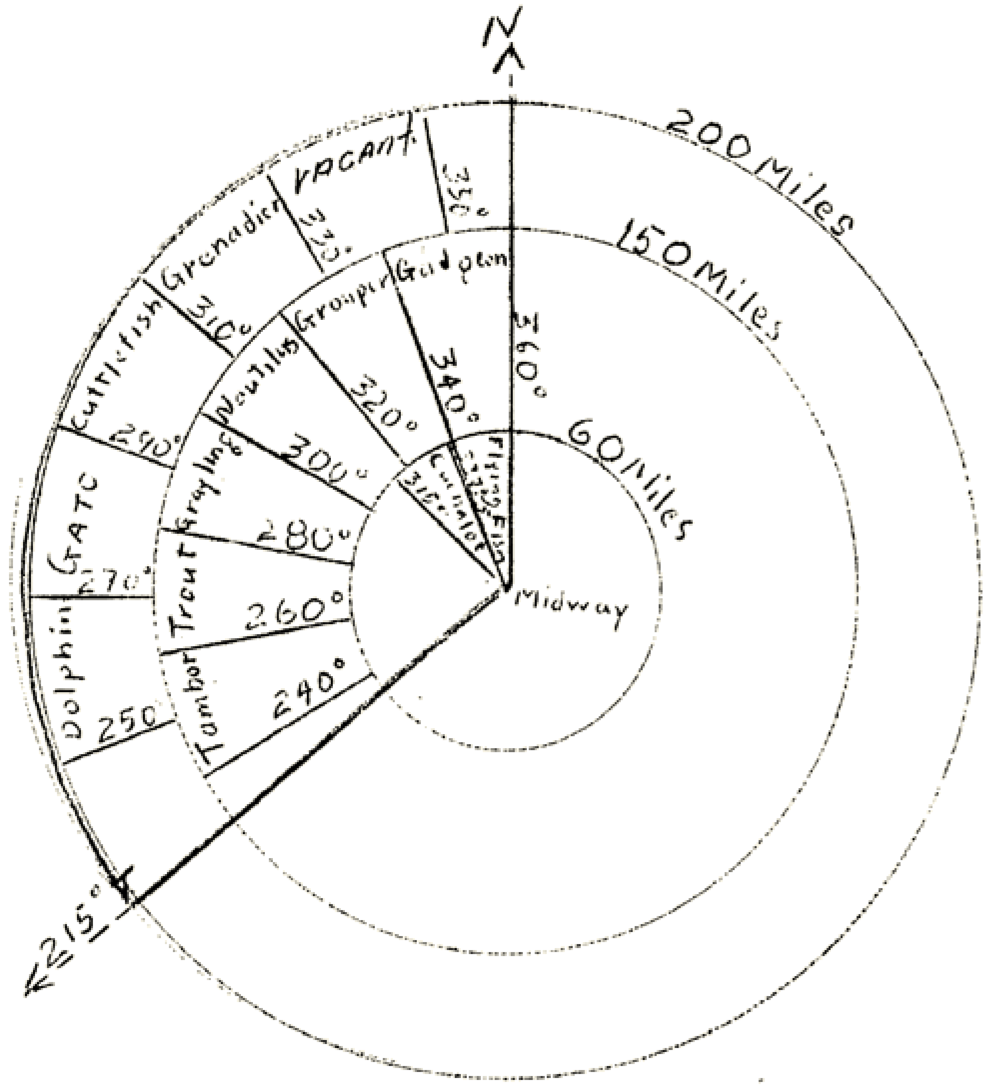
THIS PLAN MUST NOT FALL INTO THE HANDS

OF THE ENEMY

S E C R E T

ANNEX "A" TO SINCPAC OPERATION PLAN NO. 29-42.

Initial Submarine Patrol Areas.



Note #1. Submarines underlined in red are on station from daylight May 26, 1942. Other stations will be taken as soon as the submarines become available.

Note #2. In the area outlined in blue aircraft approach and bombing of submarines is forbidden. (See Operation Plan No. 29-42, para. 3(b)).

APPENDIX "1", TO SINCPAC OPERATION PLAN No. 29-42

SECRET

PLAN FOR OILERS

1. CIMARRON and PLATTE are accompanying Task Force Sixteen. On completion of fueling northeast of MIDWAY about 31 May they will be released. They will then retire with their destroyer escorts to the southeast and carry out the following plan:

(a) Pass through Point A in company at 0300 GCT two June.

(b) (1) CIMARRON proceed through points B, C, D, B in that order, arriving at B at 2000 GCT two June, four June and every second day thereafter.

(2) PLATTE proceed along the line AD and join Task Force SEVENTEEN as may be directed by Commander Task Force SEVENTEEN. If no randezvous is made, proceed ariund the same circuit DBC arriving at B at 2000 GCT on add days of the month. If PLATTE is utilized by Task Force SEVENTEEN, proceed thereafter to the BCD area and adjust position and speed to proceed around the circuit, arriving at B at 2000 GCT on odd days of the month.

2. Points: A - 31° N, 167° W.

B - 28° N, 167° W.

C - 26° N, 163° W.

D - 29° N, 163° W.

M. F DRAEMEL,
Chief of Staff.

DISTRIBUTION:
Comtaskfor 17
Comtaskfor 11
Comtaskfor 7 (12)
Comtaskfor 9 (5)

P. C. CROSLEY,
Flag Secretary.

Cincpac File No.

UNITED STATES PACIFIC FLEET
U. S. S. PENNSYLVANIA, Flagship
FLAGSHIP OF THE COMMANDER-IN-CHIEF

SECRET

APPENDIX "2" TO SINCPAC OPERATION PLAN 29-42

INITIAL AREAS FOR STRIKING FORCES

1. Task Force SIXTEEN initially operate north of latitude 32° and west of longitude 173° W.

2. Task Force SEVENTEEN initially operate north of latitude 32° and east of longitude 173° W.

3. Both task forces during each local forenoon approach Point "Luck" (Lat. 32 N, Long. 173 W) and exchange communications by plane if desired.

4. The above is not intended to restrict the operation of either force in any manner but to avoid having embarrassing or premature contact made with own forces.

M. F. DRAEMEL,
Chief of Staff.

DISTRIBUTION:
Comtaskfor 17 (1)
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Comtaskfor 16 (by despatch)

P. C. CROSLEY,
Flag Secretary.

Appendix 4 – Admiral Nimitz' Letter to His Commanders, 28 May 1942[27]

Cincpac File No.

UNITED STATES PACIFIC FLEET
U. S. S. PENNSYLVANIA, Flagship
FLAGSHIP OF THE COMMANDER-IN-CHIEF

A16-3/A4-3/
FF12(12)/(16)

Serial 0114 W.

May 28, 1942.

S E C R E T

From: Commander-in-Chief, United States Pacific Fleet.
To: Commander Striking Forces (Operation Plan 29-42).

Subject: Letter of Instructions.

1. In carrying out the task assigned in Operation Plan 29-42 you will be governed by the principle of calculated risk, which you shall interpret to mean the avoidance of expo-sure of your force to attack by superior enemy forces without good prospect of inflicting, as a result of such exposure, greater damage to the enemy. This applies to a landing phase as well as during preliminary air attacks.

C. W. NIMITZ.

Copy to:

Cominch.
ComTaskFor. 16. (Delivered by hand by War Plans)
ComTaskFor. 17. (Delivered by hand by War Plans)
ComTaskFor. 11. (Hold until arrival Pearl)

P. C. CROSLEY,
Flag Secretary.

[27] Accessed online 21 Feb 13 at: http://midway1942.org/docs/usn_doc_24.shtml.

Appendix 5 – Background Movie Questions

Name: ___________________________________

“Tora, Tora, Tora” – Questions 2

1. Commander Fuchida, who will lead the strike on Pearl Harbor is disappointed because what priority targets are missing from Pearl Harbor?

____

2. What is Colonel Bratton, the intelligence officer, convinced the Japanese will do on Sunday, November 30^{th}, 1941?

____

3. To whom does the President send a personal message just before the attack?

____

4. What does Lieutenant Commander Kramer’s wife get him to eat when the two of them are driving around Washington on the night before the attack?

____

5. On the morning of the attack, what is General Marshal, the Army Chief of Staff, doing?

____

6. On the morning of the attack, what does a U.S. destroyer see when patrolling at the entrance to Pearl Harbor?

____

7. On the morning of the attack, two U.S. Army radar operators spot the Japanese strike force on radar coming in. What does the officer at the command center think it is that the radar operators see?

____

8. Just as the attack occurs, what message does Commander Fuchida send to his Admiral?

____

9. Just as the attack occurs, what message does an American naval officer send to all commands?

____

10. What U.S. battleship blows up under the attack?

____

11. Why are Commander Genda and Commander Fuchida angry at the conclusion of the attack?

____

12. At the conclusion of the movie, Admiral Yamamoto states that he fears the Japanese have awakened a sleeping giant and filled him with a terrible resolve. Who is the sleeping giant?

____

Name: ______________________________

"Tora, Tora, Tora" – Questions 2

1. Commander Fuchida, who will lead the strike on Pearl Harbor is disappointed because what priority targets are missing from Pearl Harbor?

   Ans: The American aircraft carriers.

   ____

2. What is Colonel Bratton, the intelligence officer, convinced the Japanese will do on Sunday, November 30th, 1941?

   Ans: Attack the United States.

   ____

3. To whom does the President send a personal message just before the attack?

   Ans: The Emperor of Japan.

   ____

4. What does Lieutenant Commander Kramer's wife get him to eat when the two of them are driving around Washington on the night before the attack?

   Ans: A hot dog and a Coke.

   ____

5. On the morning of the attack, what is General Marshal, the Army Chief of Staff, doing?

   Ans: Riding his horse.

   ____

6. On the morning of the attack, what does a U.S. destroyer see when patrolling at the entrance to Pearl Harbor?

   Ans: A submarine.

   ---

7. On the morning of the attack, two U.S. Army radar operators spot the Japanese strike force on radar coming in. What does the officer at the command center think it is that the radar operators see?

   Ans: A flight of B-17's flying in from the United States.

   ---

8. Just as the attack occurs, what message does Commander Fuchida send to his Admiral?

   Ans: "Tora, Tora, Tora."

   ---

9. Just as the attack occurs, what message does an American naval officer send to all commands?

   Ans: "Air raid, Pearl Harbor. This is no drill."

   ---

10. What U.S. battleship blows up under the attack?

    Ans: U.S.S. Arizona.

    ---

11. Why are Commander Genda and Commander Fuchida angry at the conclusion of the attack?

Ans: Admiral Nagumo refuses to launch a second strike.

---

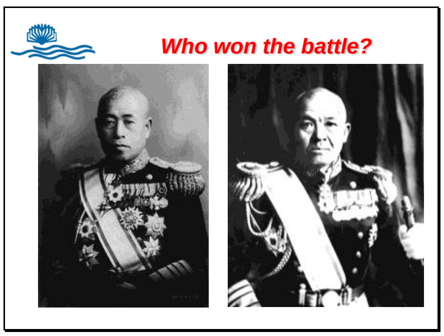
12. At the conclusion of the movie, Admiral Yamamoto states that he fears the Japanese have awakened a sleeping giant and filled him with a terrible resolve. Who is the sleeping giant?

Ans: The United States.

Appendix 6 – Classroom Slide Handouts

Slide 1

Slide 2



Slide 3

Slide 4

Who won the battle?

Captain Joseph Rochefort, USN

Officer in Charge, Joint Intelligence Center, Pacific

Leader of the cryptologists who broke into the Imperial Japanese Navy's operations code and discovered that the Japanese planned to attack Midway Island.

Since Julius Caesar's day over 2000 years ago, armies and navies have used codes to protect important messages.

http://illuminations.nctm.org/ActivityDetail.aspx?id=5

Cryptologists have come up with ever more ingenious ways to crack those codes.

Slide 5

Look at this message...

20-15-4-1-25 25-15-21 23-9-12-12 1-20-20-1-3-11 13-9-4-23-1-25 9-19-12-1-14-4

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

20-15-4-1-25 25-15-21 23-9-12-12 1-20-20-1-3-11 13-9-4-23-1-25 9-19-12-1-14-4

T-O-D-A-Y Y-O-U W-I-L-L A-T-T-A-C-K M-I-D-W-A-Y I-S-L-A-N-D

Slide 6

Now look at this one...

upebz xf xjmm buubdl njexbz jtmboe

today we will attack midway island

Match the corresponding letters. What pattern do you see?

Each letter is "shifted" one position to the right (t becomes u, o becomes p, etc.)

You can "shift" as many postions as you like up to what number?

Slide 7

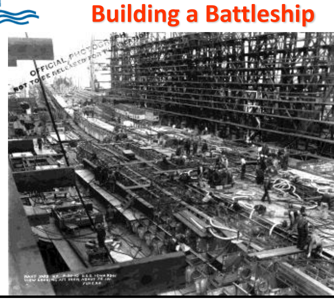
What is a Battleship?

Slide 8

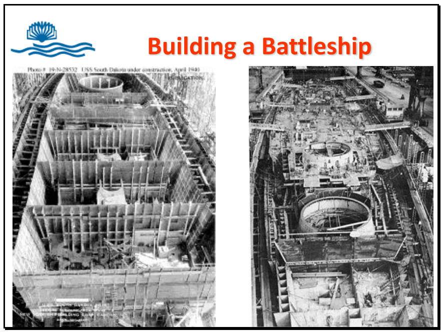
U.S.S. *Massachusetts*

Slide 9

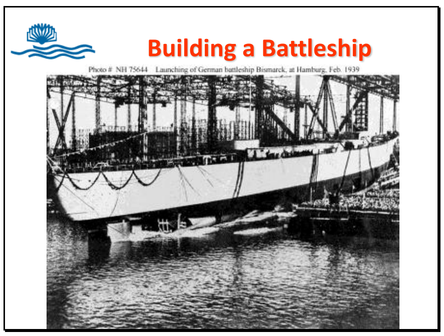
Building a Battleship



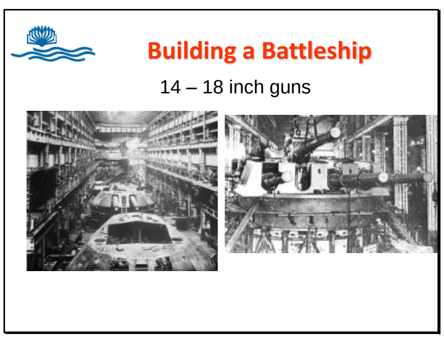
Slide 10



Slide 11



Slide 12



Slide 13

Building a Battleship

Slide 14

How Much Did One Cost?

- Hull, machinery, habitability, armament, armor
- $100,000,000 or $9,320,000,000 in today's dollars (an aircraft carrier costs approximately $6,200,000,000 to build).

Slide 15

Staying Power

- A function of how well a ship can resist harm from attack by bombs, shells, or torpedoes.
- Variables include:
  - How much armor does the ship have – above and below the waterline?
  - How fast is the ship? – with more speed, it can better evade bombs, shells, and torpedoes.

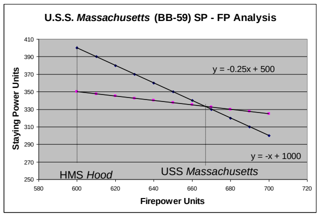
Slide 16

Firepower

- A function of how well a ship can inflict damage on another.
- Variables include:
  - Number and size of weapons such as guns or torpedoes.
  - Accuracy of weapons.
  - Speed of the ship – its ability to maneuver so as to inflict damage on the enemy while evading the enemy's weapons.

Slide 17

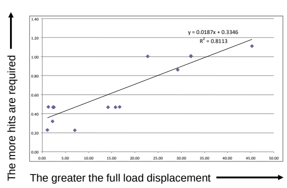
SP vs. FP



Slide 18

Testing the Hypothesis

The greater the full load displacement of a ship (a measure of how big the ship is) the more shells, bombs, or torpedoes it takes to put her out of action.



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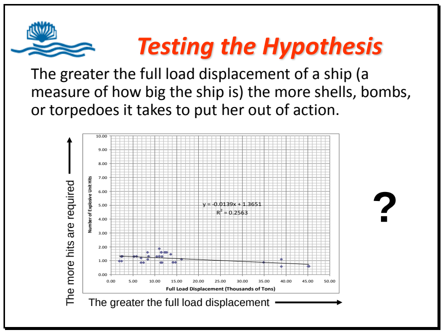

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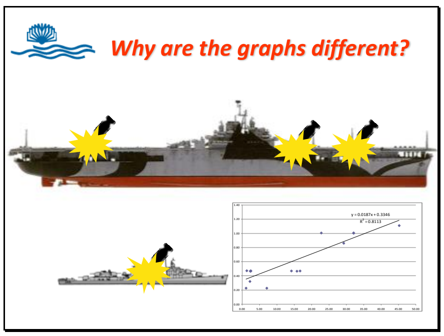

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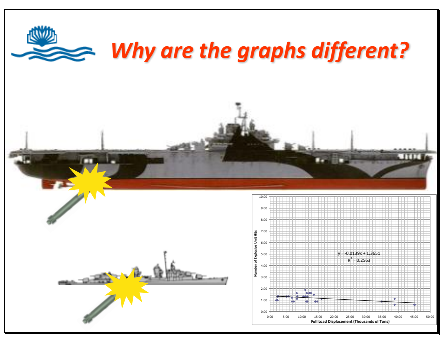

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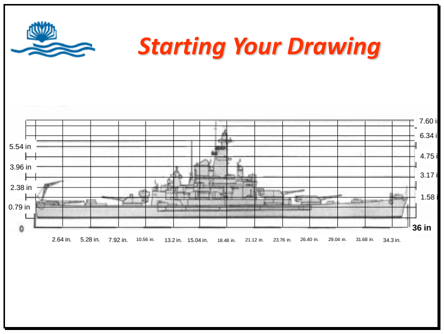


Starting Your Drawing
7.60
6.34
5.54 in
4.75
3.96 in
3.17
2.38 in
1.58
0.79 in
0
36 in
2.64 in.
5.28 in.
7.92 in.
10.56 in.
13.2 in.
15.04 in.
18.48 in.
21.12 in.
23.76 in.
26.40 in.
29.04 in.
31.68 in.
34.3 in.

Name: ______________________________

9th Grade Final Project - End of Project Reflection
Insert this into your final project journal

1. The project was designed to help you develop the following skills:

   a. **Responsibility** - being responsible for getting all work done.

   b. **Self-confidence** - having the confidence that you will get work done.

   c. **Time management** - being responsible for getting work done on time.

   d. **Resiliency** - sticking with a problem until you solve it.

   e. **Team work** - working together without cheating to get work done.

   f. **Leadership** - inspiring people and being responsible for their actions.

   g. **Mathematical computation -** finding answers to complex problems.

   h. **Mathematical reasoning -** discovering how to set up a mathematical means to solve a complex problem.

   Circle two skills that you believe you developed the most fully during the project.

   For each skill, give one example of a piece of work that demonstrates that you developed that skill.

   ____

   ____

   ____

   ____

   ____

   ____

2. It is very easy to become frustrated when completing a task seems impossible.

   a. Provide one example of a task upon which you became frustrated and describe how you overcame that frustration.

   b. Provide one example of how you helped another student overcome his / her frustration and complete a task. How did you help that student?

3. The following is a list of all the project tasks. Circle the one you are most proud of:

Breaking the Code	**Staying Power / Firepower System of Equations**
16 inch Gun Data Analysis	**Ship Stability Problems**
***Massachusetts* Scale Drawing**	***Massachusetts* Scale Model**
Maneuvering Board Exercise	**Comm. Signal Exercise**
War Game Map	**War Game**

Describe why you are proud of this work:

____

____

____

____

____

4. If you could do one task over again because you are not happy with the outcome, which would it be?

Breaking the Code	**Staying Power / Firepower System of Equations**
16 inch Gun Data Analysis	**Ship Stability Problems**
***Massachusetts* Scale Drawing**	***Massachusetts* Scale Model**
Maneuvering Board Exercise	**Comm. Signal Exercise**
War Game Map	**War Game**

Why would you want to do this over again?

5. Describe what you enjoyed most during the final project this trimester.

6. Describe what you liked least during the final project this trimester.

Name: ______________________________

Date: _______________________________

Advisor: _____________________________

Geometry – Developing Similar Shapes Review

1. **Geometry of the Ship Unit.** A shipping container has the following dimensions:

$$20' \times 8' \times 8'$$

You wish to construct a model whose length is on greater than 0.2 feet.

a. What is the scale factor you will need to apply to the dimensions of the actual shipping container to find the dimensions of your model?

Scale factor is: ____

b. What are the dimensions of the model container box?

Dimensions: ____

2. **Geometry of the Ship Unit.** You wish to construct a model of a battleship which has the following dimensions:

Length overall:	680 ft., 9.813 in.
Maximum beam:	108 ft., 2.250 in.
Maximum draft:	36 ft., 9.000 in.

a. If you want your model's length overall to be no more than 36 inches (3 feet), what scale factor will you need to apply to the actual ship dimensions to find the dimensions of your model?

Scale factor is: ____

b. What are the dimensions of the model battleship?

Length overall: ___________________

Maximum beam: ___________________

Maximum draft: ___________________

3. **Navigation Unit.** The distance from the island of O'ahu in Hawai'i to the island of Midway in the Central Pacific Ocean is 1300 nautical miles. On a game board, the distance is 13 feet. What is the scale factor used in the chart (***remember to convert nautical miles to yards and yards to feet or vice versa***).

Scale factor is: ____

4. **Congruence and Similarity Unit.** You wish to construct a model of the Solar System that is no greater than 25 feet in length with the Sun at one end and Pluto at the other. Complete the following table to determine the distances of each model planet from the model Sun:

Body	**Planet Distance from Sun (km)**	**Scale Factor**	**Model Distance (m)**
Sun			
Mercury	57,950,000		
Venus	108,110,000		
Earth	149,570,000		
Mars	227,840,000		
Jupiter	778,140,000		
Saturn	1,427,000,000		
Uranus	2,870,300,000		
Neptune	4,499,900,000		
Pluto	5,913,000,000		

Name: ______________________________

Date: ______________________________

Advisor: ______________________________

Geometry – Developing Similar Shapes Review

1. **Geometry of the Ship Unit.** A shipping container has the following dimensions:

$$20' \times 8' \times 8'$$

You wish to construct a model whose length is on greater than 0.2 feet.

a. What is the scale factor you will need to apply to the dimensions of the actual shipping container to find the dimensions of your model?

Scale factor is: **0.01**

b. What are the dimensions of the model container box?

Dimensions: $(\mathbf{20' * 0.01}) \times (\mathbf{8' * 0.01}) \times (\mathbf{8' * 0.01}) = \mathbf{0.2' \times 0.08' \times 0.08'}$

2. **Geometry of the Ship Unit.** You wish to construct a model of a battleship which has the following dimensions:

Length overall:	680 ft., 9.813 in.
Maximum beam:	108 ft., 2.250 in.
Maximum draft:	36 ft., 9.000 in.

a. If you want your model's length overall to be no more than 36 inches (3 feet), what scale factor will you need to apply to the actual ship dimensions to find the dimensions of your model (**remember to convert feet to inches or inches to feet)**?

Scale factor is: **0.0044**

b. What are the dimensions of the model battleship?

Length overall: **36 inches**

Maximum beam: **5.71 inches**

Maximum draft: **1.94**

3. **Navigation Unit.** The distance from the island of O'ahu in Hawai'i to the island of Midway in the Central Pacific Ocean is 1300 nautical miles. On a game board, the distance is 13 feet. What is the scale factor used in the chart (***remember to convert nautical miles to yards and yards to feet or vice versa***).

   Scale factor is: $\mathbf{1.67 \times 10^{-6}}$

4. **Congruence and Similarity Unit.** You wish to construct a model of the Solar System that is no greater than 25 feet in length with the Sun at one end and Pluto at the other. Complete the following table to determine the distances of each model planet from the model Sun:

Body	Planet Distance from Sun (km)	Scale Factor	Model Distance (m)
Sun			
Mercury	57,950,000	0.000000004227972	0.25
Venus	108,110,000	0.000000004227972	0.46
Earth	149,570,000	0.000000004227972	0.63
Mars	227,840,000	0.000000004227972	0.96
Jupiter	778,140,000	0.000000004227972	3.29
Saturn	1,427,000,000	0.000000004227972	6.03
Uranus	2,870,300,000	0.000000004227972	12.14
Neptune	4,499,900,000	0.000000004227972	19.03
Pluto	5,913,000,000	0.000000004227972	25

Preliminary Instruction: Vector Analysis: Maneuvering Board

Slide 1

Geometry – Vector Analysis

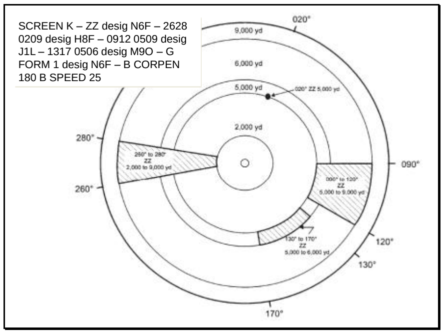
Using Maneuvering Boards
2012 – 2013 School Year

Slide 2

Slide 3

Slide 4

Slide 5



Slide 6

Basic Vector Definitions

A vector is a quantity that has both magnitude and direction. The magnitude is a scalar quantity, a scalar being defined as a quantity which may be completely specified by a number and perhaps a unit. Common textbook representations of vectors include boldfaced letters and boldface with an arrow above them. For example a displacement vector of 30 meters east could be represented in a variety of ways:

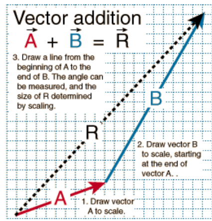
$\mathbf{D}$ $\vec{\mathbf{D}}$ or a graphical representation $\xrightarrow{30\text{ m}}$

The magnitude of the vector might be represented by absolute value signs around the vector symbol, or just the letter without the boldface.

magnitude of $\vec{\mathbf{D}} = |\vec{\mathbf{D}}| = D = 30$ meters

A vector might also be expressed in terms of unit vectors.

Slide 7



Slide 8

Ralston (3 – minute) Rule Practice

Distance Traveled	Time	Speed in Knots
1000 yards	3 minutes	
3560 yards	3 minutes	
4423 yards	3 minutes	
1200 yards	3 minutes	
5000 yards	6 minutes	
1520 yards	9 minutes	
2000 yards	4 minutes	
8000 yards	12 minutes	
1500 yards	5 minutes	
7500 yards	10 minutes	
15000 yards	18 minutes	
3000 yards	4.5 minutes	
3000 yards	3.3 minutes	
1050 yards	1 minute	
1590 yards	2 minutes	

Slide 9

What is your course and speed?

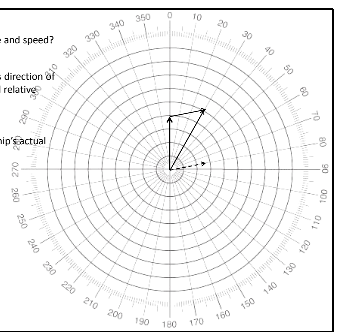
000°T at 20 knots.

What is other ship's direction of relative motion and relative speed?

080° at 12.5 knots.

What is the other ship's actual course and speed?

030°T at 25 knots.



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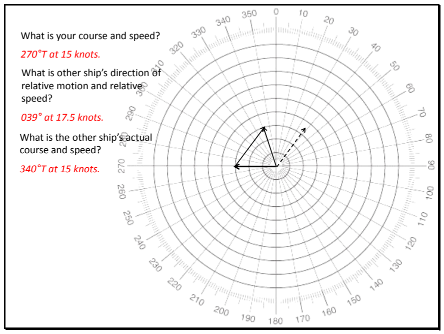

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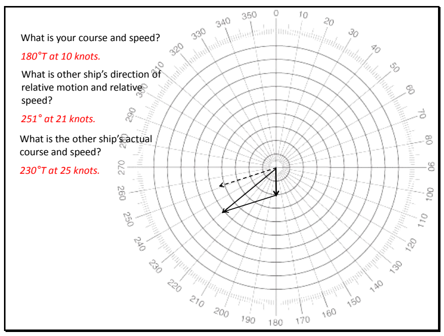

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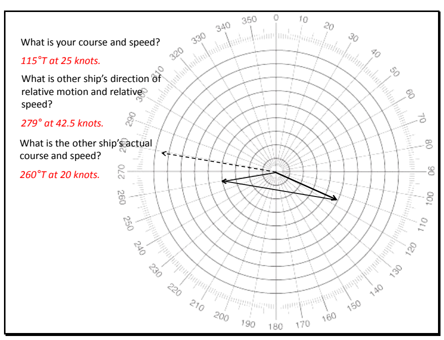

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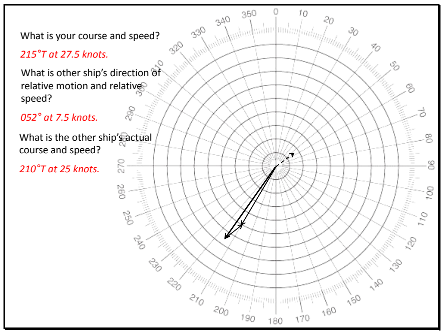

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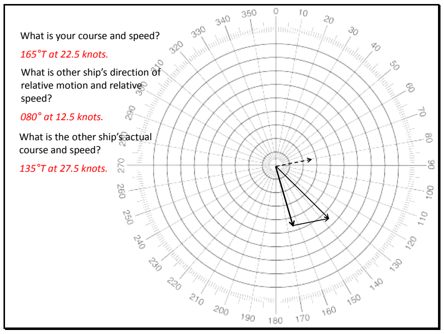

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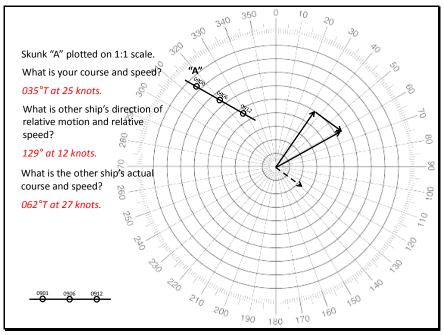

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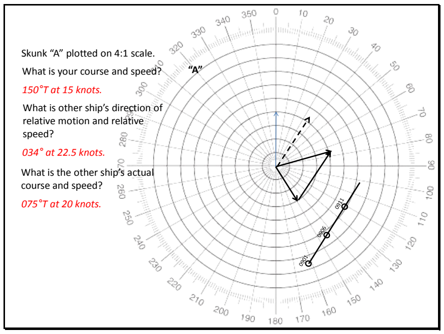

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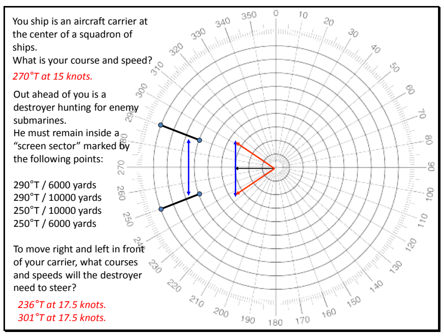


Name:_________________________________

Maneuvering Board Problem Set 1

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

   Problem 1:

   a. Plot a vector to depict own-ship course of 050°T at 15 knots.

   b. Plot a relative motion vector for the other ship of 150° at 10 knots.

   c. What is the other ship's actual course and speed?

   ___________°T at ___________ knots 088°T, 17.5 knots

   Problem 2:

   a. Plot a vector to depict own-ship course of 235°T at 10 knots.

   b. Plot a relative motion vector for the other ship of 165° at 15 knots.

   c. What is the other ship's actual course and speed?

   ___________°T at ___________ knots 192°, 21 knots

   Problem 3:

   a. Plot a vector to depict own-ship course of 320°T at 20 knots.

   b. Plot a relative motion vector for the other ship of 120° at 5 knots.

   c. What is the other ship's actual course and speed?

   ___________°T at ___________ knots 325°T, 15 knots

Problem 4:

a. Plot a vector to depict own-ship course of 135°T at 20 knots.

b. Plot a true course and speed vector for the other ship of 180° at 15 knots.

c. What direction of relative motion and relative speed of the other ship?

____________°T at ____________ knots 267°T, 14.5 knots

Problem 5:

a. Plot a vector to depict own-ship course of 020°T at 20 knots.

b. Plot a true course and speed vector for the other ship of 180° at 25 knots.

c. What direction of relative motion and relative speed of the other ship?

____________°T at ____________ knots 189°T, 44 knots

Problem 6:

a. Plot a vector to depict own-ship course of 230°T at 5 knots.

b. Plot a true course and speed vector for the other ship of 340° at 5 knots.

c. What direction of relative motion and relative speed of the other ship?

____________°T at ____________ knots 016°T, 8 knots

2. Find the speed in knots using the Ralston (3-minute) Rule:

Distance Traveled	Time	Speed in Knots
2300 yards	3 minutes	23
1850 yards	3 minutes	18.5
5423 yards	3 minutes	54.23
3631 yards	3 minutes	36.31
1250 yards	6 minutes	6.25
860 yards	9 minutes	2.87
907 yards	4 minutes	6.8
4567 yards	12 minutes	11.42
888 yards	5 minutes	5.33
1267 yards	10 minutes	3.81
15000 yards	36 minutes	12.5
3000 yards	10.5 minutes	8.57
3000 yards	8 minutes	11.25
1050 yards	2 minutes	15.75
1590 yards	1 minute	47.70

Name:______________________________

Maneuvering Board Problem Set 1

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

   Problem 1:

   a. Plot a vector to depict own-ship course of 050°T at 15 knots.

   b. Plot a relative motion vector for the other ship of 150° at 10 knots.

   c. What is the other ship's actual course and speed?

   ____________°T at ____________ knots

   Problem 2:

   d. Plot a vector to depict own-ship course of 235°T at 10 knots.

   e. Plot a relative motion vector for the other ship of 165° at 15 knots.

   f. What is the other ship's actual course and speed?

   ____________°T at ____________ knots

   Problem 3:

   a. Plot a vector to depict own-ship course of 320°T at 20 knots.

   b. Plot a relative motion vector for the other ship of 120° at 5 knots.

   c. What is the other ship's actual course and speed?

   ____________°T at ____________ knots

Problem 4:

a. Plot a vector to depict own-ship course of 135°T at 20 knots.

b. Plot a true course and speed vector for the other ship of 180° at 15 knots.

c. What direction of relative motion and relative speed of the other ship?

___________°T at ___________ knots

Problem 5:

a. Plot a vector to depict own-ship course of 020°T at 20 knots.

b. Plot a true course and speed vector for the other ship of 180° at 25 knots.

c. What direction of relative motion and relative speed of the other ship?

___________°T at ___________ knots

Problem 6:

a. Plot a vector to depict own-ship course of 230°T at 5 knots.

b. Plot a true course and speed vector for the other ship of 340° at 5 knots.

c. What direction of relative motion and relative speed of the other ship?

___________°T at ___________ knots

2. Find the speed in knots using the Ralston (3-minute) Rule:

Distance Traveled	Time	Speed in Knots
2300 yards	3 minutes	
1850 yards	3 minutes	
5423 yards	3 minutes	
3631 yards	3 minutes	
1250 yards	6 minutes	
860 yards	9 minutes	
907 yards	4 minutes	
4567 yards	12 minutes	
888 yards	5 minutes	
1267 yards	10 minutes	
15000 yards	36 minutes	
3000 yards	10.5 minutes	
3000 yards	8 minutes	
1050 yards	2 minutes	
1590 yards	1 minute	

Name:________________________________

Maneuvering Board Problem Set 2

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

   a. On radar, you note the following bearings and ranges for Skunk "A":

Time	Bearing	Range
0900	330°T	7000 yards
0903	336°T	6200 yards
0906	343°T	5500 yards

Your course and speed is 000°T at 20 knots.

Skunk "A"'s direction of relative motion and relative speed is:

____________°T at ____________ knots 113°T at 10 knots

Skunk "A"'s course and speed is:

____________°T at ____________ knots 029°T at 18 knots

   b. On radar, you note the following bearings and ranges for Skunk "B":

Time	Bearing	Range
0915	270°T	4000 yards
0918	232°T	4000 yards
0921	207°T	5450 yards

Your course and speed is 240°T at 15 knots.

Skunk "B"'s direction of relative motion and relative speed is:

____________°T at ____________ knots 162°T at 25 knots

Skunk "B"'s course and speed is:

____________°T at ____________ knots 190°T at 33 knots

2. Find the speed in knots using the Ralston (3-minute) Rule:

Distance Traveled	Time	Speed in Knots
1250 yards	3 minutes	12.5
860 yards	3 minutes	8.6
907 yards	3 minutes	9.07
4567 yards	3 minutes	45.67
888 yards	6 minutes	4.44
1267 yards	9 minutes	4.22
15000 yards	4 minutes	11.25
3000 yards	12 minutes	7.5
3000 yards	5 minutes	18
1050 yards	10 minutes	3.15
1590 yards	36 minutes	1.33
2300 yards	10.5 minutes	6.57
1850 yards	8 minutes	6.94
5423 yards	2 minutes	8.13
3631 yards	1 minute	108.93

Name:______________________________

Maneuvering Board Problem Set 2

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

   a. On radar, you note the following bearings and ranges for Skunk "A":

Time	Bearing	Range
0900	330°T	7000 yards
0903	336°T	6200 yards
0906	343°T	5500 yards

   Your course and speed is 000°T at 20 knots.

   Skunk "A"'s direction of relative motion and relative speed is:

   ____________°T at ____________ knots

   Skunk "A"'s course and speed is:

   ____________°T at ____________ knots

   b. On radar, you note the following bearings and ranges for Skunk "B":

Time	Bearing	Range
0915	270°T	4000 yards
0918	232°T	4000 yards
0921	207°T	5450 yards

   Your course and speed is 240°T at 15 knots.

   Skunk "B"'s direction of relative motion and relative speed is:

   ____________°T at ____________ knots

   Skunk "B"'s course and speed is:

   ____________°T at ____________ knots

2. Find the speed in knots using the Ralston (3-minute) Rule:

Distance Traveled	Time	Speed in Knots
1250 yards	3 minutes	
860 yards	3 minutes	
907 yards	3 minutes	
4567 yards	3 minutes	
888 yards	6 minutes	
1267 yards	9 minutes	
15000 yards	4 minutes	
3000 yards	12 minutes	
3000 yards	5 minutes	
1050 yards	10 minutes	
1590 yards	36 minutes	
2300 yards	10.5 minutes	
1850 yards	8 minutes	
5423 yards	2 minutes	
3631 yards	1 minute	

Name:________________________________

Maneuvering Board Problem Set 3

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

   a. On radar, you note the following bearings and ranges for Skunk "A":

Time	Bearing	Range
1503	085°T	8000 yards
1509	086.5°T	7000 yards
1515	088°T	6000 yards

   Your course and speed is 180°T at 15 knots.

   Skunk "A"'s direction of relative motion and relative speed is:

   ___________°T at ___________ knots   256°T at 5 kts.

   Skunk "A"'s course and speed is:

   ___________°T at ___________ knots   199°T at 17.5 kts.

   b. Your ship, U.S.S. *Enterprise* (CV-6) is at the center of a squadron of ships which, together are on a course of 180°T at 15 knots. Directly in front of your ship is U.S.S. *Cushing* (DD-376). *Cushing* is patrolling a "screen sector" looking for enemy submarines that is marked out by the following points:

   Point 1   200°T / 6000 yards from *Enterprise*

   Point 2   200°T / 10000 yards from *Enterprise*

   Point 3   160°T / 10000 yards from *Enterprise*

   Point 4   160°T / 6000 yards from *Enterprise*

   What courses must *Cushing* to move from right to left and back across her sector at an actual speed of 25 knots.

   127°T and 234°T

Name:________________________________

Maneuvering Board Problem Set 3

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

    a. On radar, you note the following bearings and ranges for Skunk "A":

Time	Bearing	Range
1503	085°T	8000 yards
1509	086.5°T	7000 yards
1515	088°T	6000 yards

    Your course and speed is 180°T at 15 knots.

    Skunk "A"'s direction of relative motion and relative speed is:

    ___________°T at ___________ knots

    Skunk "A"'s course and speed is:

    ___________°T at ___________ knots

    b. Your ship, U.S.S. *Enterprise* (CV-6) is at the center of a squadron of ships which, together are on a course of 180°T at 15 knots. Directly in front of your ship is U.S.S. *Cushing* (DD-376). *Cushing* is patrolling a "screen sector" looking for enemy submarines that is marked out by the following points:

Point 1	200°T / 6000 yards from *Enterprise*
Point 2	200°T / 10000 yards from *Enterprise*
Point 3	160°T / 10000 yards from *Enterprise*
Point 4	160°T / 6000 yards from *Enterprise*

    What courses must *Cushing* to move from right to left and back across her sector at an actual speed of 25 knots.

Name:_________________________________

Maneuvering Board Practice Assessment

Show all work on a maneuvering board. Place answers on this sheet.

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

   a. On radar, you note the following bearings and ranges for Skunk "A":

Time	Bearing	Range
1230	270°T	7000 yards
1233	247°T	6000 yards
1236	221°T	6300 yards

Your course and speed is 300°T at 20 knots.

Skunk "A"'s direction of relative motion and relative speed is:

____________°T at ____________ knots 149°T / 28 kts.

Skunk "A"'s course and speed is:

____________°T at ____________ knots 194°T / 14 kts.

   b. Your ship, U.S.S. *Hornet* (CV-8) is at the center of a squadron of ships which, together are on a course of 300°T at 20 knots. Directly in front of your ship is U.S.S. *Fletcher* (DD-445). *Fletcher* is patrolling a "screen sector" looking for enemy submarines that is marked out by the following points:

Point 1	330°T / 4000 yards from *Hornet*
Point 2	330°T / 6000 yards from *Hornet*
Point 3	010°T / 6000 yards from *Hornet*
Point 4	010°T / 4000 yards from *Hornet*

What courses must *Fletcher* to move from right to left and back across her sector at an actual speed of 25 knots in one direction and 15 knots in the other. 290°T at 25 kts. and 320°T at 15 kts.

YOU HAVE 10 MINUTES TO ANSWER EACH QUESTION

Name:________________________________

Maneuvering Board Practice Assessment

Show all work on a maneuvering board. Place answers on this sheet.

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

   a. On radar, you note the following bearings and ranges for Skunk "A":

Time	Bearing	Range
1230	270°T	7000 yards
1233	247°T	6000 yards
1236	221°T	6300 yards

Your course and speed is 300°T at 20 knots.

Skunk "A"'s direction of relative motion and relative speed is:

___________°T at ___________ knots

Skunk "A"'s course and speed is:

___________°T at ___________ knots

   b. Your ship, U.S.S. *Hornet* (CV-8) is at the center of a squadron of ships which, together are on a course of 300°T at 20 knots. Directly in front of your ship is U.S.S. *Fletcher* (DD-445). *Fletcher* is patrolling a "screen sector" looking for enemy submarines that is marked out by the following points:

Point 1	330°T / 4000 yards from *Hornet*
Point 2	330°T / 6000 yards from *Hornet*
Point 3	010°T / 6000 yards from *Hornet*
Point 4	010°T / 4000 yards from *Hornet*

What courses must *Fletcher* to move from right to left and back across her sector at an actual speed of 25 knots in one direction and 15 knots in the other.

YOU HAVE 10 MINUTES TO ANSWER EACH QUESTION

Name:________________________________

Maneuvering Board Assessment I

Show all work on a maneuvering board. Place answers on this sheet.

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

   a. On radar, you note the following bearings and ranges for Skunk "A":

Time	Bearing	Range
1915	170°T	7000 yards
1918	168°T	6100 yards
1921	165°T	5200 yards

Your course and speed is 003°T at 20 knots.

Skunk "A"'s direction of relative motion and relative speed is:

___________°T at ___________ knots   003°T / 10 kts.

Skunk "A"'s course and speed is:

___________°T at ___________ knots   090°T / 18 kts.

   b. Your ship, U.S.S. *Lexington* (CV-2) is at the center of a squadron of ships which, together are on a course of 120°T at 20 knots. Directly in front of your ship is U.S.S. *Fletcher* (DD-445). *Fletcher* is patrolling a "screen sector" looking for enemy submarines that is marked out by the following points:

Point 1	100°T / 6000 yards from *Lexington*
Point 2	100°T / 10000 yards from *Lexington*
Point 3	140°T / 6000 yards from *Lexington*
Point 4	140°T / 10000 yards from *Lexington*

What courses must *Fletcher* to move from right to left and back across her sector at an actual speed of 25 knots. 085°T / 155°T

YOU HAVE 10 MINUTES TO ANSWER EACH QUESTION

2. Find the speed in knots using the Ralston (3-minute) Rule:

Distance Traveled	Time	Speed in Knots
3000 yards	3 minutes	30
1050 yards	3 minutes	10.5
1590 yards	3 minutes	15.9
2300 yards	3 minutes	23
1850 yards	6 minutes	9.25
5423 yards	9 minutes	18.08
3631 yards	4 minutes	27.23
1250 yards	12 minutes	3.13
860 yards	5 minutes	5.16
907 yards	10 minutes	2.72
4567 yards	36 minutes	3.81
888 yards	10.5 minutes	2.66
1267 yards	8 minutes	4.75
15000 yards	2 minutes	225
3000 yards	1 minute	90

Name:_______________________________

Maneuvering Board Assessment I

Show all work on a maneuvering board. Place answers on this sheet.

1. Use a maneuvering board, ruler, and dividers to answer the following questions:

   a. On radar, you note the following bearings and ranges for Skunk "A":

Time	Bearing	Range
1915	170°T	7000 yards
1918	168°T	6100 yards
1921	165°T	5200 yards

   Your course and speed is 003°T at 20 knots.

   Skunk "A"'s direction of relative motion and relative speed is:

   ___________°T at ___________ knots

   Skunk "A"'s course and speed is:

   ___________°T at ___________ knots

   b. Your ship, U.S.S. *Lexington* (CV-2) is at the center of a squadron of ships which, together are on a course of 120°T at 20 knots. Directly in front of your ship is U.S.S. *Fletcher* (DD-445). *Fletcher* is patrolling a "screen sector" looking for enemy submarines that is marked out by the following points:

Point 1	100°T / 6000 yards from *Lexington*
Point 2	100°T / 10000 yards from *Lexington*
Point 3	140°T / 6000 yards from *Lexington*
Point 4	140°T / 10000 yards from *Lexington*

What courses must *Fletcher* to move from right to left and back across her sector at an actual speed of 25 knots.

YOU HAVE 10 MINUTES TO ANSWER EACH QUESTION

3. Find the speed in knots using the Ralston (3-minute) Rule:

Distance Traveled	Time	Speed in Knots
3000 yards	3 minutes	
1050 yards	3 minutes	
1590 yards	3 minutes	
2300 yards	3 minutes	
1850 yards	6 minutes	
5423 yards	9 minutes	
3631 yards	4 minutes	
1250 yards	12 minutes	
860 yards	5 minutes	
907 yards	10 minutes	
4567 yards	36 minutes	
888 yards	10.5 minutes	
1267 yards	8 minutes	
15000 yards	2 minutes	
3000 yards	1 minute	

Preliminary Instruction
Navigation Review

b. Latitude and Longitude:

i. Latitude:

Variables:

Latitude (La)
Observed Altitude of Sun (O)
Sun's Declination (D)

Formula:

$$La = 90° - O + D$$

(Sun in Northern Hemisphere)

$$La = 90° - O - D$$

(Sun in Southern Hemisphere)

ii. Longitude:

Variables:

Longitude: (Lo)
GMT of LAN (GMT)
Result of Subtraction (S)
Hours from Subtraction (H)
Minutes from Subtraction (M)

Formulas:

3. $S = 12{:}00 - GMT\ (12{:}00 > GMT)$
   $S = GMT - 12{:}00\ (12{:}00 < GMT)$

4. $Lo = H * 15° + \frac{M}{(4' per\ °)}$

c. True Bearing:

Variables:

True Bearing (TB °T)
Magnetic Bearing (M °E or °W)
Declination (D °E or °W)

Formulas:

TB °T $= \pm$M $\pm$ D + 360° (if initial result is negative.

g. Distance Traveled:

Variables:

Distance Traveled (D nm)
Speed in Knots (S nm/hr)
Time of Journey (T hrs.)

Formulas:

$$D\ nm = \left(\frac{S\ nm}{hr}\right) * T\ hrs.$$

h. Fuel Burned:

Variables:

Fuel Burned on Journey (F gal.)
Gallons Burned per NM (G gal.)
Distance Traveled (D nm)

Formulas:

$$F\ gal. = \left(\frac{G\ gal}{nm}\right) * D\ nm$$

i. Percentage of Fuel Burned:

Variables:

Percentage of Fuel Burned (B)
Fuel Burned on Journey (F gal.)
Total Fuel Carried (T gal.)

Formulas:

$$B = \frac{F\ gal.}{T\ gal.}$$

j. Percentage of Fuel Remaining:

Variables:

Percentage of Fuel Remaining (P)
% Fuel Onboard fm Previous Day (PD)
Percentage of Fuel Burned Today (B)

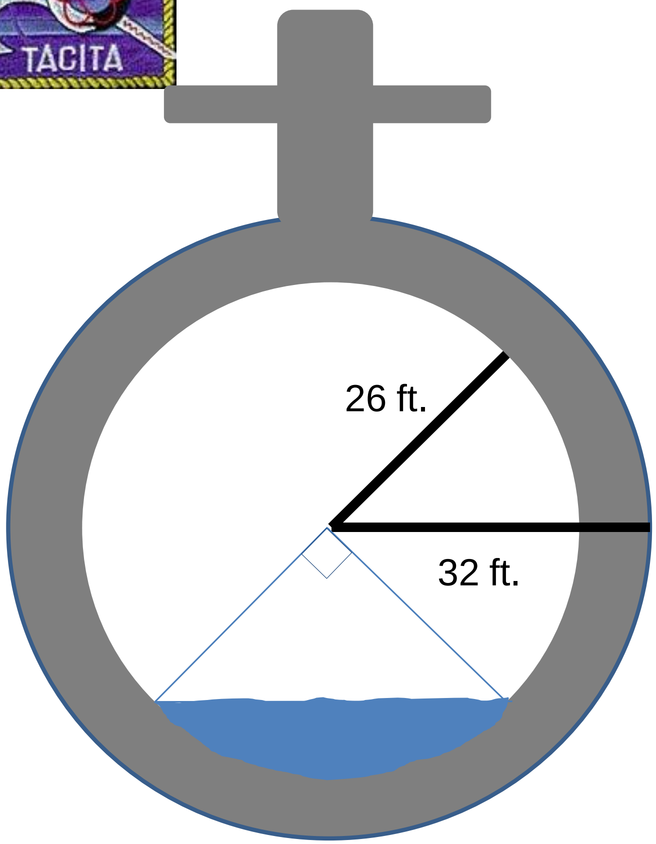
Formulas:

$$P = PD - B$$

Loss of U. S. S. Thresher

April 10th, 1963

Loss of U. S. S. Thresher



1. If the reactor / engine compartment is 100 ft. long, what is the volume of the flooded portion?

2. How many gallons of water flooded into the compartment? (7.48 gallons per cu. ft.)

3. What was the added weight to the submarine? (8.35 gallons to one pound)

Name: ______________________

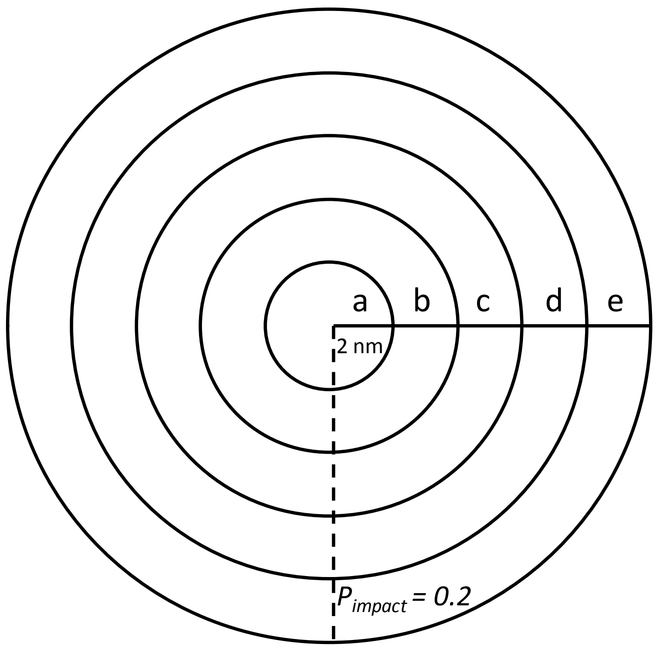
Preliminary Instruction: Practical Problems Involving Circles

Solve All Problems on Separate Sheets of Paper

1. A test missile is fired from the Pacific Missile Test Center at Edwards Air Force Base in California to an impact area in the Pacific Ocean. The impact area depicted below is a circle 20 nautical miles (nm) in diameter.

   - There are five circular regions in the impact area, labeled *a, b, c, d, e*. Each is 2 (nm) in width.

   - The probability that the missile will impact in any one of the five circular regions (P_{impact}) is 20% (0.20). Therefore, the probability that the missile will impact somewhere in the impact area is P_{impact}(a) + P_{impact}(b) + P_{impact}(c) + P_{impact}(d) + P_{impact}(e) $= 0.20 + 0.20 + 0.20 + 0.20 + 0.20 = 1.00 = 100\%$.

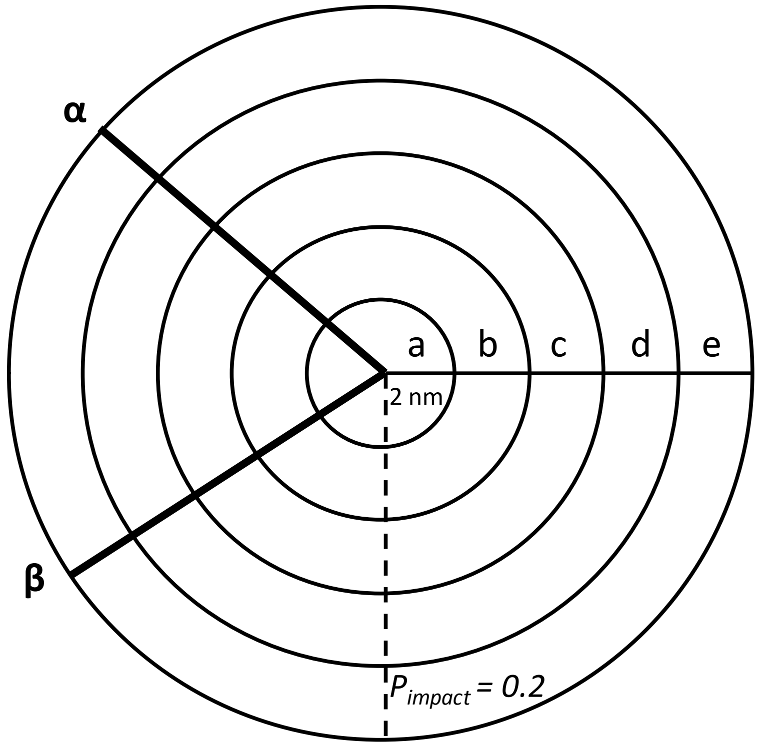


   a. What is the total impact area?

   b. What is the area of region c?

   c. What is the probability that the missile will impact no closer than 6 nm to the center of the impact area?

2. The measure of the central angle intercepting the minor arc αβ is 85°.



a. What is the length of minor arc αβ?

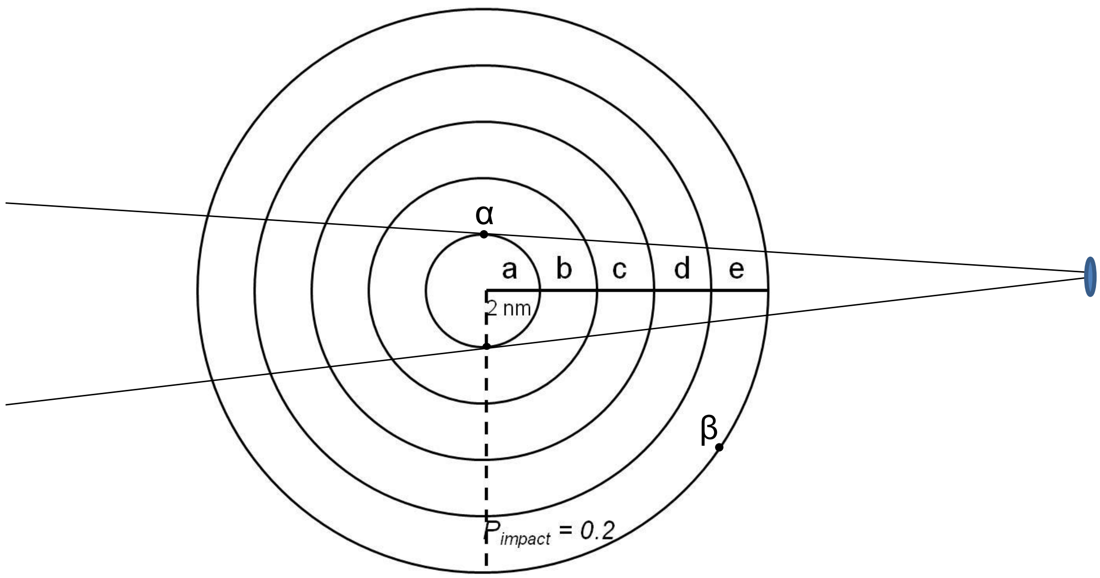
b. What is the probability that the missile will impact in the region between the two radii that intercept minor arc αβ?

c. What is the probability that the missile will impact in the region between the two radii that intercept minor arc αβ no closer than 6 nm from the center?

Preliminary Instruction: Practical Problems Involving Circles

Solve All Problems on Separate Sheets of Paper

A Chinese surveillance ship is sitting outside of the impact area using radar to observe the missile's impact.



1. If the measures of the two intercepted arcs on the outer edge of region *e* are 25° and 20° respectively, what is the measure of the angle formed by the radar beam?

2. If the radar beam covers one fifth of the impact area every second, what is the probability that the Chinese surveillance ship will see the missile just as it hits the ocean?

3. If the length of the radar beam from the Chinese surveillance ship to point α is 20 nm, what is the distance of the Chinese surveillance ship from the center of the impact area?

4. If the Chinese surveillance ship were to move from its present position to the edge of the impact area at point β and shine its radar beam toward the center of the impact area, what would be the measure and arc length of the arc intercepted on the outer edge of region *e*?

Name: ___________________________________

9^{th} Grade Final Project Pre – Assessment

Show all work on separate sheets of paper attached to this one.

1. **Data Analysis and Linear Modeling.**

   A. Enter the following data into Microsoft Excel or Open Office Spreadsheet.
   B. Graph the data as a scatter plot, labeling the graph and both axes.
   C. Fit a linear function to the graph.
   D. Display the linear equation and the R^2 value on the graph.
   E. Predict the value of "y" when $x = 2015$.
   F. Print and attach the graph with equation, R^2 value, and answer to "E" included.

x = Year	**y = Altitude (in km)**
1998	400
1999	380
2000	360
2001	380
2002	390
2003	380
2004	360
2005	350
2006	340
2007	330
2008	340
2009	360
2010	340
2011	340

2. **Systems of Equations: Algebra students only should answer this question.**

A. Enter the following data into Microsoft Excel or Open Office Spreadsheet.
B. Graph both data sets as a scatter plot.
C. Fit linear functions to the two graphs.
D. Display the linear equations on the graph.
E. The two linear equations are a system of equations. Rewrite them and then solve using any method with which you are familiar.
F. Attach the graph to this document.

x	**y1**	**y2**
-10	-15	34
-9	-13	31
-8	-11	28
-7	-9	25
-6	-7	22
-5	-5	19
-4	-3	16
-3	-1	13
-2	1	10
-1	3	7
0	5	4
1	7	1
2	9	-2
3	11	-5
4	13	-8
5	15	-11
6	17	-14

7	19	-17
8	21	-20
9	23	-23
10	25	-26

Solution to the system:

x = _________

y = _________

3. **Geometry and Scaling.**

A. You want to draw a scaled picture of a ship which has an actual length of 700 feet. The picture will be 36 inches in length. Determine the scale factor to convert all dimensions of the ship from actual size to your drawing size.

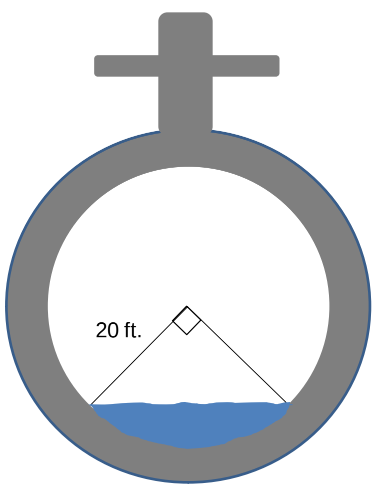
Ans.: ____

B. **Geometry students only should answer this question.** You want to fill a container box which has dimensions of 20 feet by 8 feet by 8 feet with eight identical cylinders which are 8 feet in height and exactly fill the box (i.e. no usable space is left over). What is the surface area and volume of one of the cylinders?

Surface area: _______________ Volume: _______________

C. **Geometry students only should answer this question.**

i. The flooded compartment of the submarine depicted below is 50 feet in length. Find the volume of the flooded portion.



Volume = ______________

4. **Navigation.**

A. From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 15th	1630	40° 23.15'	23° 08' S		

B. U.S.S. *Rentz* travelled 450 nm in 24 hours on one engine.

i. At what speed was she travelling? ___________________________

ii. How many gallons of fuel did she burn? ______________________

PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth

“A Voyage to the South Seas”

Logarithms, Planar Trigonometry and Spherical Trigonometry

Thomas R. Beall
Captain, U. S. Navy (Ret.)

Introduction

In the eighteenth century, European nation-states developed the organization and finances necessary to create large, standing military and naval forces. In Great Britain, for example, the Royal Navy evolved into the largest industrial organization in human history prior to the Industrial Revolution with hundreds of ships and a shore establishment of dockyards, workshops, and hospitals for the care and feeding of thousands of personnel. This massive organization was designed with one purpose in mind, to keep warships at sea for years at a time to guard Britain's growing empire.

The ability to keep ships at sea indefinitely hinged on the Royal Navy's ability to keep its sailors healthy. Throughout the century, the Royal Navy developed methods of personal hygiene, nutrition, and medical care that, although primitive by today's standards, were a vast improvement over what Europeans had experienced for centuries. British seaman, already a sturdy lot, were now fed and cared-for so that they could endure the hardships of life at sea indefinitely.

Many of these improvements took place in the decades after 1740 when Great Britain was more often at war than at peace. In 1744, Commodore George Anson returned from a world cruise during which he seized one of the great Spanish treasure ships in Philippine waters at the cost of all but one of his five ships and over 5/6 of his men who perished from sickness, exposure, and starvation. In 1771, Captain James Cook, commanding H.M. Bark *Endeavour* returned from his first world cruise having lost none of his men to any of these causes. Cook and his successors, including Captain William Bligh and Captain George Vancouver, learning from Anson's experience, took personal interest in caring for their men and preserving their health.

With these improvements, long voyages became less the epic adventures of old and more of an accepted although still challenging effort. As it became possible to mount such voyages, European nations sought to learn as much as possible about the far Pacific which, prior to the mid-eighteenth century, remained largely uncharted by Europeans. Expeditions such as Captain Cook's were mounted to discover and chart new lands for eventual settlement or colonial exploitation. The methods mariners such as Cook used to navigate and manage their ships and survey new lands involved much of the mathematics we will learn in this course.

Suggested Reading

I will offer extra credit for reading and reporting on one of these books. See me to arrange it.

Alexander, Caroline. *The Bounty: The True Story of the Mutiny on the Bounty*. New York: Penguin Books, 2004.

Beaglehole, John C. *The Life of Captain James Cook*. Palo Alto, CA: Stanford University Press, 1992.

Beaglehole, John C. *The Journals of Captain James Cook on His Voyages of Discovery*, 3 volumes in 4 books. London: The Hakluyt Society / Cambridge University Press, 1968 – 1974.

Nordhoff, Charles and James Norman Hall. *The Bounty Trilogy*. Boston: Little Brown and Company, 1985.

O'Brian, Patrick. *The Golden Ocean.* New York: W. W. Norton, 2012.

Rodger, N. A. M. *The Wooden World: An Anatomy of the Georgian Navy.* New York: W. W. Norton, 1996.

Williams, Glyndwr. *The Prize of All the Oceans: The Dramatic True Story of Commodore Anson's Voyage Round the World and How He Seized the Spanish Treasure Galleon.* New York: Viking Press, 2000.

Class Organization

Throughout this series of units, students will role-play the roles of officers of an 18th century Royal Navy ship on a voyage of exploration. In addition to traditional mathematics exercises, students will perform mathematical tasks associated with ship management, navigation, and coastal surveying.

Students will be assigned to five-man crews. On initial assignment, crew members will each hold the rank of midshipman. Upon successful completion of the Lieutenant's Examination, crew members will be assigned to one of the following positions based upon ranking of examination grades:

Exam Ranking	**Position**	**Responsibility**	**Eligibility**
1st	Captain	• Completion of team work • Weekly grade of team members • Submission of one complete chart project • Submission of satisfactory journal	
2nd	1st Lieutenant	• Quality assurance of team member journals • Daily grade of team member journals • Checking and correction of daily team work • Submission of one complete chart project • Submission of satisfactory journal	
3rd	Master	• Completion of daily navigation and survey problems • Submission of one complete chart project • Submission of satisfactory journal	
4th	Purser	• Completion of daily ship logistics problems • Submission of one complete chart project • Submission of satisfactory journal	
5th	Quartermaster	• Assisting master in completion of daily navigation and survey problems • Submission of one complete chart project • Submission of satisfactory journal	

Group Assignments

	H.M.S. *Endeavour*	**H.M.S. *Adventure***	**H.M.S. *Resolution***	**H.M.S. *Bounty***	**H.M.S. *Pearl***
Captain					
1st Lieutenant					
Master					
Purser					
Quartermaster					
	H.M.S. *Centurion*	**H.M.S. *Severn***	**H.M.S. *Wager***	**H.M.S. *Gloucester***	**H.M.S. *Sirius***
Captain					
1st Lieutenant					
Master					
Purser					
Quartermaster					
	H.M.S. *Dolphin*	**H.M.S. *Golden Hind***	**H.M.S. *Providence***	**H.M.S. *Paramour***	**H.M.S. *Tyral***
Captain					
1st Lieutenant					
Master					
Purser					
Quartermaster					

Examination for Lieutenant[28]

The following is derived from British Admiralty regulations from the nineteenth century. Young men seeking to be sea-going officers would normally sign on to a warship under the patronage of the Captain or another officer. They would be required to serve satisfactorily six years at sea, including two in the officer cadet rank of "midshipman", to be eligible for examination for a commission as a lieutenant. A commission was granted upon satisfactory completion of an oral or written examination.

The Lords Commissioners of the Admiralty have established the following Regulations for examination for a commission in H.M. Navy.

1. No Person will be nominated for a commission in H.M. Navy who will be under 13, or above 17 years of age at the time of Examination.

2. Every midshipman, on obtaining a nomination, will be required to pass an Examination within three months of nomination.

3. The midshipman must be in good health and fit for the Service, that is, free from impediment of speech, defect of vision, rupture, or other physical inefficiency.

4. Midshipmen will be required:

   a. To write English correctly and in a legible hand,
   b. To have a satisfactory knowledge of Arithmetic, including Proportion,
   c. To have a satisfactory knowledge of Algebra, including Fractions and Simple Equations,
   d. To have a satisfactory knowledge of the use of the Globes, with correct definitions of Latitude, Longitude, Azimuth, Amplitude, and the other Circles of the Sphere,
   e. To have a satisfactory knowledge of Vulgar and Decimal Fractions.
   f. To have a satisfactory knowledge of The First Book of Euclid,
   g. To have a satisfactory knowledge of A practical knowledge of the Elements of Plane Trigonometry, and its application to the Numerical Solution of Easy and Useful Problems.

The following exam covers 9th grade math, encompassing the above topics.

[28] Derived from Admiralty Circular 288 of 23 February 1857, found at: http://freepages.genealogy.rootsweb.ancestry.com/~pbtyc/Navy_List_1870/Mid_Exams.html.

Name: ______________________

Date: ______________________

Advisor: ______________________

Examination for Lieutenant

1. This examination will be given over five days. Midshipmen may not take the examination from the classroom overnight but may review and study each night to better complete the examination.

2. **ALL WORK MUST BE SHOWN ON SEPARATE SHEETS OF PAPER ATTACHED TO THE EXAMINATION PAPER.**

3. This examination is divided into seven sections:

   a. Operations with Real Numbers.
   b. Navigation Problems.
   c. Algebraic Equations.
   d. Operations with Exponents.
   e. Surface Area and Volume of 3-dimensional Shapes.
   f. Plane Geometry.
   g. Trigonometric Functions.

4. Each section has three parts:

   a. The first part of each section is "2-level problems." All "2-level problems" must be completed correctly to pass this examination.

   b. The second part of each section is "3-level problems." In addition to satisfactory completion of all "2-level problems", all "3-level problems" must be completed correctly to achieve a rating of "Good Standing" on this examination. No midshipman will be appointed to the ranks of "Captain" or "First Lieutenant" who has not achieved a rating of at least "Good Standing".

   c. The third part of each section is "4-level problems." In addition to satisfactory completion of all "2-level problems" and "3-level problems", all "4-level problems" must be completed correctly to achieve a rating of "Honors" on this examination.

Name: ___________________________

Date:____________________________

Advisor:____________________________

Examination for Lieutenant – Final Grades

Topic	**Grade**			
a. Operations with Real Numbers.	1	2	3	4
b. Navigation Problems.	1	2	3	4
c. Algebraic Equations.	1	2	3	4
d. Operations with Exponents	1	2	3	4
e. Surface Area and Volume of 3-dimensional Shapes.	1	2	3	4
f. Plane Geometry.	1	2	3	4
g. Trigonometric Functions.	1	2	3	4

Final Overall Grade ***Fail*** ***Partially Satisfactory*** ***Good Standing*** ***Honors***

Position Assigned **Captain** **1st Lt.** **Master** **Purser** **QM**

1. **Operations with Real Numbers.** The following problems involve addition, subtraction, multiplication and division of real numbers. Solve them without a calculator, *showing your work.*

* 2 – level Problems *

$-7 + (-15) =$ ______ $-5 + (-2) =$ ______ $8 + (-15) =$ ________

$28 + (-50) =$ ______ $-29 + 14 =$ ______ $-43 + (-82) =$ ______

$\frac{11}{25} + \left(-\frac{13}{25}\right) =$ ______ $-\frac{1}{2} + \frac{1}{3} =$ ______ $-\frac{3}{4} + \frac{1}{5} =$ ______

$12 - 7 =$ ______ $-9 - 4 =$ ______ $-57 - 14 =$ ______

* 3 – level Problems *

$\frac{1}{5} - \frac{4}{5} =$ ______ $\frac{3}{16} - \left(-\frac{7}{16}\right) =$ ______ $\frac{1}{2} - \frac{1}{3} =$ ______

$990 - (-155) =$ ______ $-30.4 - 2.05 =$ ______ $-64.5 + 65 =$ ______

$(-3)(-8) =$ ______ $(-5)(6) =$ ______ $(3)(-17) =$ ______

$1257 \div 1 =$ ______ $(1257)(0) =$ ______ $0 \div 3 =$ ______

$\left(\frac{1}{2}\right)\left(\frac{3}{4}\right) =$ ______ $\frac{1}{2} \div \frac{3}{4} =$ ______ $\left(\frac{1}{2}\right)\left(\frac{4}{3}\right) =$ ______

2. **Navigation Problems.** Solve these problems without a calculator, *showing your work on separate sheets of paper.*

   a. **2-level Problems.** Compute your ship's longitude for the dates below.

Date	GMT of LAN	Longitude
December 8th	20:08	
December 9th	20:13	
December 10th	20:15	
December 11th	20:19	

   b. **3-level Problems.** Compute ship's latitude at noon for the following:

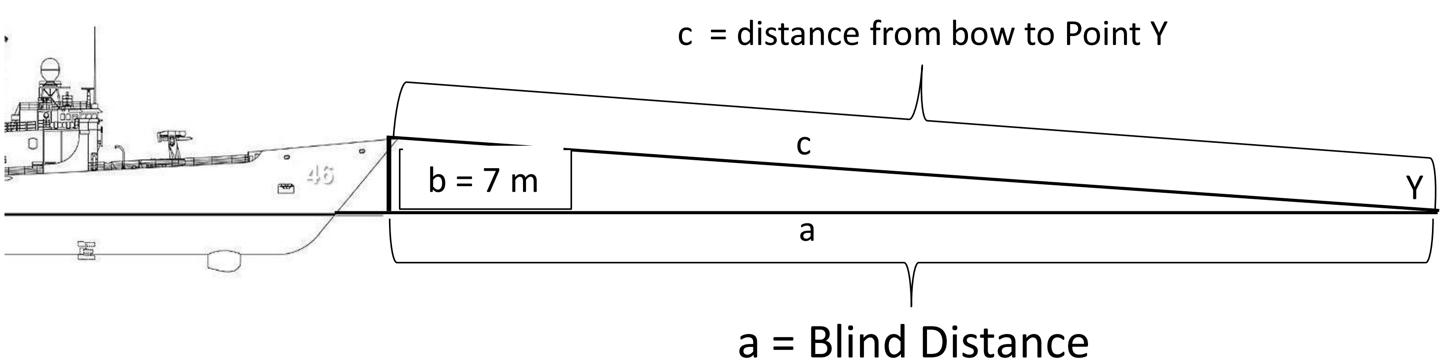
Date	Sun's Altitude	Sun's Declination	Latitude
Dec 8	55°55' N	22°46' S	
Dec 9	55°43' N	22°52' S	
Dec 10	55°32' N	22°57' S	
Dec 11	55°21' N	23°02' S	

Compute the average speed traveled on each watch in nautical miles per hour.

Time	Noon – 4 pm	4 – 8 pm
1 bell	70 yards	98 yards
2 bells	72.2 yards	93.5 yards
3 bells	74 yards	93.5 yards
4 bells	76.3 yards	89.2 yards
5 bells	81 yards	91 yards
6 bells	89.2 yards	88.4 yards
7 bells	96 yards	85 yards
8 bells	95.3 yards	85.2 yards
Average Speed		

c. **4-level Problem.** Find the blind distance (a) for the ship in the following diagram given $b = 7\ meters$ and $c = 8b\ meters$:

Blind distance in meters is: ____



3. **Algebraic Equations.** Solve these problems without a calculator, *showing your work on separate sheets of paper.*

   c. **2-level Problems.** Solve for the variable in each equation:

$x + 7 = 10$ $\quad$ $-9 + x = 5$ $\quad$ $k + 6.8 = -4.2$

$8x = 72$ $\quad$ $\frac{q}{7} = 8$ $\quad$ $-\frac{x}{8} = 44$

$66 = 2f - 4$ $\quad$ $-4 = \frac{x}{2} - 5$ $\quad$ $n - 6 = 2n - 14$

   d. **3-level Problems.** Solve for the variable in each equation:

$8x - 1.5 = 2x$ $\quad$ $6p - 12 = -4p + 18$ $\quad$ $-10x + 3 = -3x + 12 - 4x$

$5(x - 3) = 10$ $\quad$ $7x - 2(x + 6) = -2$ $\quad$ $3 - 4x = 5(x + 6)$

$x + \frac{5}{8} + \frac{3x}{4} = \frac{2}{3} + 5x$ $\quad$ $2x + 3 = 3x + 5$ $\quad$ $5m - 3(2m - 3) = 2(m + 3)$

c. 4-level Problem. Calculating Fuel Consumption

Background. Being very careful, a destroyer-type ship with a full load of fuel can remain at sea without refueling for approximately three weeks. It is the job of the ship's Chief Engineer (CHENG) and his engineers to monitor fuel use and report to the ship's Captain daily on how much has been burned, how fast it was burned and how much is left. With this information, the Captain can decide if he will keep his ship on its mission or take time out to refuel from a tanker at sea.

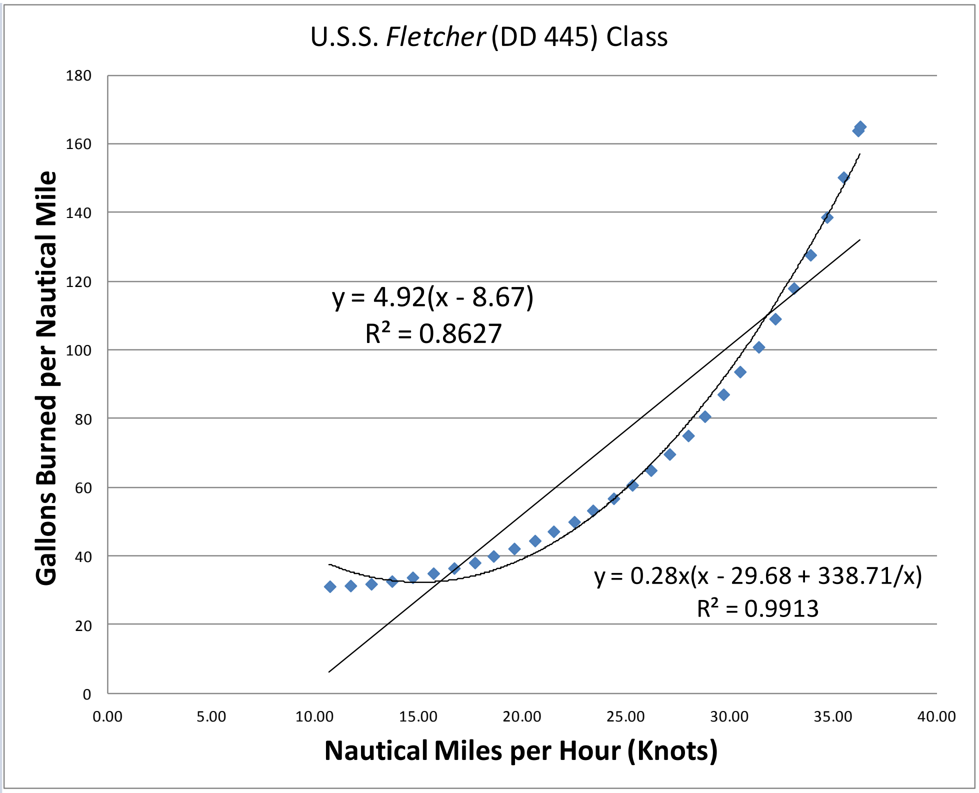
A typical destroyer is pictured below. The

U.S.S. *Fletcher* (DD 445)

The following table provides the data that the Chief Engineer and his engineers use to make their recommendations to the Captain.[29] The table shows that at a given speed in nautical miles per hour (knots), the ship will burn a certain number of gallons. This information is graphically depicted in the scatter plot next to the table.

[29] *From War Service Fuel Consumption of U.S. Naval Surface Vessels* (FTP 218), UNITED STATES FLEET, Headquarters of the Commander in Chief, 1945. Online at: http://www.ibiblio.org/hyperwar/USN/ref/Fuel/.

Knots	Gallons per NM
10.70	31.20
11.70	31.40
12.70	31.90
13.70	32.70
14.70	33.80
15.70	35.00
16.70	36.50
17.70	38.10
18.60	40.00
19.60	42.20
20.60	44.50
21.50	47.20
22.50	50.00
23.40	53.30
24.40	56.80
25.30	60.70
26.20	65.00
27.10	69.70
28.00	75.10
28.80	80.70
29.70	87.10
30.50	93.70
31.40	100.90
32.20	109.10
33.10	118.00
33.90	127.70
34.70	138.70
35.50	150.30
36.20	163.90
36.30	165.10



One line and one polynomial curve are fitted to the graph. Note that both have an R-squared value greater than 0.5. What this means is that either the linear equation or the curve's equation could be used to predict gallons burned for this class of destroyer. We will use both in the following exercises.

Linear Equation: $y = 4.92(x - 8.67)$

Polynomial Equation: $y = 0.28x(x - 29.68 + \frac{338.71}{x})$

Problem Set. Complete the table below by predicting the gallons burned per nautical mile for the given speeds using each equation.

Example: For $x = 10\ knots$ using the linear equation:

$$y_1 = 4.92(10 - 8.67) = 4.92(1.33) = 6.54\ gallons\ per\ nm.$$

$x = Knots$	$y_1 = \frac{Gallons}{NM} =$ $4.92(x - 8.67)$	$y_2 = \frac{Gallons}{NM} =$ $0.28x(x - 29.68 + \frac{338.71}{x})$	**Difference**
10			
15			
20			
25			
30			
35			
40			

Which equation provides a better predictor of fuel use (particularly at lower and high speeds? Why?

____

____

____

____

____

____

____

4. **Operations with Exponents.**

a. Complete these problems correctly for a “2”.

i. $(6^3)(6^4)$ ii. $(2y^{12}z^6)(7y^9z^2)$ iii. $(5r^6)^2$ iv. 56^0

b. In addition to problems in part a, complete these problems correctly for a “3”.

Simplify.

i. $\frac{3^8}{3^2}$ ii. $(-7x^{12}y^8)^4$ iii. $\frac{18p^9q^{11}}{3p^6q^6}$ iv. $\frac{\left(g^6h^7i^{10}\right)^3}{g^3h^4i^2}$

Write each number in scientific notation or decimal notation.

v. 6,510,000 vi. 8.5×10^4 vii. 16,567,832

c. In addition to problems in parts 1 and 2, complete this problems correctly for a “4”.

i. Does the equation $y = 9 * (1 - 0.65)^t$ model exponential growth or exponential decay? Why?

5. **Surface Area and Volume of 3-dimensional Shapes.** Solve these problems using a calculator if you choose, *showing your work on separate sheets of paper.*

* 2 – level Problems *

B. **Definitions (Matching).**

vii.	Chord:	_____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.
viii.	Diameter:	_____ The distance around a closed curve such as a circle.
ix.	Volume:	_____ Any line segment from the center of a circle to its perimeter.
x.	Radius:	_____ The sum of the areas of the surfaces of a three-dimensional object.
xi.	Surface Area:	_____ The amount of space, inside and out, a solid body occupies.
xii.	Circumference:	_____ Line segment whose endpoints lie on the circle.

C. **Formulas.**

i. What is the formula for the **circumference** of a circle? ________________

ii. What is the formula for the **area** of a circle? ________________

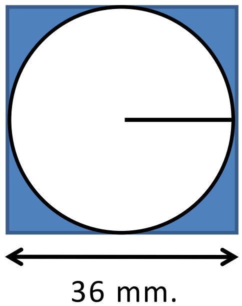
iii. What is the formula for the **surface area** of a rectangular prism? ________________

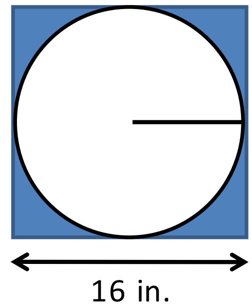
iv. What is the formula for the **volume** of a rectangular prism? ________________

v. What is the formula for the **surface area** of a cylinder? ________________

vi. What is the general formula for the **surface area** of a regular prism? ________________

D. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)





a. Circle Area: _______

b. Square Area: _______

c. Ratio: _______

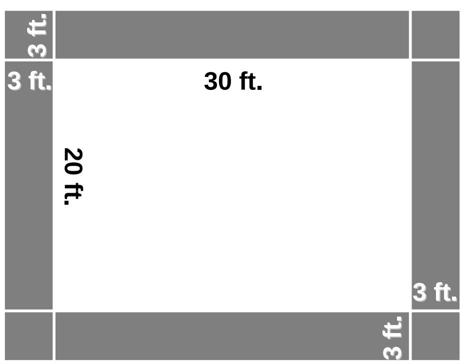
d. Circle Area: _______

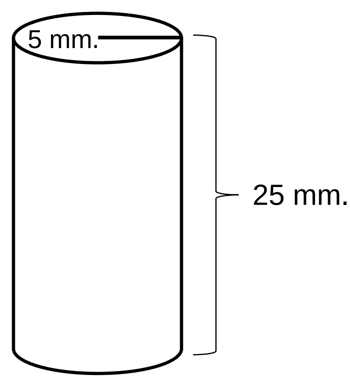
e. Square Area: _______

f. Ratio: _______

* 3 – level Problems *

E. Find the area and volume (if asked for) of the following shapes:





a. Total Area: _______

b. Area of Shaded Region: _______

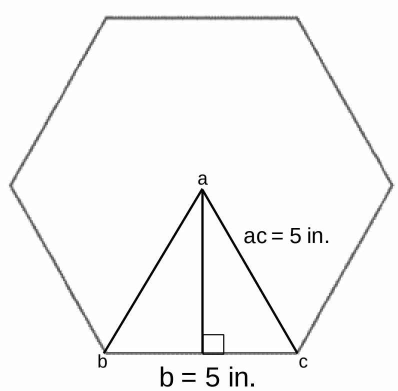
c. Surface Area: _______

d. Volume: _______

Regular hexagonal prism with this hexagon as its base and a height of 40 in.

e. Surface Area: _______

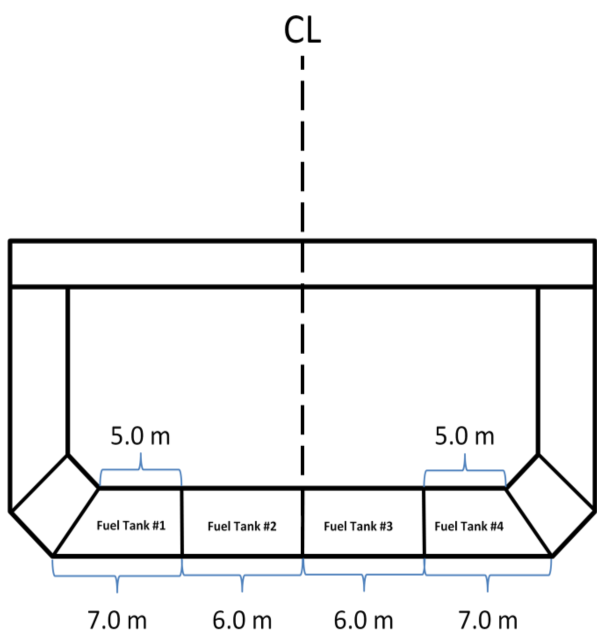
f. Volume: _______



* 4 – level Problems *
Complete the following table:

Container Ship Load Planning Sheet

Hold Number	Hold Length	Hold Width	Hold Height	Hold Volume	Water Necessary to Fill Hold
1	20.0 m	5.0 m / 7.0 m	4.0 m		
2	20.0 m	6.0 m	4.0 m		
3	20.0 m	6.0 m	4.0 m		
4	20.0 m	5.0 m / 7.0 m	4.0 m		



Height of all tanks is 4.0 m
Length of all tanks is 20.0 m
1 cu. m. = 1000 liters

6. **Plane Geometry.**

a. **2-level Problems.** Indicate whether the following statements are **True** or **False**.

- Things which equal the same thing also equal one another. **True / False**

- Even if equals are added to equals, then the wholes man not be equal. **True / False**

- The whole is greater than the part. **True / False**

- The sum of the interior angles of a triangle is equal to 360°. **True / False**

- An acute angle has a measure greater than 90°. **True / False**

- The length of the hypotenuse of a right triangle is equal to the square root of the sum of the squares of the other two sides. **True / False**

- The angles opposite the congruent sides of an isosceles triangle are also congruent. **True / False**

- If two straight lines intersect, then the sum of their vertical angles equals 180°. **True / False**

- A circle is uniquely defined by its radius. **True / False**

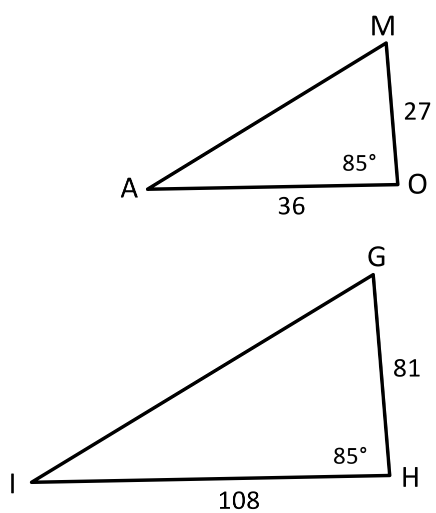
- A tangent to a circle is a line that intersects the circle at more than one point. **True / False**

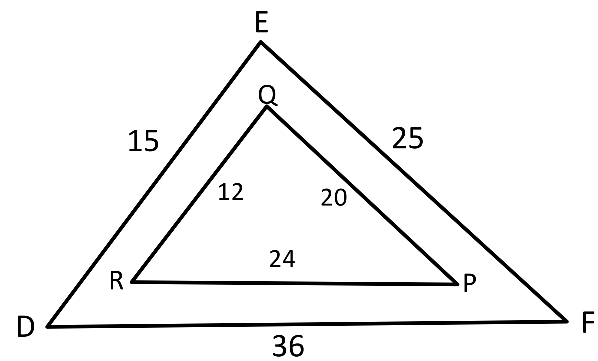
- A central angle is an angle whose sides are chords of the circle. **True / False**

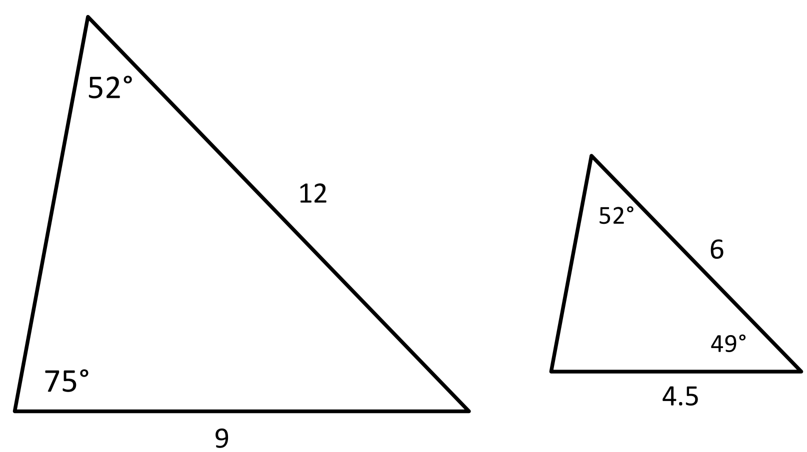
b. **3-level Problems.**

Determining Similarity. For the given triangles:

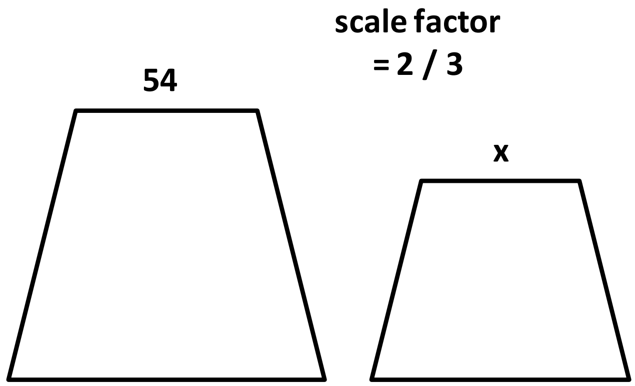
- State whether they are similar.
- If they are, state what triangle similarity postulate or theorem supports your conclusion.
- If not, state why not.
- Justify your conclusion by indicating corresponding angles and / showing proportionality of the sides.

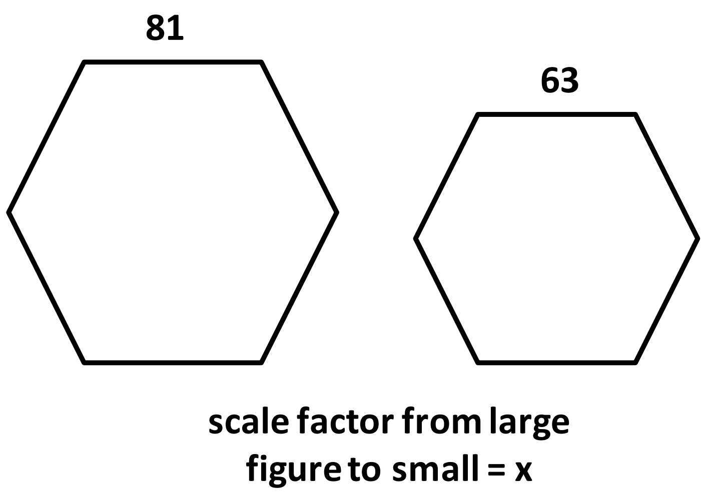






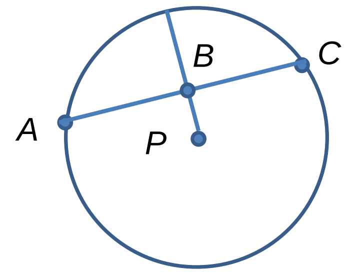
Scale Factors. In each pair, the polygons are similar. Find either the scale factor or the indicated side length.





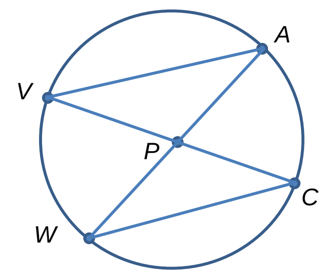
Circles.

Chord AC is perpendicular to the radius. What is the relationship between segments AB and AC?



____

Arc AC = 60°, arc VA = 120°. Find the measures of the following angles:

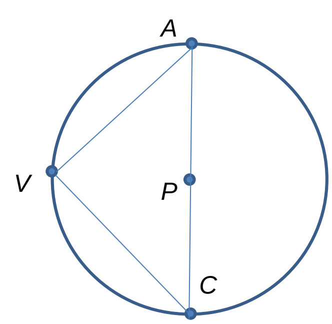


i. ∠AVC: ______________________________________

ii. ∠APV: ______________________________________

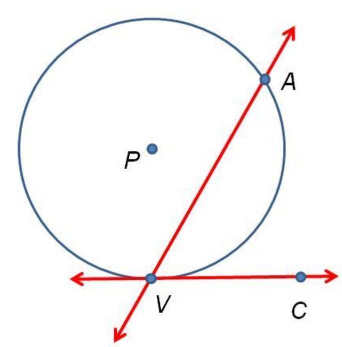
iii. ∠CPW: ____________________________________

iv. ∠WPV: ____________________________________



If segment AC is a diameter, what is the measure of ∠AVC? What is the measure of arc AC?

____

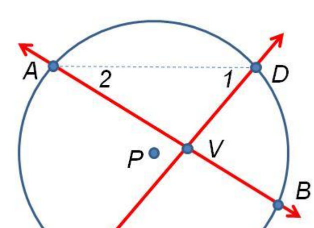


If arc AV = 120°, what is the measure of ∠AVC?

____

Point V is called the __________________________________

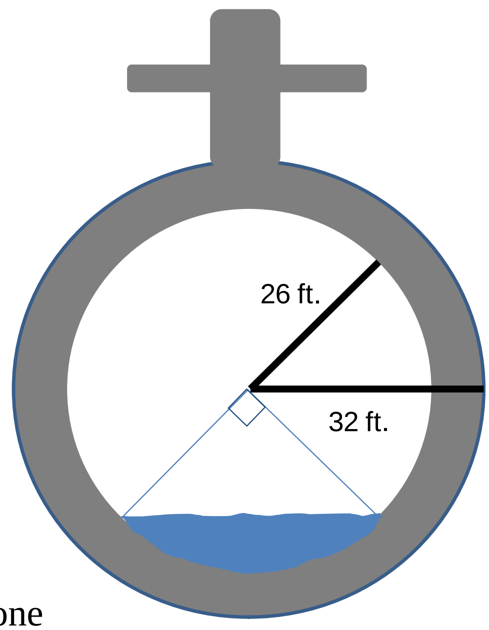
The measure of arc AC = 110° and the measure of arc DB =



70°. What is the measure of ∠DVB? ________________

What is the measure of ∠BVC? ____________________

c. **4-level Problem.** Solve the following three problems concerning a flooded submarine.



i. If the reactor / engine compartment is 100 ft. long, what is the volume of the flooded portion?

____

ii. How many gallons of water flooded into the compartment?
(7.48 gallons per cu. ft.)

____

iii. What was the added weight to the submarine? (one gallon weighs 8.35 pounds)

____

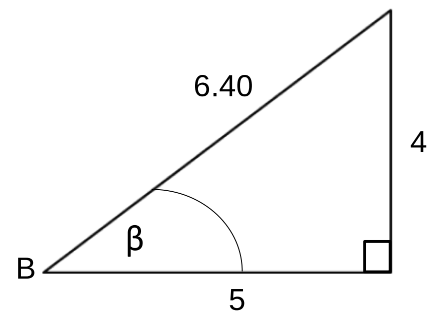
7. **Trigonometry.**

2-level problems. Find the sine (sin), cosine (cos) and tangent (tan) for each triangle.

a. sin β = ________

cos β = ________

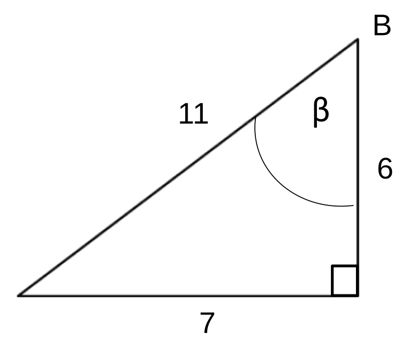
tan β = ________



b. sin β = ________

cos β = ________

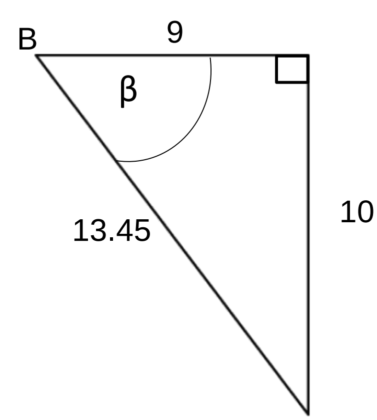
tan β = ________



c. sin β = ________

cos β = ________

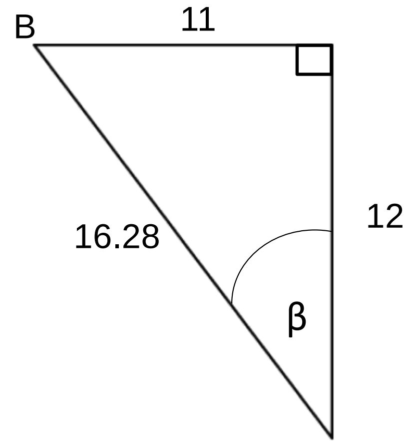
m∠B = ________



d. sin β = ________

tan β = ________

m∠B = ________

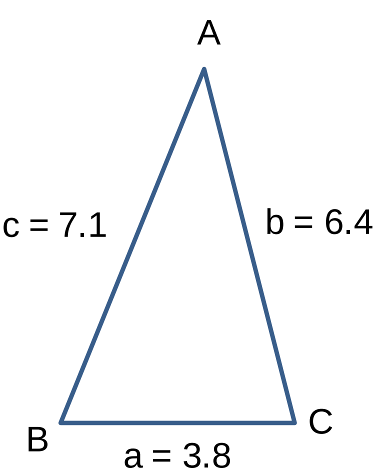


3-level Problems. Find the indicated measures using the Law of Cosines and / or Law of Sines.

a. $a = 1$ $b = 13$ $c =$ ________ $m\angle C = 20°$

b. $a = 4$ $b = 7$ $c = 5$ $m\angle C =$ ________

4-level Problem. Solve the triangle.

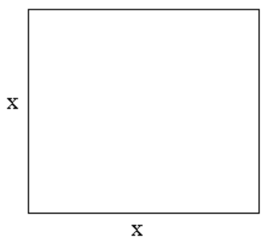


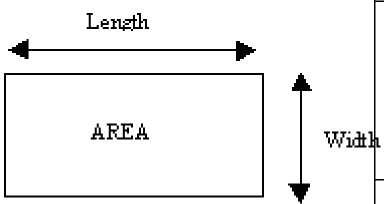
a. $m\angle A =$ ________________

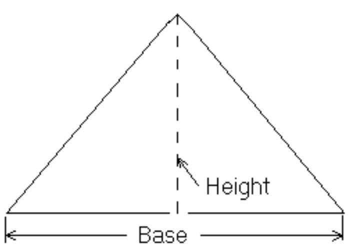
b. $m\angle B =$ ________________

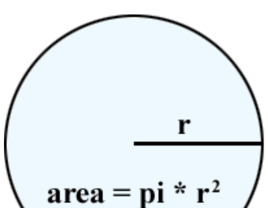
c. $m\angle C =$ ________________

Formulas

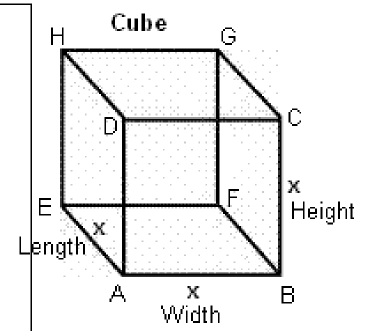


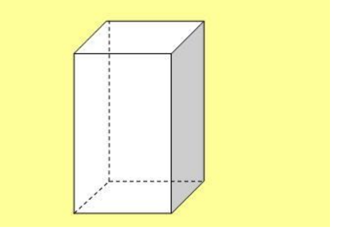


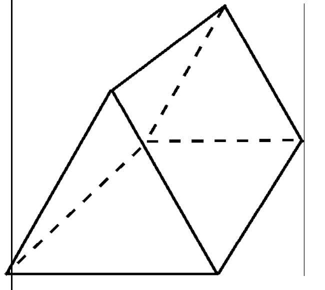


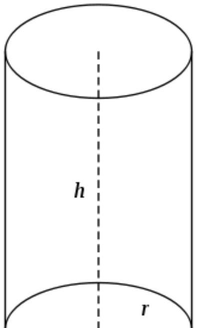


Area of a Square:	$\boldsymbol{A = s^2}$ where s = the length of a side.	***Right Square Prism:*** • $\boldsymbol{SA = 2(s^2) + 4(sH)}$ • $\boldsymbol{V = s^2 H}$ ...where s = the length of a side and H = height of the prism. ***Right Square Pyramid:*** • $\boldsymbol{SA = s^2 + 4\left(\frac{1}{2}b \times sh\right)}$ • $\boldsymbol{V = \frac{1}{3}sH}$ ...where s = the length of a side and H = height of the prism.
Area of a Rectangle:	$\boldsymbol{A = l \times w}$ where l = the length of the rectangle and w = the width.	***Right Rectangular Prism:*** • $\boldsymbol{SA = 2\big((lw) + (lH) + (wH)\big) = 2B + PH}$ • $\boldsymbol{V = l \times w \times H = PH}$ ...where l = the length, w = the width, P = perimeter of the base, H = height of the prism.
Area of a Triangle:	$\boldsymbol{A = \frac{1}{2}bh}$ where b = the base of the triangle and h = the height of the triangle.	***Right Triangular Prism:*** • $\boldsymbol{SA = 2\left(\frac{1}{2}bh\right) + (S_1 + S_2 + S_2)H}$ • $\boldsymbol{V = \left(\frac{1}{2}bh\right)H}$ ...where S_i is the length of side i, b is the length of the base of the base triangle, h is the height of the base triangle, and H is the height of the prism. ***Right Triangular Pyramid:*** • $\boldsymbol{SA = \frac{1}{2}bh + 3(\frac{1}{2}b \times sh)}$ • $\boldsymbol{V = \frac{1}{3}\left(\frac{1}{2}bh\right)H}$ ...b is the length of the base of the base triangle, h is the height of the base triangle, sh is the height of each side triangle, and H is the height of the pyramid.
Area of a Circle:	$\boldsymbol{A = \pi r^2}$ where r = the radius of the circle.	***Right Cylinder:*** • $\boldsymbol{SA = 2\pi r^2 + 2\pi rH}$

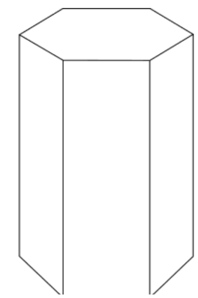


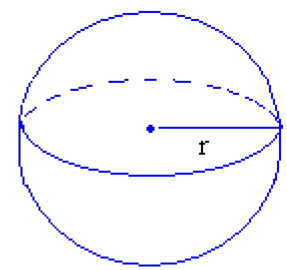


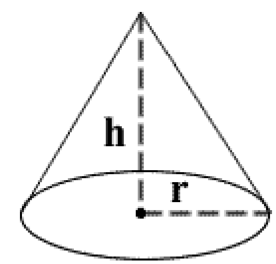




Circumference: $\boldsymbol{C = 2\pi r}$ ***Arc Length:*** $L = \frac{M}{360}°(2\pi r)$	• $\boldsymbol{V = \pi r^2 H}$ ...where r is the radius of the base circle and H is the height of the cylinder. ***Right Cone:*** • $\boldsymbol{SA = \pi r^2 + \pi r \times \sqrt{r^2 + H^2}}$ • $\boldsymbol{V = \frac{1}{3}\pi r^2 H}$ ...where r is the radius of the base circle and H is the height of the cylinder. ***Sphere:*** • $\boldsymbol{SA = 4\pi r^2}$ • $\boldsymbol{V = \frac{4}{3}\pi r^3}$ ...where r is the radius of the sphere.
Area of a Regular Hexagon: $\boldsymbol{A = \frac{1}{2}aP}$ where a = the apothem and P = the perimeter of the base.	***Right Regular Hexagonal Prism:*** • $\boldsymbol{SA = 2 \times (\frac{1}{2}aP) + PH}$ • $\boldsymbol{V = \frac{1}{2}aP \times H}$ ...where a = the apothem, P = the perimeter of the base, and H = the height of the prism.
General Formulas	***Prism and Cylinder:*** • $\boldsymbol{SA = 2B + PH}$ • $\boldsymbol{V = BH}$ ...where B is the area of the base, P is the perimeter of the base, and H is the height of the prism. ***Pyramid:*** • $\boldsymbol{SA = B + N(\frac{1}{2}b \times sh)}$ • $\boldsymbol{V = \frac{1}{3}BH}$ ...where B is the area of the base, N is the number of sides, b is the length of the base of a side triangle, and sh is the slant height of the side triangle, and H is the height of the pyramid.
Trigonometric Formulas	***Law of Sines***

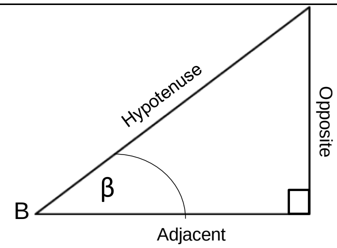




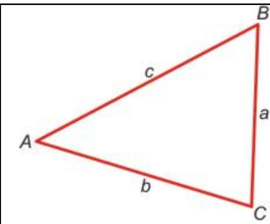


$$\sin\beta = \frac{opposite}{hypotenuse}$$

$$\cos\beta = \frac{adjacent}{hypotenuse}$$



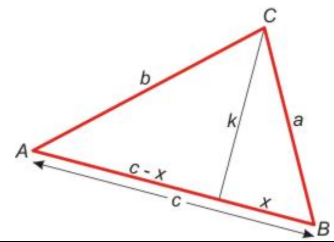
$$\frac{\sin\beta}{\cos\beta} = \frac{\frac{opposite}{hypotenuse}}{\frac{adjacent}{hypotenuse}} = \frac{opposite}{adjacent} = \tan\beta$$



For ***any triangle*** ΔABC with sides *a*, *b*, and *c*:

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

Law of Cosines



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Determining Latitude:

$$Latitude\ (at\ noon) = 90° - Sun's\ altitude + Sun's\ declination$$
(Sun over Northern Hemisphere)

$$Latitude\ (at\ noon) = 90° - Sun's\ altitude - Sun's\ declination$$
(Sun over Southern Hemisphere)

$$Latitude\ (at\ night\ in\ Northern\ Hemisphere) = Altitude\ of\ Polaris$$

Determining Longitude

For east longitude:

(1) $12{:}00 - Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) = h{:}mm$

(2) $h \times \frac{15°}{hour} = degrees\ east\ of\ 0°\ longitude$

(3) $mm \div \frac{4'}{1°} = additional\ degrees\ east\ of\ 0°\ longitude$

(4) $add\ the\ two\ degrees\ east\ together\ to\ get\ ship's\ longitude\ east$

For west longitude:

(1) $Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) - 12{:}00 = h{:}mm$

(2) $h \times \frac{15°}{hour} = degrees\ west\ of\ 0°\ longitude$

(3) $mm \div \frac{4'}{1°} = additional\ degrees\ west\ of\ 0°\ longitude$

(4) $add\ the\ two\ degrees\ west\ together\ to\ get\ ship's\ longitude\ west$

Answer Key

Operations	Navigation			Equations		Exponents
-22.00	122.00		3.00			6^7
-7.00	123.25		14.00			14y^21z^8
-7.00	123.75		-11.00			25r^12
-22.00	124.75		9.00			1.00
-15.00	34.05	11.19	56.00			
-125.00	34.17	11.25	-352.00			3^6
-0.08	34.28	11.31	35.00			-7^x^48y^32'
-0.17	34.39	11.37	2.00			6p^3q^5
-0.55			8.00			g^15h^17i^28
5.00	4.91					
-13.00	5.43		0.25			
-71.00			3.00			
	55.56		-3.00			
-0.60			5.00			
0.63			2.00			
0.17			-3.00			
1145.00						
-32.45			-2.00			
0.50						
24.00			10.00	6.54	39.73	
-30.00			15.00	31.14	33.18	
-51.00			20.00	55.74	40.63	
1257.00			25.00	80.34	62.08	
0.00			30.00	104.94	97.53	
0.00			35.00	129.54	146.97	
0.38			40.00	154.14	210.42	
0.67						
0.67						
3D Shapes						
1017.36	200.96	19266.00				
1296.00	256.00	144109.68				
0.79	0.79	1203315.83				
936.00	942.00					
336.00	1962.50					
1329.90						
2598.00						
80.00	480000.00					

Examination for Lieutenant
Reflection

Answer these questions completely in your journal. Your journal grade will be based, in part, on how seriously you take writing reflections such as this one.

1. Which section did you find easiest?

2. Which section did you find most difficult?

3. Would a classroom review help you to improve in any particular topic? Which topic?

4. How difficult was it to remain focused on the examination?

5. What did you do to remain focused?

6. What would have helped you to remain focused?

Journal

All midshipmen, regardless of rank shall keep a daily journal which will constitute one-half of the final unit grade. The journal will include the following information:

1. Date: dd mmm yy, i.e. 01 Sep 14
2. Altitude of the Sun (provided)
3. Declination of the Sun (from http://www.wsanford.com/~wsanford/exo/sundials/DEC_Sun.html)
4. Latitude
5. Longitude
6. Time of Sunrise, Local Apparent Noon, and Sunset (from http://aa.usno.navy.mil/data/docs/RS_OneDay.php)
7. Class work
8. Reflection on day's class work

The journal will be maintained in a bound government record book, formatted by the student, as shown below:

Date	Sun's Altitude	Sun's Declination	Latitude	Longitude	Sunrise	GMT of Local Apparent Noon	Sunset	Class Work and Daily Reflection
								This section will include all work shown to solve latitude and longitude problems as well as daily class work problems. It will also include a daily, brief reflection on the work done in class on the given day.

Journal Grading Rubric

	4	3	2	1
Daily Entry	• Today's date properly noted. • All warm-up problems completed correctly. • All classwork problems completed correctly.	• Today's date properly noted. • All warm-up problems completed correctly. • All classwork problems attempted (not just copied down).	• Date is inconsistently entered. • All warm-up problems attempted. • All classwork problems attempted (not just copied down).	• Today's date is not entered or... • Student fails to attempt either the warm-up and / or the classwork.
Neatness and Professionalism	Journal is exceptionally neat and well-organized, making it easy and pleasurable to read.	Journal is neat and well organized making it easy to read.	Journal is legible but occasionally entries are written sloppily making them difficult to read.	Student obviously shows little or no interest in marinating a neat and professional journal.

Daily Grade is average of the two scores from above rubric.

Daily Grade = (Daily Entry + Neatness and Professionalism) / 2

Captain's Weekly Grade assigned by group "Captain" is average of the two scores from the above rubric and the Daily Grades.

Captain's Weekly Grade = (Daily Entry + Neatness and Professionalism + 4 Daily Grades) / 6

Bi-weekly Grade is average of Captain's Weekly Grades for previous two weeks and my grade.

Final Journal Grade is 1/2 your trimester grade.

Unit 1: Logarithms

I. **Lesson I: Introduction.**

A. **Mini-lesson 1.**[30] In Algebra I, we learned about exponents. For example:

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

Suppose, however, that instead of trying to find what 2 raised to the 4th power equaled, we wanted to find what power we needed to raise 2 to in order to get 16:

$$2^x = 16$$

We already know the answer, of course:

$$2^x = 16 \Rightarrow x = 4$$

Another way to write this would be in logarithmic notation:

$$log_2 16 = x \text{ or } log_2 16 = 4$$

[30] Refer to Kahn Academy lesson at: https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/logarithms.

B. **Class Work Exercise 1 (copy the problems and record your answers in your journal):** Change the following exponential equations into logarithmic equations.

1. $2^3 = 8$ $\qquad$ $log_2 8 = 3$

2. $2^5 = 32$ $\qquad$ ________________

3. $2^7 = 128$ $\qquad$ ________________

4. $3^4 = 81$ $\qquad$ $log_3 81 = 4$

5. $3^6 = 2187$ $\qquad$ ________________

6. $4^3 = 64$ $\qquad$ ________________

7. $5^{10} = 9765625$ $\qquad$ ________________

C. **Mini-lesson 2.**[31] Of course, it is also possible to convert logarithmic equations to exponential equations.

$$log_3 81 = 4 \Rightarrow 3^4 = 81$$

Writing the expression $log_3 81$ is the same thing as asking the question, "*To what power do I have to raise* 3 *to get* 81*?*"

If we don't know the answer to the question, we could employ trial and error:

$$log_4 256 = x$$

We would say this as, "*To what power to I need to raise* 4 *to get* 256*?*"

[31] Refer to Kahn Academy lesson at: https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/logarithms.

$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$

So $log_4 256 = 4$.

D. **Class Work Exercise 2 (copy the problems and record your answers in your journal):** Convert the logarithmic expression into words and find the value of *x* in each logarithmic equation.

1. $log_6 1296 = x$

   To what power do I have to raise 6 *to get* 1296*?*
   $6^1 = 6$
   $6^2 = 36$
   $6^3 = 216$
   $6^4 = 1296$

2. $log_7 49 = x$

3. $log_3 243 = x$

4. $log_4 4096 = x$

5. $log_{10} 100000 = x$

6. $log_{27} 19683 = x$

7. $log_{100} 10000 = x$

8. $log_{100} 1 = x$

D. **Mini-lesson 3.**[32]

1. How would we evaluate:

$$log_8 2 = x$$

To what power do I have to raise 8 *to get* 2*?*

$$8^x = 2$$

Recall that $2^3 = 8$. This implies that:

$$8^{\frac{1}{3}} = 2$$

Therefore, $x = \frac{1}{3}$.

2. Suppose we were asked to evaluate:

$$log_2\left(\frac{1}{8}\right) = x$$

To what power do I have to raise 2 *to get* $\frac{1}{8}$*?*

$$2^x = \frac{1}{8}$$

We know that $2^3 = 8$. Therefore:

$$\frac{1}{2^3} = \frac{1}{8}$$

Recall from Algebra I that:

[32] Refer to https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/fancier-logarithm-expressions.

$$\frac{1}{2^3} = 2^{-3}$$

so:

$$2^{-3} = \frac{1}{8}$$

and, therefore,

$$log_2\left(\frac{1}{8}\right) = -3$$

3. Suppose we were asked to evaluate:

$$log_8\left(\frac{1}{2}\right) = x$$

To what power do I have to raise 8 *to get* $\frac{1}{2}$*?*

$$8^x = \frac{1}{2}$$

We know that $8^{\frac{1}{3}} = 2$. Therefore:

$$\frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}$$

Recall from Algebra I that:

$$\frac{1}{8^{\frac{1}{3}}} = 8^{-\frac{1}{3}}$$

so:

$$8^{-\frac{1}{3}} = \frac{1}{2}$$

and, therefore,

$$log_8\left(\frac{1}{2}\right) = -\frac{1}{3}$$

E. **Class Work Exercise 3 (copy the problems and record your answers in your journal).** Convert the logarithmic expression into words and find the value of *x* in each logarithmic equation.

1. $log_9 3 = x$

   To what power do I have to raise 9 *to get* 3*?*

$$9^x = 3$$
$$9^{\frac{1}{3}} = 3$$
$$x = \frac{1}{3}$$

2. $log_4 \frac{1}{256} = x$

3. $log_{10} \frac{1}{10000} = x$

4. $log_4 \frac{1}{16} = x$

5. $log_{16} 4 = x$

6. $log_{16} \frac{1}{4} = x$

7. $log_{256}\frac{1}{16} = x$

F. **Mini-lesson 4: Solving exponential equations with logarithms.**[33]

How would we solve for *t* in the following equation:

$$10^{2t-3} = 7$$

...which, in words, can be expressed as, "10 *raised to the power of* $2t$ – 3 *equals* 7."

We can start by rewriting the equation in its logarithmic form by first asking the question, "*To what power do I have to raise* 10 *to get* 7?" We already know that the answer is $2t - 3$ so we can rewrite the exponential equation in its logarithmic form as:

$$log_{10}7 = 2t - 3$$

Notice that now we can solve for *t* as in any algebraic equation:

$$\begin{array}{c} log_{10}7 = 2t - 3 \\ \underline{\quad +3 \qquad\quad +3} \end{array}$$

$$\frac{(log_{10}7 + 3)}{2} = \frac{2t}{2}$$

$$\frac{log_{10}7 + 3}{2} = t$$

G. **Class Work Exercise 4 (copy the problems and record your answers in your journal).** Solve for the variable in each exponential equation by converting to logarithmic form to solve.

[33] https://www.khanacademy.org/math/algebra2/logarithms-tutorial/logarithm_basics/v/exponential-equation.

Unit 1: Logarithms

1. $10^{2t-3} = 7$

2. $8^{4t-7} = 12$

3. $14^{8r+17} = 1000$

4. $123678^{1023x+150000} = .00000045$

5. $\frac{3}{56}^{9-14u} = 765423$

II. Lesson II: Historical Use of Logarithms.

A. **Mini-lesson 1:** Logarithms were used before the development of calculators to ease numerical calculation by substituting multiplication and division of large numbers with addition and subtraction of small numbers. The most common system of logarithms used were those founded on base 10 with all numbers are regarded to be powers of 10. Therefore,

$10^0 = 1$	0 is the logarithm of 1
$10^1 = 10$	1 is the logarithm of 10
$10^2 = 100$	2 is the logarithm of 100
$10^3 = 1000$	3 is the logarithm of 1000
$10^4 = 10000$	4 is the logarithm of 10000

et cetera.

It stands to reason, therefore, that the logarithms of numbers between 1 and 10 are between 0 and 1, the logarithms of numbers between 10 and 100 are between 1 and 2, and so on.

For numbers less than 0, logarithms are expressed as negative numbers.

$10^{-1} = 0.1$	-1 is the logarithm of 0.1
$10^{-2} = 0.01$	-2 is the logarithm of 0.01
$10^{-3} = 0.001$	-3 is the logarithm of 0.001
$10^{-4} = 0.0001$	-4 is the logarithm of 0.0001

et cetera.

In the absence of calculators, pre-printed tables were normally used to determine the logarithm for any number. For example, Table 1 displays logarithms for all numbers between 1 and 360.

Unit 1: Logarithms

N	LOG	N	LOG	N	LOG	N	LOG	N	LOG	N	LOG
1	0	31	1.491362	61	1.78533	91	1.959041	121	2.082785	151	2.178977
2	0.30103	32	1.50515	62	1.792392	92	1.963788	122	2.08636	152	2.181844
3	0.477121	33	1.518514	63	1.799341	93	1.968483	123	2.089905	153	2.184691
4	0.60206	34	1.531479	64	1.80618	94	1.973128	124	2.093422	154	2.187521
5	0.69897	35	1.544068	65	1.812913	95	1.977724	125	2.09691	155	2.190332
6	0.778151	36	1.556303	66	1.819544	96	1.982271	126	2.100371	156	2.193125
7	0.845098	37	1.568202	67	1.826075	97	1.986772	127	2.103804	157	2.1959
8	0.90309	38	1.579784	68	1.832509	98	1.991226	128	2.10721	158	2.198657
9	0.954243	39	1.591065	69	1.838849	99	1.995635	129	2.11059	159	2.201397
10	1	40	1.60206	70	1.845098	100	2	130	2.113943	160	2.20412
11	1.041393	41	1.612784	71	1.851258	101	2.004321	131	2.117271	161	2.206826
12	1.079181	42	1.623249	72	1.857332	102	2.0086	132	2.120574	162	2.209515
13	1.113943	43	1.633468	73	1.863323	103	2.012837	133	2.123852	163	2.212188
14	1.146128	44	1.643453	74	1.869232	104	2.017033	134	2.127105	164	2.214844
15	1.176091	45	1.653213	75	1.875061	105	2.021189	135	2.130334	165	2.217484
16	1.20412	46	1.662758	76	1.880814	106	2.025306	136	2.133539	166	2.220108
17	1.230449	47	1.672098	77	1.886491	107	2.029384	137	2.136721	167	2.222716
18	1.255273	48	1.681241	78	1.892095	108	2.033424	138	2.139879	168	2.225309
19	1.278754	49	1.690196	79	1.897627	109	2.037426	139	2.143015	169	2.227887
20	1.30103	50	1.69897	80	1.90309	110	2.041393	140	2.146128	170	2.230449
21	1.322219	51	1.70757	81	1.908485	111	2.045323	141	2.149219	171	2.232996
22	1.342423	52	1.716003	82	1.913814	112	2.049218	142	2.152288	172	2.235528
23	1.361728	53	1.724276	83	1.919078	113	2.053078	143	2.155336	173	2.238046
24	1.380211	54	1.732394	84	1.924279	114	2.056905	144	2.158362	174	2.240549
25	1.39794	55	1.740363	85	1.929419	115	2.060698	145	2.161368	175	2.243038
26	1.414973	56	1.748188	86	1.934498	116	2.064458	146	2.164353	176	2.245513
27	1.431364	57	1.755875	87	1.939519	117	2.068186	147	2.167317	177	2.247973
28	1.447158	58	1.763428	88	1.944483	118	2.071882	148	2.170262	178	2.25042
29	1.462398	59	1.770852	89	1.94939	119	2.075547	149	2.173186	179	2.252853
30	1.477121	60	1.778151	90	1.954243	120	2.079181	150	2.176091	180	2.255273
N	LOG	N	LOG	N	LOG	N	LOG	N	LOG	N	LOG
181	2.257679	211	2.324282	241	2.382017	271	2.432969	301	2.478566	331	2.519828
182	2.260071	212	2.326336	242	2.383815	272	2.434569	302	2.480007	332	2.521138
183	2.262451	213	2.32838	243	2.385606	273	2.436163	303	2.481443	333	2.522444
184	2.264818	214	2.330414	244	2.38739	274	2.437751	304	2.482874	334	2.523746
185	2.267172	215	2.332438	245	2.389166	275	2.439333	305	2.4843	335	2.525045
186	2.269513	216	2.334454	246	2.390935	276	2.440909	306	2.485721	336	2.526339
187	2.271842	217	2.33646	247	2.392697	277	2.44248	307	2.487138	337	2.52763
188	2.274158	218	2.338456	248	2.394452	278	2.444045	308	2.488551	338	2.528917
189	2.276462	219	2.340444	249	2.396199	279	2.445604	309	2.489958	339	2.5302
190	2.278754	220	2.342423	250	2.39794	280	2.447158	310	2.491362	340	2.531479
191	2.281033	221	2.344392	251	2.399674	281	2.448706	311	2.49276	341	2.532754
192	2.283301	222	2.346353	252	2.401401	282	2.450249	312	2.494155	342	2.534026
193	2.285557	223	2.348305	253	2.403121	283	2.451786	313	2.495544	343	2.535294
194	2.287802	224	2.350248	254	2.404834	284	2.453318	314	2.49693	344	2.536558
195	2.290035	225	2.352183	255	2.40654	285	2.454845	315	2.498311	345	2.537819
196	2.292256	226	2.354108	256	2.40824	286	2.456366	316	2.499687	346	2.539076
197	2.294466	227	2.356026	257	2.409933	287	2.457882	317	2.501059	347	2.540329
198	2.296665	228	2.357935	258	2.41162	288	2.459392	318	2.502427	348	2.541579
199	2.298853	229	2.359835	259	2.4133	289	2.460898	319	2.503791	349	2.542825
200	2.30103	230	2.361728	260	2.414973	290	2.462398	320	2.50515	350	2.544068
201	2.303196	231	2.363612	261	2.416641	291	2.463893	321	2.506505	351	2.545307
202	2.305351	232	2.365488	262	2.418301	292	2.465383	322	2.507856	352	2.546543
203	2.307496	233	2.367356	263	2.419956	293	2.466868	323	2.509203	353	2.547775
204	2.30963	234	2.369216	264	2.421604	294	2.468347	324	2.510545	354	2.549003
205	2.311754	235	2.371068	265	2.423246	295	2.469822	325	2.511883	355	2.550228
206	2.313867	236	2.372912	266	2.424882	296	2.471292	326	2.513218	356	2.55145
207	2.31597	237	2.374748	267	2.426511	297	2.472756	327	2.514548	357	2.552668
208	2.318063	238	2.376577	268	2.428135	298	2.474216	328	2.515874	358	2.553883
209	2.320146	239	2.378398	269	2.429752	299	2.475671	329	2.517196	359	2.555094
210	2.322219	240	2.380211	270	2.431364	300	2.477121	330	2.518514	360	2.556303

Table 1: Logarithms Between 1 and 360

B. **Class Work Exercise 1: (copy the problems and record your answers in your journal).** Using table 1, find the logarithms of the following numbers:

1. 265
2. 123
3. 357
4. 21
5. 86
6. 198
7. 312
8. 59
9. 66
10. 360

C. **Mini-lesson 2: Interpolating Non-Integer Logarithms from Tables.**

To find the logarithm of a simple or vulgar fraction,[34] we find the difference between the logarithm of the numerator and the denominator. For example:

$$\log_{10}\left(\frac{3}{16}\right) = \log_{10}(3) - \log_{10}(16) = 0.477121 - 1.20412 = -0.726999$$

To find the logarithm of a non-integer number, we first must learn the following rule:

The integer to the left of the decimal point of a logarithm is equal to one less than the number of integers in the original number.

For example, given the number 175 which has three integers, “1” and “7” and “5”, the number to the left of the decimal point in the logarithm will be $3 - 1 = 2$. From table 1, we can see that $\log_{10}(175) = 2.243038$.

To determine the logarithm of 17.5, we find the logarithm of 175 and change the integer to the left of the decimal point to reflect the number of integer numbers to the left of the decimal in 17.5.

[34] A simple fraction, also known as a vulgar fraction is one in which the numerator and the denominator are both integers.

Unit 1: Logarithms

1. $\log_{10}(175) = 2.243038$.

2. Number of integers to left of the decimal point in 17.5 is 2. $2 - 1 = 1$.

3. $\log_{10}(17.5) = 1.243038$.

D. **Class Work Exercise 2: (copy the problems and record your answers in your journal).** Using Table 1, find the number which corresponds to the logarithm indicated.

1. $\log_{10}(25) =$ _______	16. $\log_{10}\left(\frac{3}{5}\right) =$ _______	28. $\log_{10}(3.5) =$ _______
2. $\log_{10}(37) =$ _______	17. $\log_{10}\left(\frac{12}{17}\right) =$ _______	29. $\log_{10}(28.9) =$ _______
3. $\log_{10}(18) =$ _______	18. $\log_{10}\left(\frac{85}{105}\right) =$ _______	30. $\log_{10}(1.87) =$ _______
4. $\log_{10}(41) =$ _______	19. $\log_{10}\left(\frac{6}{98}\right) =$ _______	31. $\log_{10}(23.9) =$ _______
5. $\log_{10}(127) =$ _______	20. $\log_{10}\left(\frac{12}{5}\right) =$ _______	32. $\log_{10}(32.9) =$ _______
6. $\log_{10}(238) =$ _______	21. $\log_{10}\left(\frac{56}{12}\right) =$ _______	33. $\log_{10}(17.0) =$ _______
7. $\log_{10}(25) =$ _______	22. $\log_{10}\left(\frac{42}{6}\right) =$ _______	34. $\log_{10}(.142) =$ _______
8. $\log_{10}(341) =$ _______	23. $\log_{10}\left(\frac{18}{36}\right) =$ _______	35. $\log_{10}(12.7) =$ _______
9. $\log_{10}(201) =$ _______	24. $\log_{10}\left(\frac{75}{225}\right) =$ _______	36. $\log_{10}(33.6) =$ _______
10. $\log_{10}(2) =$ _______	25. $\log_{10}\left(\frac{87}{4}\right) =$ _______	37. $\log_{10}(1.97) =$ _______
11. $\log_{10}(65) =$ _______	26. $\log_{10}\left(\frac{23}{52}\right) =$ _______	38. $\log_{10}(.98) =$ _______
12. $\log_{10}(95) =$ _______	27. $\log_{10}\left(\frac{88}{11}\right) =$ _______	39. $\log_{10}(.04) =$ _______
13. $\log_{10}(72) =$ _______		40. $\log_{10}(27.9) =$ _______
14. $\log_{10}(102) =$ _______		41. $\log_{10}(34.1) =$ _______
15. $\log_{10}(18) =$ _______		42. $\log_{10}(1.85) =$ _______

E. Mini-lesson 3: Interpolating Logarithms from a Table.[35]

The Table of Logarithms depicted on the next several pages will be used for all subsequent problems.

N	0	1	2	3	4	5	6	7	8	9
1.00	0	0.000434	0.000868	0.001301	0.001734	0.002166	0.002598	0.003029	0.003461	0.003891
1.01	0.004321	0.004751	0.005181	0.005609	0.006038	0.006466	0.006894	0.007321	0.007748	0.008174
1.02	0.0086	0.009026	0.009451	0.009876	0.0103	0.010724	0.011147	0.01157	0.011993	0.012415
1.03	0.012837	0.013259	0.01368	0.0141	0.014521	0.01494	0.01536	0.015779	0.016197	0.016616
1.04	0.017033	0.017451	0.017868	0.018284	0.0187	0.019116	0.019532	0.019947	0.020361	0.020775
1.05	0.021189	0.021603	0.022016	0.022428	0.022841	0.023252	0.023664	0.024075	0.024486	0.024896
1.06	0.025306	0.025715	0.026125	0.026533	0.026942	0.02735	0.027757	0.028164	0.028571	0.028978
1.07	0.029384	0.029789	0.030195	0.0306	0.031004	0.031408	0.031812	0.032216	0.032619	0.033021
1.08	0.033424	0.033826	0.034227	0.034628	0.035029	0.03543	0.03583	0.03623	0.036629	0.037028
1.09	0.037426	0.037825	0.038223	0.03862	0.039017	0.039414	0.039811	0.040207	0.040602	0.040998
1.10	0.041393	0.045323	0.049218	0.053078	0.056905	0.060698	0.064458	0.068186	0.071882	0.075547
1.20	0.079181	0.082785	0.08636	0.089905	0.093422	0.09691	0.100371	0.103804	0.10721	0.11059
1.30	0.113943	0.117271	0.120574	0.123852	0.127105	0.130334	0.133539	0.136721	0.139879	0.143015
1.40	0.146128	0.149219	0.152288	0.155336	0.158362	0.161368	0.164353	0.167317	0.170262	0.173186
1.50	0.176091	0.178977	0.181844	0.184691	0.187521	0.190332	0.193125	0.1959	0.198657	0.201397
1.60	0.20412	0.206826	0.209515	0.212188	0.214844	0.217484	0.220108	0.222716	0.225309	0.227887
1.70	0.230449	0.232996	0.235528	0.238046	0.240549	0.243038	0.245513	0.247973	0.25042	0.252853
1.80	0.255273	0.257679	0.260071	0.262451	0.264818	0.267172	0.269513	0.271842	0.274158	0.276462
1.90	0.278754	0.281033	0.283301	0.285557	0.287802	0.290035	0.292256	0.294466	0.296665	0.298853
2.00	0.30103	0.303196	0.305351	0.307496	0.30963	0.311754	0.313867	0.31597	0.318063	0.320146
2.10	0.322219	0.324282	0.326336	0.32838	0.330414	0.332438	0.334454	0.33646	0.338456	0.340444
2.20	0.342423	0.344392	0.346353	0.348305	0.350248	0.352183	0.354108	0.356026	0.357935	0.359835
2.30	0.361728	0.363612	0.365488	0.367356	0.369216	0.371068	0.372912	0.374748	0.376577	0.378398
2.40	0.380211	0.382017	0.383815	0.385606	0.38739	0.389166	0.390935	0.392697	0.394452	0.396199
2.50	0.39794	0.399674	0.401401	0.403121	0.404834	0.40654	0.40824	0.409933	0.41162	0.4133
2.60	0.414973	0.416641	0.418301	0.419956	0.421604	0.423246	0.424882	0.426511	0.428135	0.429752
2.70	0.431364	0.432969	0.434569	0.436163	0.437751	0.439333	0.440909	0.44248	0.444045	0.445604
2.80	0.447158	0.448706	0.450249	0.451786	0.453318	0.454845	0.456366	0.457882	0.459392	0.460898
2.90	0.462398	0.463893	0.465383	0.466868	0.468347	0.469822	0.471292	0.472756	0.474216	0.475671
3.00	0.477121	0.478566	0.480007	0.481443	0.482874	0.4843	0.485721	0.487138	0.488551	0.489958
3.10	0.491362	0.49276	0.494155	0.495544	0.49693	0.498311	0.499687	0.501059	0.502427	0.503791
3.20	0.50515	0.506505	0.507856	0.509203	0.510545	0.511883	0.513218	0.514548	0.515874	0.517196
3.30	0.518514	0.519828	0.521138	0.522444	0.523746	0.525045	0.526339	0.52763	0.528917	0.5302
3.40	0.531479	0.532754	0.534026	0.535294	0.536558	0.537819	0.539076	0.540329	0.541579	0.542825

[35] http://k12math.com/math-concepts/algebra/logarithms/log_tables_slide_rule.htm.

3.50	0.544068	0.545307	0.546543	0.547775	0.549003	0.550228	0.55145	0.552668	0.553883	0.555094
3.60	0.556303	0.557507	0.558709	0.559907	0.561101	0.562293	0.563481	0.564666	0.565848	0.567026
3.70	0.568202	0.569374	0.570543	0.571709	0.572872	0.574031	0.575188	0.576341	0.577492	0.578639
3.80	0.579784	0.580925	0.582063	0.583199	0.584331	0.585461	0.586587	0.587711	0.588832	0.58995
3.90	0.591065	0.592177	0.593286	0.594393	0.595496	0.596597	0.597695	0.598791	0.599883	0.600973
4.00	0.60206	0.603144	0.604226	0.605305	0.606381	0.607455	0.608526	0.609594	0.61066	0.611723
4.10	0.612784	0.613842	0.614897	0.61595	0.617	0.618048	0.619093	0.620136	0.621176	0.622214
4.20	0.623249	0.624282	0.625312	0.62634	0.627366	0.628389	0.62941	0.630428	0.631444	0.632457
4.30	0.633468	0.634477	0.635484	0.636488	0.63749	0.638489	0.639486	0.640481	0.641474	0.642465
4.40	0.643453	0.644439	0.645422	0.646404	0.647383	0.64836	0.649335	0.650308	0.651278	0.652246
4.50	0.653213	0.654177	0.655138	0.656098	0.657056	0.658011	0.658965	0.659916	0.660865	0.661813
4.60	0.662758	0.663701	0.664642	0.665581	0.666518	0.667453	0.668386	0.669317	0.670246	0.671173
4.70	0.672098	0.673021	0.673942	0.674861	0.675778	0.676694	0.677607	0.678518	0.679428	0.680336
4.80	0.681241	0.682145	0.683047	0.683947	0.684845	0.685742	0.686636	0.687529	0.68842	0.689309
4.90	0.690196	0.691081	0.691965	0.692847	0.693727	0.694605	0.695482	0.696356	0.697229	0.698101
5.00	0.69897	0.699838	0.700704	0.701568	0.702431	0.703291	0.704151	0.705008	0.705864	0.706718
5.10	0.70757	0.708421	0.70927	0.710117	0.710963	0.711807	0.71265	0.713491	0.71433	0.715167
5.20	0.716003	0.716838	0.717671	0.718502	0.719331	0.720159	0.720986	0.721811	0.722634	0.723456
5.30	0.724276	0.725095	0.725912	0.726727	0.727541	0.728354	0.729165	0.729974	0.730782	0.731589
5.40	0.732394	0.733197	0.733999	0.7348	0.735599	0.736397	0.737193	0.737987	0.738781	0.739572
5.50	0.740363	0.741152	0.741939	0.742725	0.74351	0.744293	0.745075	0.745855	0.746634	0.747412
5.60	0.748188	0.748963	0.749736	0.750508	0.751279	0.752048	0.752816	0.753583	0.754348	0.755112
5.70	0.755875	0.756636	0.757396	0.758155	0.758912	0.759668	0.760422	0.761176	0.761928	0.762679
5.80	0.763428	0.764176	0.764923	0.765669	0.766413	0.767156	0.767898	0.768638	0.769377	0.770115
5.90	0.770852	0.771587	0.772322	0.773055	0.773786	0.774517	0.775246	0.775974	0.776701	0.777427
6.00	0.778151	0.778874	0.779596	0.780317	0.781037	0.781755	0.782473	0.783189	0.783904	0.784617
6.10	0.78533	0.786041	0.786751	0.78746	0.788168	0.788875	0.789581	0.790285	0.790988	0.791691
6.20	0.792392	0.793092	0.79379	0.794488	0.795185	0.79588	0.796574	0.797268	0.79796	0.798651
6.30	0.799341	0.800029	0.800717	0.801404	0.802089	0.802774	0.803457	0.804139	0.804821	0.805501
6.40	0.80618	0.806858	0.807535	0.808211	0.808886	0.80956	0.810233	0.810904	0.811575	0.812245
6.50	0.812913	0.813581	0.814248	0.814913	0.815578	0.816241	0.816904	0.817565	0.818226	0.818885
6.60	0.819544	0.820201	0.820858	0.821514	0.822168	0.822822	0.823474	0.824126	0.824776	0.825426
6.70	0.826075	0.826723	0.827369	0.828015	0.82866	0.829304	0.829947	0.830589	0.83123	0.83187
6.80	0.832509	0.833147	0.833784	0.834421	0.835056	0.835691	0.836324	0.836957	0.837588	0.838219
6.90	0.838849	0.839478	0.840106	0.840733	0.841359	0.841985	0.842609	0.843233	0.843855	0.844477
7.00	0.845098	0.845718	0.846337	0.846955	0.847573	0.848189	0.848805	0.849419	0.850033	0.850646
7.10	0.851258	0.85187	0.85248	0.85309	0.853698	0.854306	0.854913	0.855519	0.856124	0.856729
7.20	0.857332	0.857935	0.858537	0.859138	0.859739	0.860338	0.860937	0.861534	0.862131	0.862728
7.30	0.863323	0.863917	0.864511	0.865104	0.865696	0.866287	0.866878	0.867467	0.868056	0.868644
7.40	0.869232	0.869818	0.870404	0.870989	0.871573	0.872156	0.872739	0.873321	0.873902	0.874482
7.50	0.875061	0.87564	0.876218	0.876795	0.877371	0.877947	0.878522	0.879096	0.879669	0.880242
7.60	0.880814	0.881385	0.881955	0.882525	0.883093	0.883661	0.884229	0.884795	0.885361	0.885926
7.70	0.886491	0.887054	0.887617	0.888179	0.888741	0.889302	0.889862	0.890421	0.89098	0.891537

7.80	0.892095	0.892651	0.893207	0.893762	0.894316	0.89487	0.895423	0.895975	0.896526	0.897077
7.90	0.897627	0.898176	0.898725	0.899273	0.899821	0.900367	0.900913	0.901458	0.902003	0.902547
8.00	0.90309	0.903633	0.904174	0.904716	0.905256	0.905796	0.906335	0.906874	0.907411	0.907949
8.10	0.908485	0.909021	0.909556	0.910091	0.910624	0.911158	0.91169	0.912222	0.912753	0.913284
8.20	0.913814	0.914343	0.914872	0.9154	0.915927	0.916454	0.91698	0.917506	0.91803	0.918555
8.30	0.919078	0.919601	0.920123	0.920645	0.921166	0.921686	0.922206	0.922725	0.923244	0.923762
8.40	0.924279	0.924796	0.925312	0.925828	0.926342	0.926857	0.92737	0.927883	0.928396	0.928908
8.50	0.929419	0.92993	0.93044	0.930949	0.931458	0.931966	0.932474	0.932981	0.933487	0.933993
8.60	0.934498	0.935003	0.935507	0.936011	0.936514	0.937016	0.937518	0.938019	0.93852	0.93902
8.70	0.939519	0.940018	0.940516	0.941014	0.941511	0.942008	0.942504	0.943	0.943495	0.943989
8.80	0.944483	0.944976	0.945469	0.945961	0.946452	0.946943	0.947434	0.947924	0.948413	0.948902
8.90	0.94939	0.949878	0.950365	0.950851	0.951338	0.951823	0.952308	0.952792	0.953276	0.95376
9.00	0.954243	0.954725	0.955207	0.955688	0.956168	0.956649	0.957128	0.957607	0.958086	0.958564
9.10	0.959041	0.959518	0.959995	0.960471	0.960946	0.961421	0.961895	0.962369	0.962843	0.963316
9.20	0.963788	0.96426	0.964731	0.965202	0.965672	0.966142	0.966611	0.96708	0.967548	0.968016
9.30	0.968483	0.96895	0.969416	0.969882	0.970347	0.970812	0.971276	0.97174	0.972203	0.972666
9.40	0.973128	0.97359	0.974051	0.974512	0.974972	0.975432	0.975891	0.97635	0.976808	0.977266
9.50	0.977724	0.978181	0.978637	0.979093	0.979548	0.980003	0.980458	0.980912	0.981366	0.981819
9.60	0.982271	0.982723	0.983175	0.983626	0.984077	0.984527	0.984977	0.985426	0.985875	0.986324
9.70	0.986772	0.987219	0.987666	0.988113	0.988559	0.989005	0.98945	0.989895	0.990339	0.990783
9.80	0.991226	0.991669	0.992111	0.992554	0.992995	0.993436	0.993877	0.994317	0.994757	0.995196
9.90	0.995635	0.996074	0.996512	0.996949	0.997386	0.997823	0.998259	0.998695	0.999131	0.999565
10.00	1	1.000434	1.000868	1.001301	1.001734	1.002166	1.002598	1.003029	1.003461	1.003891

Table 2: Table of Logarithms between 1.000 and 1.009 and between 1.01 and 10.00

Suppose we want to find a logarithm for a number outside the range of the table – 345. To find its logarithm, we first rewrite the number in scientific notation:

$$345 = 3.45 \times 10^2$$

Recall from above that the logarithm of a product equals the sum of the logarithms of the factors:

$$log_{10}(3.45 \times 10^2) = log_{10}(3.45) + log_{10}(10^2)$$

From the table, we know that $log_{10}(3.45) = 0.537819$ and $log_{10}(10^2) = 2$, so:

Unit 1: Logarithms

$$log_{10}(345) = 2.537819$$

Note that:

$$log_{10}(3.45) = 0.537819$$

$$log_{10}(34.5) = 0.537819 + log_{10}(10^1) = 1.537819$$

$$log_{10}(345) = 0.537819 + log_{10}(10^2) = 2.537819$$

$$log_{10}(3450) = 0.537819 + log_{10}(10^3) = 3.537819$$

What relationship do you see between the original numbers, the powers of 10 in each equation and the logarithm answers?

The original number is 3.45 raised to a power of 10, the power of 10 in the equation is the power of 10 to which 3.45 has been raised, and the integer to the left of the decimal in each answer is the power of 10 to which the original number was raised. This integer is called the logarithm's *characteristic.*

Note also that the integers to the right of the decimal in each answer are the same as in all the other answers. This series of integers is called the logarithm's *mantissa*.

$$\underbrace{2.}_{\textbf{\textit{characteristic}}}\underbrace{537819}_{\textbf{\textit{mantissa}}}$$

F. **Class Work Exercise 3: (copy the problems and record your answers in your journal).** Use table 2 to find the logarithms of the following numbers. In your answers, identify the characteristic and the mantissa.

1. 575
2. 38
3. 967
4. 103
5. 1030
6. 10300

G. **Mini-lesson 4: Adding and Subtracting Logarithms**.[36]

Before the advent of electronic calculators, logarithms were used to simplify complex calculations. For example, suppose we want to multiply 175 and 231 together. Without a calculator, this involves a number of calculation steps. Suppose, however, we converted each number to its base 10 logarithm. Using table 2 we find that:

$$log_{10}(175) = log_{10}(1.75 \times 10^2) = log_{10}(1.75) + log_{10}(10^2) = 0.243038 + 2 = 2.243038$$

$$log_{10}(231) = log_{10}(231 \times 10^2) = log_{10}(231) + log_{10}(10^2) = 0.363612 + 2 = 2.363612$$

To multiply 175 and 231 together, we use the property of logarithmic addition:

$$log_{10}(ab) = log_{10}(a) + log_{10}(b)$$

so we can find 175×231 using their logarithms:

$$log_{10}(175 \times 231) = log_{10}(175) + log_{10}(231)$$

$$2.243038 + 2.363612 = 4.60665$$

[36] http://k12math.com/math-concepts/algebra/logarithms/log_tables_slide_rule.htm.

The next step, converting the answer from a logarithm to its original number will be taken up in the next lesson.

H. **Class Work Exercise 4: (copy the problems and record your answers in your journal).** Find the logarithm of the product of the two factors given using Table 2.

1. 657×238
2. 160×345
3. 98×230
4. 7210×127
5. 1234×67

I. **Mini-lesson 5:** To find the answer to the product of 175 and 231, we need to convert the logarithm 4.60665 to its original number. We know that the characteristic is 4 so the answer will be a number $\times 10^4$. We go to table 2 and find those numbers closest to the mantissa 0.60665. This number falls between 0.659916 and 0.660865 or between the numbers 4.04 and 4.05.

So the product of 175 and 231 falls between 4.04×10^4 and 4.05×10^4 or between 40400 and 40500. The product as found on your calculator is 40425.

Division of large numbers works the same way. Let's divide 231 by 175 using logarithms. To do that, we use the property of logarithmic subtraction:

$$log_{10}(a \div b) = log_{10}(a) - log_{10}(b)$$

$$log_{10}(231 \div 175) = log_{10}(231) - log_{10}(175)$$

$$2.363612 - 2.243038 = 0.120574$$

From table 2, we find that the number associated with the logarithm 0.120574 is exactly 1.32. Using a calculator, we find that dividing 231 and 175, we find:

$$231 \div 175 = 1.32$$

J. **Class Work Exercise 5: (copy the problems and record your answers in your journal).** Find either the exact value of the quotient, or, from table 2, the values the quotient falls between.

1. $167 \div 45$
2. $1256 \div 456$
3. $37 \div 2$
4. $14567 \div 800$
5. $97 \div 14$

K. **Mini-lesson 6: Converting Logarithms Back to Natural Numbers.** The final step in making logarithms useful is converting them back to natural numbers after we have performed addition or subtraction on them. For example, suppose we added two logarithms together and found that they equaled 5.892095. Referring to our Table of Logarithms, we would look for the mantissa of this number and find that the natural number corresponding to 0.892095 is 7.80. Since the characteristic of the original logarithm is 5, to find the final answer, multiply 7.80 by 10^5:

$$7.80 \times 10^5 = 78000$$

Recalling a previous example, we found that, to find the product of 175 and 231, we could convert both numbers to logarithms and find the logarithmic sum:

$$log_{10}(175 \times 231) = log_{10}(175) + log_{10}(231)$$

$$2.243038 + 2.363612 = 4.606650$$

The final step is to convert the answer, 4.606650, back into a natural number. Consulting our Table of Logarithms, we find that the mantissa, 0.606650 is not present. We proceed as follows:

1. Find the two mantissas on either side of 0.60665.

$$0.606381 < 0.606650 < 0.607455$$

2. Find the difference between 0.606381 and 0.607455.

$$0.607455 - 0.606381 = 0.001074$$

3. Find the difference between 0.606381 and 0.606650.

$$0.606650 - 0.606381 = 0.000269$$

4. Divide the second difference by the first difference.

$$0.000269 \div 0.001074 = 0.25$$

5. Find the natural numbers corresponding to 0.606381 and 0.607455. These are 4.04 and 4.05. Therefore, we know that:

$$4.04 < 10^{0.60665} < 4.05$$

   Where $10^{0.60665}$ is the natural number corresponding to the logarithm 0.60666.

6. Our next step is to multiply the difference of 4.04 and 4.05 by 0.25 and add that result to 4.04.

$$0.25 \times (4.05 - 4.04) + 4.04 = 4.0425$$

7. Since the characteristic of our original logarithm, 4.606650, is 4, we multiply 4.0425 by 10^4 to get our final answer:

$$4.0425 \times 10^4 = 40425$$

8. We can check our final answer by multiplying 175 and 231 on a calculator.

$$175 \times 231 = 40425$$

L. **Class Work Exercise 6: (copy the problems and record your answers in your journal).** Find the natural number products of the following using logarithms.

1. 657×238
2. 160×345
3. 98×230
4. 7210×127

M. **Mini-lesson 7: Logarithms and Navigation.**[37] Recall from our course in basic navigation that, in the Northern Hemisphere, given the Sun's declination on a given day and its observed altitude above the horizon at the observer's local apparent noon (LAN), we could determine our latitude as follows:

$$Latitude = 90° - Sun's\ altitude + Sun's\ declination$$

(Sun over Northern Hemisphere)

$$Latitude = 90° - Sun's\ altitude - Sun's\ declination$$

(Sun over Southern Hemisphere)

[37] Nathaniel Bowditch, *The New American Practical Navigator*, 3rd edition (New York, Edmund M. Blunt, 1811).

For example, if we are sailing in the Northern Hemisphere and, on a given day when the Sun's declination is 19° 47' S and we observe the Sun at LAN to be 40° 13' above the horizon, we would calculate the latitude as follows:

$$\begin{array}{r} 90^\circ = 89^\circ\, 60' \\ \underline{\quad -40^\circ\, 13'} \\ 49^\circ\, 47' \\ \underline{\quad -19^\circ\, 47'} \\ 30^\circ\, 00'\ \text{N} \end{array}$$

Unfortunately, this only works if we can observe the Sun at LAN. If, however, the Sun is obscured at that time, we must resort to one of the more complex methods such as determining latitude using two solar altitude observations taken at times other than LAN when the Sun is visible.

In order to accomplish more complex calculations, navigators relied upon tables of trigonometric functions such as table 3 below. In this table, find the degree of latitude down the left hand column, and the minutes across the top. For example, the sine of 8° 12' is 0.142629.

x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	0	0.001745	0.003491	0.00524	0.006981	0.008727	0.01047	0.01222	0.01396	0.01571
1	0.01745	0.019197	0.020942	0.02269	0.024432	0.026177	0.02792	0.02967	0.03141	0.03316
2	0.0349	0.036644	0.038388	0.04013	0.041876	0.043619	0.04536	0.04711	0.04885	0.05059
3	0.05234	0.054079	0.055822	0.05756	0.059306	0.061049	0.06279	0.06453	0.06627	0.06802
4	0.06976	0.071497	0.073238	0.07498	0.076719	0.078459	0.0802	0.08194	0.08368	0.08542
5	0.08716	0.088894	0.090633	0.09237	0.094108	0.095846	0.09758	0.09932	0.10106	0.10279
6	0.10453	0.106264	0.107999	0.10973	0.111469	0.113203	0.11494	0.11667	0.1184	0.12014
7	0.12187	0.123601	0.125333	0.12707	0.128796	0.130526	0.13226	0.13399	0.13572	0.13745
8	0.13917	0.140901	0.142629	0.14436	0.146083	0.147809	0.14954	0.15126	0.15299	0.15471
9	0.15643	0.158158	0.159881	0.1616	0.163326	0.165048	0.16677	0.16849	0.17021	0.17193
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	0.17365	0.175367	0.177085	0.1788	0.180519	0.182236	0.18395	0.18567	0.18738	0.1891
11	0.19081	0.192522	0.194234	0.19595	0.197657	0.199368	0.20108	0.20279	0.2045	0.2062
12	0.20791	0.209619	0.211325	0.21303	0.214735	0.21644	0.21814	0.21985	0.22155	0.22325

13	0.22495	0.226651	0.228351	0.23005	0.231748	0.233445	0.23514	0.23684	0.23853	0.24023
14	0.24192	0.243615	0.245307	0.247	0.24869	0.25038	0.25207	0.25376	0.25545	0.25713
15	0.25882	0.260505	0.262189	0.26387	0.265556	0.267238	0.26892	0.2706	0.27228	0.27396
16	0.27564	0.277315	0.278991	0.28067	0.282341	0.284015	0.28569	0.28736	0.28903	0.2907
17	0.29237	0.29404	0.295708	0.29738	0.299041	0.300706	0.30237	0.30403	0.3057	0.30736
18	0.30902	0.310676	0.312335	0.31399	0.315649	0.317305	0.31896	0.32061	0.32227	0.32392
19	0.32557	0.327218	0.328867	0.33051	0.332161	0.333807	0.33545	0.3371	0.33874	0.34038
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	0.34202	0.34366	0.345298	0.34694	0.348572	0.350207	0.35184	0.35348	0.35511	0.35674
21	0.35837	0.359997	0.361625	0.36325	0.364877	0.366501	0.36813	0.36975	0.37137	0.37299
22	0.37461	0.376224	0.377841	0.37946	0.38107	0.382683	0.3843	0.38591	0.38752	0.38912
23	0.39073	0.392337	0.393942	0.39555	0.397148	0.398749	0.40035	0.40195	0.40355	0.40514
24	0.40674	0.40833	0.409923	0.41151	0.413104	0.414693	0.41628	0.41787	0.41945	0.42104
25	0.42262	0.424199	0.425779	0.42736	0.428935	0.430511	0.43209	0.43366	0.43523	0.4368
26	0.43837	0.439939	0.441506	0.44307	0.444635	0.446198	0.44776	0.44932	0.45088	0.45244
27	0.45399	0.455545	0.457098	0.45865	0.4602	0.461749	0.4633	0.46484	0.46639	0.46793
28	0.46947	0.471012	0.472551	0.47409	0.475624	0.477159	0.47869	0.48022	0.48175	0.48328
29	0.48481	0.486335	0.48786	0.48938	0.490904	0.492424	0.49394	0.49546	0.49697	0.49849
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
30	0.5	0.501511	0.50302	0.50453	0.506034	0.507538	0.50904	0.51054	0.51204	0.51354
31	0.51504	0.516533	0.518027	0.51952	0.52101	0.522499	0.52399	0.52547	0.52696	0.52844
32	0.52992	0.531399	0.532876	0.53435	0.535827	0.5373	0.53877	0.54024	0.54171	0.54317
33	0.54464	0.546102	0.547563	0.54902	0.550481	0.551937	0.55339	0.55484	0.5563	0.55775
34	0.55919	0.560639	0.562083	0.56353	0.564967	0.566406	0.56784	0.56928	0.57071	0.57215
35	0.57358	0.575005	0.576432	0.57786	0.579281	0.580703	0.58212	0.58354	0.58496	0.58637
36	0.58779	0.589196	0.590606	0.59201	0.593419	0.594823	0.59623	0.59763	0.59902	0.60042
37	0.60182	0.603208	0.604599	0.60599	0.607376	0.608761	0.61015	0.61153	0.61291	0.61429
38	0.61566	0.617036	0.618408	0.61978	0.621148	0.622515	0.62388	0.62524	0.6266	0.62796
39	0.62932	0.630676	0.632029	0.63338	0.634731	0.636078	0.63742	0.63877	0.64011	0.64145
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	0.64279	0.644124	0.645458	0.64679	0.64812	0.649448	0.65077	0.6521	0.65342	0.65474
41	0.65606	0.657375	0.658689	0.66	0.661312	0.66262	0.66393	0.66523	0.66653	0.66783
42	0.66913	0.670427	0.671721	0.67301	0.674302	0.67559	0.67688	0.67816	0.67944	0.68072
43	0.682	0.683274	0.684547	0.68582	0.687088	0.688355	0.68962	0.69088	0.69214	0.6934
44	0.69466	0.695913	0.697165	0.69842	0.699663	0.700909	0.70215	0.7034	0.70463	0.70587
45	0.70711	0.70834	0.709571	0.7108	0.712026	0.71325	0.71447	0.71569	0.71691	0.71813
46	0.71934	0.720551	0.72176	0.72297	0.724172	0.725374	0.72658	0.72777	0.72897	0.73016
47	0.73135	0.732543	0.73373	0.73492	0.736097	0.737277	0.73846	0.73963	0.74081	0.74198

48	0.74315	0.744312	0.745476	0.74664	0.747798	0.748956	0.75011	0.75126	0.75242	0.75356
49	0.75471	0.755853	0.756995	0.75813	0.759271	0.760406	0.76154	0.76267	0.7638	0.76492
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	0.76604	0.767165	0.768284	0.7694	0.770513	0.771625	0.77273	0.77384	0.77494	0.77605
51	0.77715	0.778243	0.779338	0.78043	0.78152	0.782608	0.78369	0.78478	0.78586	0.78694
52	0.78801	0.789084	0.790155	0.79122	0.79229	0.793353	0.79442	0.79547	0.79653	0.79758
53	0.79864	0.799685	0.800731	0.80178	0.802817	0.803857	0.80489	0.80593	0.80696	0.80799
54	0.80902	0.810042	0.811064	0.81208	0.813101	0.814116	0.81513	0.81614	0.81715	0.81815
55	0.81915	0.820152	0.821149	0.82214	0.823136	0.824126	0.82511	0.8261	0.82708	0.82806
56	0.82904	0.830012	0.830984	0.83195	0.832921	0.833886	0.83485	0.83581	0.83676	0.83772
57	0.83867	0.83962	0.840567	0.84151	0.842452	0.843391	0.84433	0.84526	0.84619	0.84712
58	0.84805	0.848972	0.849893	0.85081	0.851727	0.85264	0.85355	0.85446	0.85536	0.85627
59	0.85717	0.858065	0.85896	0.85985	0.860742	0.861629	0.86251	0.8634	0.86428	0.86515
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	0.86603	0.866897	0.867765	0.86863	0.869495	0.870356	0.87121	0.87207	0.87292	0.87377
61	0.87462	0.875465	0.876307	0.87715	0.877983	0.878817	0.87965	0.88048	0.8813	0.88213
62	0.88295	0.883766	0.884581	0.88539	0.886204	0.887011	0.88782	0.88862	0.88942	0.89021
63	0.89101	0.891798	0.892586	0.89337	0.894154	0.894934	0.89571	0.89649	0.89726	0.89803
64	0.89879	0.899558	0.900319	0.90108	0.901833	0.902585	0.90334	0.90408	0.90483	0.90557
65	0.90631	0.907044	0.907777	0.90851	0.909236	0.909961	0.91068	0.9114	0.91212	0.91283
66	0.91355	0.914254	0.91496	0.91566	0.916363	0.91706	0.91776	0.91845	0.91914	0.91982
67	0.92051	0.921185	0.921863	0.92254	0.92321	0.92388	0.92455	0.92521	0.92587	0.92653
68	0.92718	0.927836	0.928486	0.92913	0.929776	0.930418	0.93106	0.93169	0.93232	0.93295
69	0.93358	0.934204	0.934826	0.93544	0.93606	0.936672	0.93728	0.93789	0.93849	0.93909
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	0.93969	0.940288	0.940881	0.94147	0.942057	0.942641	0.94322	0.9438	0.94438	0.94495
71	0.94552	0.946085	0.946649	0.94721	0.947768	0.948324	0.94888	0.94943	0.94997	0.95052
72	0.95106	0.951594	0.952129	0.95266	0.953191	0.953717	0.95424	0.95476	0.95528	0.95579
73	0.95631	0.956814	0.957319	0.95782	0.958323	0.95882	0.95931	0.95981	0.96029	0.96078
74	0.96126	0.961741	0.962218	0.96269	0.963163	0.96363	0.9641	0.96456	0.96502	0.96547
75	0.96593	0.966376	0.966823	0.96727	0.967709	0.968148	0.96858	0.96902	0.96945	0.96987
76	0.9703	0.970716	0.971134	0.97155	0.971961	0.97237	0.97278	0.97318	0.97358	0.97398
77	0.97437	0.974761	0.975149	0.97554	0.975917	0.976296	0.97667	0.97705	0.97742	0.97778
78	0.97815	0.978509	0.978867	0.97922	0.979575	0.979925	0.98027	0.98062	0.98096	0.98129
79	0.98163	0.981959	0.982287	0.98261	0.982935	0.983255	0.98357	0.98389	0.9842	0.9845
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	0.98481	0.985109	0.985408	0.9857	0.985996	0.986286	0.98657	0.98686	0.98714	0.98741
81	0.98769	0.98796	0.988228	0.98849	0.988756	0.989016	0.98927	0.98953	0.98978	0.99002

82	0.99027	0.990509	0.990748	0.99098	0.991216	0.991445	0.99167	0.99189	0.99212	0.99233
83	0.99255	0.992757	0.992966	0.99317	0.993373	0.993572	0.99377	0.99396	0.99415	0.99434
84	0.99452	0.994703	0.994881	0.99506	0.995227	0.995396	0.99556	0.99573	0.99588	0.99604
85	0.9962	0.996345	0.996493	0.99664	0.996779	0.996917	0.99705	0.99719	0.99731	0.99744
86	0.99756	0.997684	0.997801	0.99792	0.998027	0.998135	0.99824	0.99834	0.99844	0.99854
87	0.99863	0.998719	0.998806	0.99889	0.998971	0.999048	0.99912	0.99919	0.99926	0.99933
88	0.99939	0.99945	0.999507	0.99956	0.99961	0.999657	0.9997	0.99974	0.99978	0.99982
89	0.99985	0.999877	0.999903	0.99993	0.999945	0.999962	0.99998	0.99999	0.99999	1
90	1	0.999998	0.999994	0.99999	0.999976	0.999962	0.99995	0.99993	0.9999	0.99988

Table 3: Table of Natural Sines

Many of the calculations, however required the use of logarithms of the trigonometric identities. From the identities in table 3, two steps were performed. For example, to find the logarithm of the sine of 85° 30’ we would first take the mantissa of the sine and then take the logarithm of the mantissa.

$$\sin(85°\ 30') = 0.996917$$

the mantissa of 0.996917 is 996917. We now take its logarithm:

$$log_{10}(996917) = 5.998659$$

We will learn how to perform the more complex double-altitudes method for determining latitude in the next unit on spherical trigonometry.

N. **Class Work Exercise 7: (copy the problems and record your answers in your journal).** Using table 3, find the logarithm of the natural sines of the latitudes indicated:

1. 17° 45’
2. 13° 24’
3. 76° 36’
4. 89° 18’
5. 05° 30’
6. 27° 54’

Unit 1 Project

I. **Background.** Navigators performed their calculations using pre-prepared mathematical tables. In this project, you will create the tables you will use in subsequent units. ***Print all tables you create and insert them into your journal. You will need these tables to pass the unit assessment.***

II. **Task 1.** In Microsoft EXCEL, recreate table 2, a table of base 10 logarithms. Print the table and place it in your journal.

III. **Task 2.** In Microsoft EXCEL, create the following tables:

A. A table of natural sines for all latitudes between 0° and 90°, for every 6 minutes of each degree – recreating table 3.
B. A table of natural cosines for all latitudes between 0° and 90°, for every 6 minutes of each degree.
C. A table of natural tangents for all latitudes between 0° and 90°, for every 6 minutes of each degree.
D. A table of natural cotangents for all latitudes between 0° and 90°, for every 6 minutes of each degree.
E. A table of natural secants for all latitudes between 0° and 90°, for every 6 minutes of each degree.
F. A table of natural cosecants for all latitudes between 0° and 90°, for every 6 minutes of each degree.

IV. **Task 3.** In Microsoft EXCEL, create a logarithmic table for each of the tables created in task 2, using the mantissas of values in each of those tables.

Unit 1: Logarithms

Unit 1 Project Rubric

Name: ________________________________

Place completed rubric in your journal with your project printouts.

	4	3	2	1
Student has created a table of base 10 logarithms between 1.000 and 1.009 and between 1.01 and 10.00.				
Student has created six tables, one for each natural trigonometric function, for all latitudes between 0° and 90°, for every 6 minutes of each degree.				
Student has created six logarithmic tables, one for each natural trigonometric function, for all latitudes between 0° and 90°, for every 6 minutes of each degree.				

Unit 1: Logarithms

Name: ___________________________

Date:____________________________

Advisor:__________________________

Unit Assessment

Show all work on separate sheets of paper attached to this one. Failure to follow this instruction will result in a failing grade for this assessment.

I. Write the following exponential equations in logarithmic form.

A. **2-level problems.**

$2^5 = 32$
$6^2 = 36$
$5^2 = 25$

B. **3-level problems.**

$7^7 = 823543$
$15^{23} = 11222741464018821716308593 75$

II. Solve for *x* in the following problems.

A. **2-level problems.**

1. $log_5 125 = x$
2. $log_{12} 144 = x$
3. $log_7 2401 = x$

B. **3-level problems.**

1. $log_9 3 = x$
2. $log_{81} 6561 = x$
3. $log_7 \left(\frac{1}{49}\right) = x$

Unit 1: Logarithms

III. Solve for the indicated variable by converting the exponential equation into logarithmic form.

A. **2-level problems.**

1. $6^{9t-7} = 36$
2. $5^{7x-5} = 25$
3. $13^{125x-248} = 169$

B. **3-level problems.**

1. $256^{18y-56} = 12345$
2. $67^{1276r-1267} = 1000000$

IV. **4-level problems.** Use the tables you created for your project to determine the logarithms for the following latitudes:

A. Cosecant of $8° \, 54'$
B. Cotangent of $87° \, 36'$
C. Tangent of $66° \, 24'$

Unit 2: Spherical and Plane Trigonometry in Navigation and Surveying

I. Lesson I: How Did 18th Century Mariners Find Their Positions at Sea?

A. **Mini-lesson 1: Finding a geographic distance along a great circle of known arc measure.** Spherical trigonometry involves the analysis of triangles overlaid on a sphere such as the Earth's surface. Trigonometric functions are used to find the lengths of sides and the coordinates of vertices on a sphere. By analyzing triangles on the Earth's surface, one can use trigonometric functions to determine geographic locations and distances.

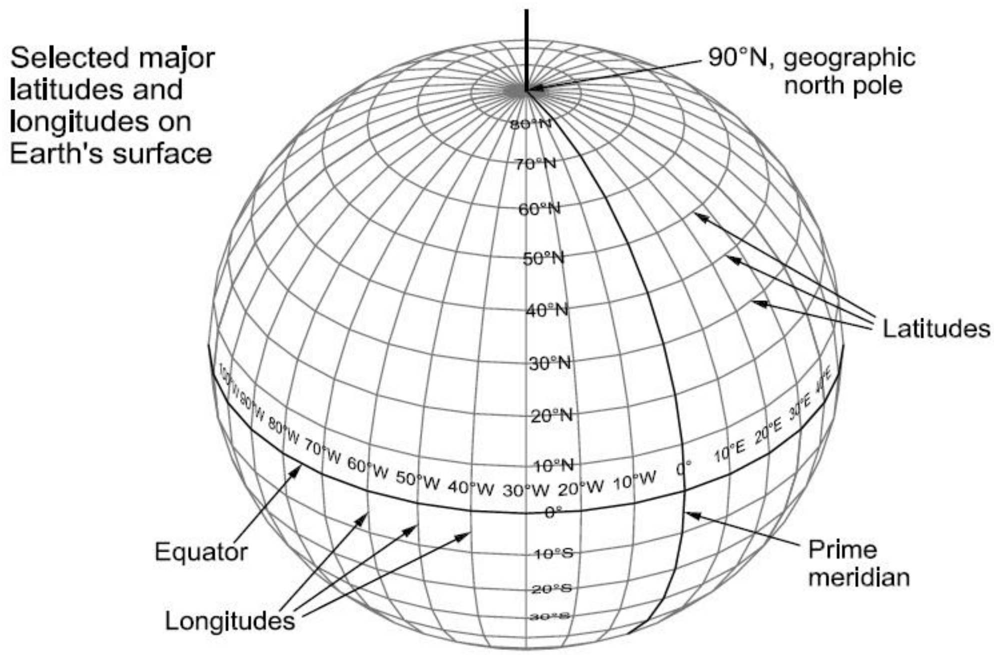


Figure 1: The Geographic Coordinate System

One of the easiest ways to form a spherical triangle on figure 1 is to use the geographic North Pole as one vertex and select the other two vertices on the equator. Let us form such a triangle, ΔABC, with vertex C at the geographic north pole, vertex A at the Equator at 0°, and vertex B at the Equator at 60° W. Recall that, along the Equator, every degree of longitude equals 60 nautical miles (nm). Therefore, the length of side AB, encompassing 60° is

$$60° \times \left(\frac{60\ nm}{1°}\right) = 3600\ nm$$

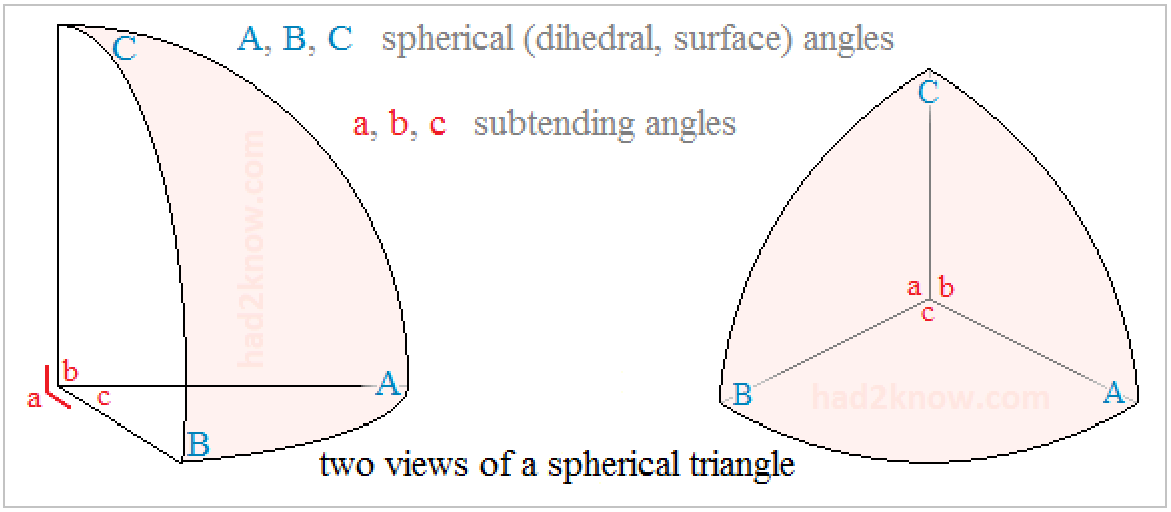


Figure 2: Spherical Triangle Anchored at Geographic North Pole, 0°, and 60° W

It is important to note that a spherical triangle, such as our ΔABC, must have the following properties:[38]

1. The three sides are all arcs of great circles, i.e. circles that encompass the radius of the sphere.
2. Any two sides are, together, longer than the third side.
3. The sum of the three angles is greater than 180° (this is the property that distinguishes a spherical triangle from a planar triangle).
4. Each individual spherical angle is less than 180°.

Recall from Geometry that the length of an arc of a circle of radius *r* is given by the solution to the formula:

$$L = \frac{degree\ measure\ of\ the\ arc}{360°} \times 2\pi r$$

We know that *r*, the radius of the Earth, is 3443.89849 nm so we can verify the distance of side AB (which is an arc of a great circle) is:

$$L = \frac{60°}{360°} \times 2\pi \times 3443.89849\ nm$$

[38] http://faculty.trinityvalleyschool.org/hoseltom/labs/Lab-01-(Spherical%20Trigonometry%20Intro).pdf.

$$L = 0.167 \times 2\pi \times 3443.89849\ nm$$

$$L \cong 0.167 \times 2 \times 3.14 \times 3443.89849\ nm$$

$$L \cong 3600\ nm$$

The arc length formula works well if we know the degree measure of the arc of the great circle representing the distance between the two points. We will need to rely on trigonometric formulae to find that distance when we do not know the degree measure of the arc.

B. **Class Work Exercise 1 (copy the problems and record your answers in your journal).** Assume ΔABC, with vertex C anchored on the geographic North Pole and vertices A and B anchored on the equator. Use the arc length formula to find the shortest distance between the two points:

1. Vertex A at 045°E, vertex B at 030°W.
2. Vertex A at 120°E, vertex B at 100°E.
3. Vertex A at 117°W, vertex B at 117°E.
4. Vertex A at 117°W, vertex B at 110°E.
5. Vertex A at 055°E, vertex B at 135°W.

C. **Mini-lesson 2: Finding geographic distance along a great circle, when its arc length is unknown.**

Referring to figure 2, assume vertex C remains anchored at the geographic North Pole but that arc BA does not lie along the equator and that arcs CB and CA are not of equal length.

If we know the latitudes of points B and A, we can determine the arc lengths of CB and CA using the arc length formula since we know the measures of ∠a and ∠b. Unfortunately, what we need to find is the arc length of BA.

Suppose B is at 40°N, 120°W and A is at 35°N, 150°W. Recall the theorem from geometry that states that the measure of an arc of a circle inscribed by the endpoints of two rays forming an angle whose vertex is the circle's

center is equal to the measure of that angle. The arc measures of CB and CA, therefore, are given by

$$m\ of\ arc\ CB = m\angle b = 90° - 40° = 50°$$

$$m\ of\ arc\ CA = m\angle a = 90° - 35° = 55°$$

Given this information, we can find the measure of ∠c and, hence, the arc length of arc BA by the following formula:

$$\cos c = \cos b \times \cos a + \sin b \times \sin a \times \cos C$$

where:

$\cos b = \cos 50° = 0.642788$
$\cos a = \cos 55° = 0.573576$
$\sin b = \sin 50° = 0.766044$
$\sin a = \sin 55° = 0.819152$
$\cos c = \cos 30° = 0.866025$

$$\cos c = 0.642788 \times 0.573576 + 0.766044 \times 0.819152 \times 0.866025$$

$$\cos c \cong 0.36869 + 0.54344 = 0.91213$$

We find the measure of *c* by taking the inverse of the cosine:

$$\cos^{-1} 0.91213 \cong 24.2°$$

Now we can use the arc length formula:

$$L \cong \frac{24.2°}{360°} \times 2 \times 3.14 \times 3443.89849\ nm$$

$$L \cong 1453.86\ nm$$

With a calculator, such computations are relatively straightforward. Mariners of the 18th century, however, did not have such devices and had to rely on logarithms to make such complex computations easier.

D. **Class Work Exercise 2 (copy the problems and record your answers in your journal).** Find the shortest distance between the indicated points using trigonometric and arc length formulae. For all problems, point C is anchored at the geographic North Pole.

1. B is at 55°N, 170°W and A is at 35°N, 150°W.
2. B is at 30°N, 160°W and A is at 35°N, 120°W.
3. B is at 25°N, 085°E and A is at 20°N, 175°W.
4. B is at 15°S, 135°E and A is at 15°N, 165°W.
5. B is at 75°N, 030°W and A is at 65°S, 150°E.

E. **Mini-lesson 3: Using Tables to Calculate Distances.** Without the use of calculators, mariners had the option to perform complex multiplication and division to find the sine and cosine of the angles and then the product and sum of those sines and cosines. A simpler method, involving more steps but simpler calculations, was to use tables of natural sines and cosines as well as logarithmic tables to perform simpler addition and subtraction operations.

Solving the problem from the last mini-lesson in this fashion:

1. Find the logarithms of the mantissas of the sines and cosines of the angles on your tables of natural sines and natural cosines:

$$log_{10}(mantissa\ of\ \cos b) = log_{10}(mantissa\ of\ \cos 50°) = 5.808067$$
$$log_{10}(mantissa\ of\ \cos a) = log_{10}(mantissa\ of\ \cos 55°) = 5.758591$$
$$log_{10}(mantissa\ of\ \sin b) = log_{10}(mantissa\ of\ \sin 50°) = 5.884254$$
$$log_{10}(mantissa\ of\ \sin a) = log_{10}(mantissa\ of\ \sin 55°) = 5.913365$$
$$log_{10}(mantissa\ of\ \cos C) = log_{10}(mantissa\ of\ \sin 30°) = 5.937531$$

2. Add the mantissas as follows:

$log_{10}(mantissa\ of\ \cos b) + log_{10}(mantissa\ of\ \cos a) = 5.808067 + 5.758591 = 11.566658$

$log_{10}(mantiss\ of\ \sin b) + log_{10}(mantiss\ of\ \sin a) + log_{10}(mantissa\ of\ \cos C) =$
$5.884254 + 5.913365 + 5.937531 = 17.73515$

3. Find the natural numbers associated with the mantissas (.566658 and .73515) in your table of logarithms. These are equivalent to:

$$10^{.56658} = 3.68687$$
$$10^{.73515} = 5.43438$$

4. Divide both answers by ten and add them together:

$$.368687 + .543448 = 0.912125$$

5. From your table of natural cosines, you will find that:

$$\cos^{-1} 0.912125 \cong 24.2°$$

6. Use the arc length formula to find the distance as before.

F. **Class Work Exercise 3 (copy the problems and record your answers in your journal).** Find the shortest distance between the indicated points using tables of natural sines, natural cosines, logarithms, and arc length formulae. For all problems, point C is anchored at the geographic North Pole.

1. B is at 55°N, 170°W and A is at 35°N, 150°W.
2. B is at 30°N, 160°W and A is at 35°N, 120°W.
3. B is at 25°N, 085°E and A is at 20°N, 175°W.
4. B is at 15°S, 135°E and A is at 15°N, 165°W.
5. B is at 75°N, 030°W and A is at 65°S, 150°E.

G. **Interlude 1: Stowing the Ship for a Long Voyage.**[39] A typical British exploration ship might carry 94 men and might load the following provisions (food and beverages) before leaving England on her voyage:[40]

- 6,000 pieces of pork in 26 barrels
- 4,000 of beef in 52 barrels
- nine tons of bread in 162 bags
- five tons of flour in 31 barrels
- three tons of sauerkraut in 60 barrels
- one ton of raisins in 20 barrels
- Sundry quantities of cheese, salt, peas, oil, sugar and oatmeal
- 250 barrels of beer
- 44 barrels of brandy
- 17 barrels of rum

A sailor was typically allocated the following rations: 1 lb. of bread and 1 lb. of sauerkraut per day plus 1 lb. of pork three times a week, and 2 lbs. of beef four times a week, He would also be allocated a gallon of beer, or a quart of brandy, or a pint of rum each day.[41]

H. **Interlude Exercise 1 (copy the problems and record your answers in your journal). The group purser is responsible for seeing that all group members complete this exercise. Show all work in your journals. You will need to consult online sources. Cite those sources fully.**

1. Each day, you must provide each man 1 lb. of bread, 1 lb. of pork or 2 lbs. of beef, and 1 lb. of sauerkraut. How many days provisions total, does your ship carry?

[39] Interludes are designed to provide a break from the main topic. They may be inserted into the schedule whenever the teacher feels one is appropriate.

[40] John C. Beaglehole, *The Journals of Captain James Cook on His Voyages of Discovery, v.1: The Voyage of the* Endeavour (London, The Hakluyt Society, 1968), p. 613.

[41] http://blogs.discovermagazine.com/80beats/2012/03/27/old-bones-tell-the-tales-and-reveal-the-diets-of-18th-century-sailors/#.U_sHusWwLYg.

2. How much cubic volume do all of the barrels of spirits (beer, brandy, rum) take up in the cargo hold?

3. Why is sauerkraut one of the most important items in a sailor's diet?

I. **Mini-lesson 4: Finding latitude by double altitudes.**[42] If the Sun is obscured by clouds at local apparent noon, we cannot be able to find the ship's latitude as we have in the past. We can, however, determine our latitude by the method of double altitudes, using the trigonometric and logarithmic operations we have learned.

1. To use the double altitude method we need to review some trigonometric functions.

   a. The secant of an angle θ (sec (θ)) is defined as:

$$\sec\theta = \left(\frac{1}{\cos\theta}\right)$$

   b. The cosecant of an angle θ (csc (θ)) is defined as:

$$\csc\theta = \left(\frac{1}{\sin\theta}\right)$$

2. **Example 1 –** Given:

   a. Sun's declination: 9.22° N
   b. First altitude: 40.38°
   c. Second altitude: 45.47°
   d. First time: 10:00 (noon – 120 minutes)
   e. Second time: 11:00 (noon – 60 minutes)

3. Use the tables to find the following:

[42] Thomas Beverley, *The Mariner's Latitude and Longitude Ready-Computer* (Cirencester, UK, John Bravender, 1836), 71.

a. Logarithm of the Secant of the declination:

$$\log_{10}(\sec 9.22°) = \log_{10} 1.013088705 = \log_{10} 130887 = 6.116897 = 0.116897$$

b. Logarithm of the Cosine of ½ the sum of the first and second altitudes:

$$\log_{10}(\cos 42.93°) = \log_{10} 0.732186 = \log_{10} 732186 = 5.864621 = 0.864621$$

c. Logarithm of the Sine of ½ the difference of the first and second altitudes:

$$\log_{10}(\sin 2.55°) = \log_{10} 0.0444912 = \log_{10} 444912 = 5.648274 = 0.648274$$

d. Logarithm of the Cosecant of ½ the sum of the first and second times (assuming 60 minutes = 1 degree of time).

$$\log_{10}(\csc 1.5°) = \log_{10} 38.201550 = \log_{10} 201550 = 4.304383 = 0.304383$$

e. Logarithm of the Cosecant of ½ the difference of the first and second times (assuming 60 minutes = 1 degree of time).

$$\log_{10}(\csc 0.5°) = \log_{10} 114.593013 = \log_{10} 593013 = 5.773065 = 0.77305$$

4. Add the answers together:

$$0.116897 + 0.864621 + 0.648274 + 0.304383 + 0.77305 = 2.707239 = 0.707239$$

5. This result is the logarithm of the Cosine of the latitude. To find the latitude, consult the Table of Logarithms of Mantissas of Natural Cosines. The mantissa 707239 falls between 59° 18’ and 59° 24’. Either answer would be adequate for most purposes of eighteenth century sailing, however, a more accurate answer can be found by interpolation as 59° 22’ N. This is our latitude.

6. **Example 2 –** Given:

    a. Sun's declination: 8° 52' N
    b. First altitude: 25° 41'
    c. Second altitude: 35° 23'
    d. First time: 9:00 (noon – 180 minutes)
    e. Second time: 11:00 (noon – 60 minutes)

7. First, convert the minutes to degrees with decimals:

$$\frac{52'}{\left(\frac{60'}{1°}\right)} = 0.87°$$

$$\frac{41'}{\left(\frac{60'}{1°}\right)} = 0.68°$$

$$\frac{23'}{\left(\frac{60'}{1°}\right)} = 0.38°$$

8. Use the tables to find the following:

    a. Logarithm of the Secant of the declination:

$$\log_{10}(\sec 8.87°) = \log_{10} 1.0121040 = \log_{10} 121040 = 5.0829289 = 0.0829289$$

    b. Logarithm of the Cosine of ½ the sum of the first and second altitudes:

$$0.5 * (35.38° + 25.68°) = 30.53°$$

$$\log_{10}(\cos 30.53°) = \log_{10} 0.861363 = \log_{10} 861363 = 5.935186 = 0.935186$$

c. Logarithm of the Sine of ½ the difference of the first and second altitudes:

$$0.5 * (35.38° - 25.68°) = 9.7°$$

$$\log_{10}(\sin 9.7°) = \log_{10} 0.168489 = \log_{10} 168489 = 5.226572 = 0.226572$$

d. Logarithm of the Cosecant of ½ the sum of the first and second times (assuming 60 minutes = 1 degree of time):

$$0.5 * (3° + 1°) = 2°$$

$$\log_{10}(\csc 2°) = \log_{10} 28.653708 = \log_{10} 653708 = 5.815384 = 0.815384$$

e. Logarithm of the Cosecant of ½ the difference of the first and second times (assuming 60 minutes = 1 degree of time).

$$0.5 * (3° - 1°) = 1°$$

$$\log_{10}(\csc 1°) = \log_{10} 57.298689 = \log_{10} 298689 = 5.475219 = 0.475219$$

9. Add the answers together:

$$0.0829289 + 0.935186 + 0.226572 + 0.815384 + 0.475219 = 2.53529 = 0.53529$$

10. This result is the logarithm of the Cosine of the latitude. To find the latitude, consult the Table of Logarithms of Mantissas of Natural Cosines. The mantissa 53529 falls between 69° 54' and 70° 00'. Either answer would be adequate for most purposes of eighteenth century sailing, however, a more accurate answer can be found by interpolation as 69° 56' 22" N. This is our latitude.

J. **Class Work Exercise 4 (copy the problems and record your answers in your journal).** Find your ship's latitude, longitude, distance travelled and average hourly speed given the following information. The previous position for each problem, taken seven days earlier, is 57.42° N, 175° W.

1. Given:

a. Sun's declination: 15° 30' N
b. First altitude: 23.40°
c. Second altitude: 67.90°
d. First time: 9:00 (noon – 180 minutes)
e. Second time: 10:00 (noon – 120 minutes)
f. GMT of LAN: 20:08

2. Given:

a. Sun's declination: 13° 12' N
b. First altitude: 20.70°
c. Second altitude: 65.80
d. First time: 08:00 (noon - 240 minutes)
e. Second time: 09:00 (noon - 180 minutes)
f. GMT of LAN: 23:40

3. Given:

a. Sun's declination: 11° 30' N
b. First altitude: 18° 42'
c. Second altitude: 45° 48'
d. First time: 10:00 (noon – 120 minutes)
e. Second time: 11:00 (noon – 60 minutes)
f. GMT of LAN: 19:36

K. **Interlude 2: Dinner for the Officers**. Life at sea in 18th century ships was harsh and demanding. Unlike many walks of life at that time, the measures of success were not the virtues of the gentleman but the abilities of the seaman. If an officer did not know how to sail and manage his ship, he was held in contempt by his peers and his sailors. And yet, officers and sailors tried with some success to maintain a domestic life at sea. Officers, in particular, strove to maintain their status as gentlemen, dining together as elites would in civil society, even if shipboard food was not up to the standard of wealthy people ashore.

Even if it was not up to wealthy standards, the food served onboard late-eighteenth century British naval ships was plentiful, for the most part nourishing, and far better that to which most of the crew was accustomed. Sailors and officers ate meat and bread daily and received enough Vitamin C in their diets to keep them free of the dreaded disease of scurvy.

We will enjoy a formal wardroom dinner much like that of the 18th century. The menu will consist of:

1. Salt beef (beef jerky).
2. Hardtack (ship's biscuit).
3. Pea Soup.
4. Limeade.
5. Recipes prepared by students from the cookbook *Saltwater Foodways*.

Students interested in preparing a recipe from the cookbook should get in touch with Captain Beall. Extra credit will be granted for contributing a recipe.

Unit 2 Project – Part 1: "A Day's Work in Navigation."

A "day's work in navigation" encompasses those tasks that a ship's Navigator performs to determine the ship's position, course made good (the actual ship's course after the effects of wind and current on the ship are factored in), the distance traveled and the speed made good (the average speed of the ship after the effects of wind and current are factored in). In the eighteenth century, the basis of the "day's work" was the sightings of the Sun made at local apparent noon (the time at which the Sun is at its maximum elevation above the horizon) or earlier if the Sun is obscured at local apparent noon but visible at other times.

Junior officers, for centuries, learned navigation from the Navigator or the ship's Captain by performing "a day's work" until they had attained proficiency. I, myself, did this as part of my training.

In this project, you and your classmates will perform "a day's work" much as junior naval officers did in the eighteenth century. In the process, you will develop your skill to perform complex arithmetic without calculators and your ability to persevere through a complex problem.

Figure 3: Junior U.S. Naval Officer Taking Sights on the Sun as Part of a "Day's Work in Navigation."

Unit 2 Project – Part 1: "A Day's Work in Navigation."

A Day's Work in Navigation - Day 1

Attach one complete copy of work to this paper.

Team members:

Name	**Grade**

Given:

Day 1 - May 3rd	
Declination	15.50 N
1st Altitude	19.40
2nd Altitude	71.90
1st Time	0900
2nd Time	1000
GMT of LAN	20:08
Position 7 days ago	33.70 N
	117 W

Find:

Latitude: ______________________________

Longitude: ______________________________

Distance Traveled: ______________________________

Average Speed: ______________________________

Unit 2 Project – Part 1: "A Day's Work in Navigation."

A Day's Work in Navigation - Day 2
Attach one complete copy of work to this paper.

Team members:

Name	**Grade**

Given:

Day 2 - May 10th	
Declination	17.45 N
1st Altitude	17.60
2nd Altitude	71.90
1st Time	0900
2nd Time	1000
GMT of LAN	20:48

Find:

Latitude: ______________________________

Longitude: ______________________________

Distance Traveled: ______________________________

Average Speed: ______________________________

Unit 2 Project – Part 1: "A Day's Work in Navigation."

A Day's Work in Navigation - Day 3
Attach one complete copy of work to this paper.

Team members:

Name	**Grade**

Given:

Day 3 - May 17th	
Declination	19.18 N
1st Altitude	16.90
2nd Altitude	69.60
1st Time	0900
2nd Time	1000
GMT of LAN	21:24

Find:

Latitude: ________________________________

Longitude: ________________________________

Distance Traveled: ________________________________

Average Speed: ________________________________

Unit 2 Project – Part 1: "A Day's Work in Navigation."

A Day's Work in Navigation - Day 4
Attach one complete copy of work to this paper.

Team members:

Name	**Grade**

Given:

Day 4 - May 24th	
Declination	20.65 N
1st Altitude	32.40
2nd Altitude	68.10
1st Time	1000
2nd Time	1100
GMT of LAN	21:56

Find:

Latitude: ______________________________

Longitude: ______________________________

Distance Traveled: ______________________________

Average Speed: ______________________________

Unit 2 Project – Part 1: "A Day's Work in Navigation."

A Day's Work in Navigation - Day 5

Attach one complete copy of work to this paper.

Team members:

Name	**Grade**

Given:

Day 4 - May 31st	
Declination	21.83 N
1st Altitude	31.50
2nd Altitude	66.70
1st Time	1000
2nd Time	1100
GMT of LAN	22:32

Find:

Latitude: ______________________________

Longitude: ______________________________

Distance Traveled: ______________________________

Average Speed: ______________________________

"A Day's Work in Navigation"
Reflection

Answer these questions completely in your journal. Your journal grade will be based, in part, on how seriously you take writing reflections such as this one.

Among the skills that will bring you success in college are:

- Ability to stick to a task until it is complete.
- Ability to complete complex tasks correctly with very limited help from a teacher.
- Ability to work with others in study groups in order to share a very challenging work load.

1. Were you able to stay focused to complete your share of the work?

2. Were you able to get the correct answers on your own, without much help from the teacher?

3. How well did your group work as a team?

4. Did everyone do his / her fair share of work?

5. How might you have worked better together?

Unit 2 Project – Part 1: "A Day's Work in Navigation."

Unit 2 Project – Part 1: Project Rubric

Name: ________________________________

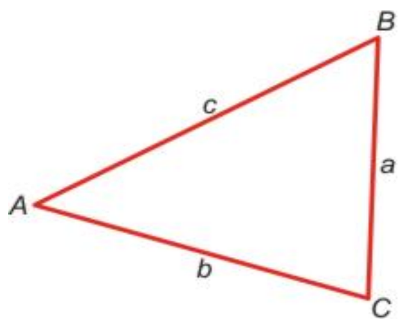
Place completed rubric in your journal.

	4	3	2	1
Student completed work assigned by team captain accurately and on time.				
Student was a supportive member of the team, helping his/her team mates when called upon.				
Student sought support and assistance from his/her team mates not the teacher.				

II. **Lesson II: How Did 18th Century Mariners Chart the Islands of the Pacific?** Mariners such as Captain James Cook used trigonometric functions, logarithms, and the method of triangulation to chart the coasts of Pacific islands. Triangulation involves dividing a coastline into a series of triangles. In any given triangle, if all angle measures and the length of one side are know, the lengths of the other two sides can be determined using either the Law of Sines or the Law of Cosines.

A. **Mini-lesson 1: Review of Trigonometric Laws.**

a. Law of Sines. For a given triangle ΔABC,



$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$

Law of Sines can be used when:

- Measures of two angles and a side (AAS and ASA) or,
- Measures of two sides and a non-included angle are known.

Example:

$m\angle A = 56°, m\angle B = 24°, b = 1.22\ cm$

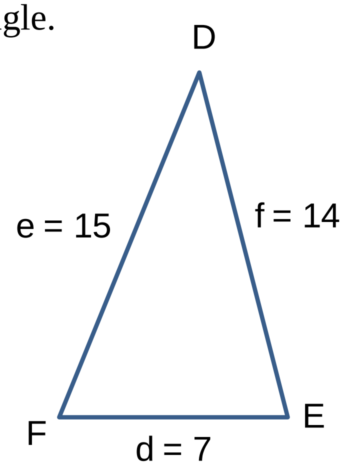
1. $m\angle C = 180° - 56° - 24° = 100°$
2. $\frac{\sin\ B}{b} = \frac{\sin\ 24°}{b} \cong \frac{0.41}{1.22} \cong 0.34$
3. $\frac{\sin\ C}{c} = \frac{\sin\ 100°}{c}$
4. $c = \frac{\sin\ 100°}{0.34} \cong 2.87$

b. Law of Cosines. For a given triangle ΔABC,

$c^2 = a^2 + b^2 - 2ab \times \cos \angle C$
$b^2 = a^2 + c^2 - 2ac \times \cos \angle B$
$a^2 = b^2 + c^2 - 2bc \times \cos \angle A$

<u>Example</u>:

1. Use the Law of Cosines to find the first angle.



$e^2 = d^2 + f^2 - 2df \cos E$
$15^2 = 7^2 + 14^2 - 2(7)(14) \cos E$
$225 = 245 - 196 \cos E$
$-20 = -196 \cos E$
$\dfrac{-20}{-196} = \cos E$
$m\angle E = \cos^{-1}(\dfrac{-20}{-196}) = 84°$

2. Use the Law of Sines to find the second angle.

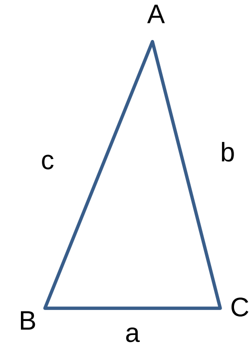
$\dfrac{\sin F}{14} = \dfrac{\sin 84°}{15}$

$\sin F = \dfrac{14 \sin 84°}{15} = 0.93$
$F = 68°$

3. Use the Triangle Sum Theorem to find the third angle.

$m\angle D = 180° - (84° + 68°) = 28°$

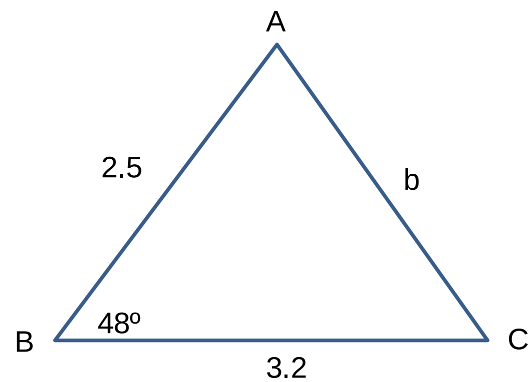
B. **Class Work Exercise 1. (copy the problems and record your answers in your journal).** Find the indicated measures using the Law of Cosines and / or Law of Sines.



1. $a = 12$ $b = 5$ $c =$ _____ $m\angle C = 68°$

2. $a = 9$ $b = 3$ $c = 8$ $m\angle C =$ _______

3. Solve the given triangle using the Law of Sines and / or the Law of Cosines.



A. $m\angle A =$ _______________

B. $m\angle C =$ _______________

C. $b =$ _______________

C. Mini-lesson 2: Principles of Nautical Surveying Using Triangulation.[43]

Triangulation is a type of surveying that uses the principles of trigonometry to determine positions on the surface of the Earth (i.e., latitude and longitude). If the three angles of a triangle and the length of one side of the triangle are known, the lengths of the other two sides of the triangle can be computed. When a series of triangles with common sides extends along a coastline or elsewhere, distances can be computed for each side of the triangles and carried forward through the triangles from the first triangle to the last.

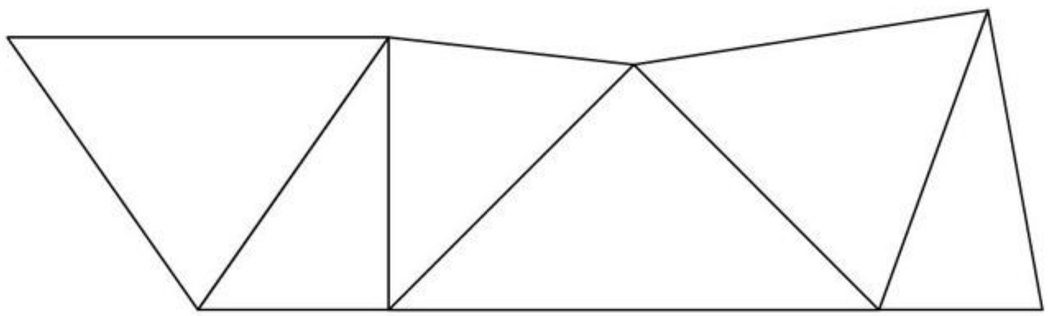


Figure 3: A series of triangles used in triangulation surveying is called an "arc of triangulation."

A series of triangles extending in an approximate straight line in this fashion is called an "arc of triangulation." A later variation of this method consisted of a chain of four-sided figures with the diagonals as well as the sides of the figures observed. Since each quadrilateral then consisted of four triangles, this provided extra redundancy, or checks, on the survey observations.

[43] http://celebrating200years.noaa.gov/foundations/spatial/side2_spatial.html.

Example:

Let: b = 1 nm = 2025 yards.
$m\angle C = 60°$
$m\angle B = 45°$

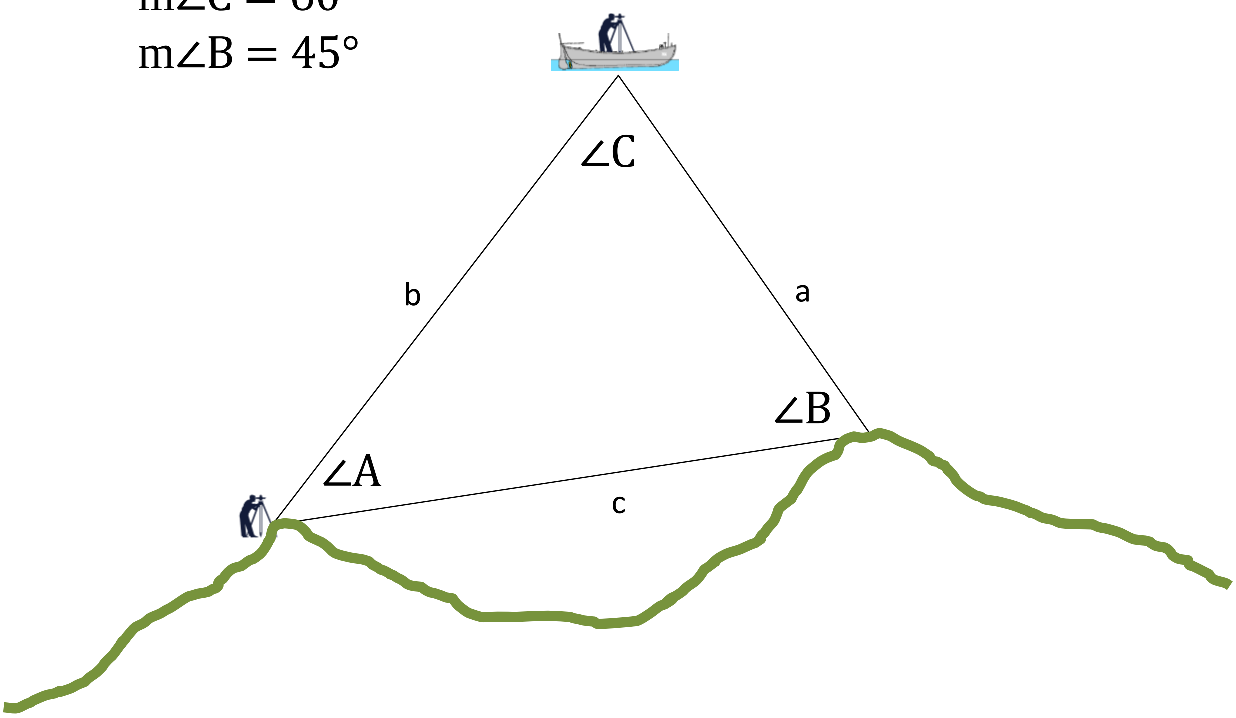


Figure 4: Solving a Triangle in a Coastal Survey

Since we know the measures of two of the angles, we can find the measure of the third by recalling that the sum of the interior angles of a triangle is 180°. Therefore,

$$m\angle A = 180° - 60° - 45° = 75°$$

We can use the Law of Sines to solve the remainder of the triangle:

$$\frac{\sin 45°}{2025\ yards} = \frac{\sin 60°}{c}, c = 2480.11\ yards$$

$$\frac{\sin 45°}{2025\ yards} = \frac{\sin 75°}{a}, a = 2766.20\ yards$$

With one triangle solved, we can continue the survey of the entire coast line, one triangle at a time.

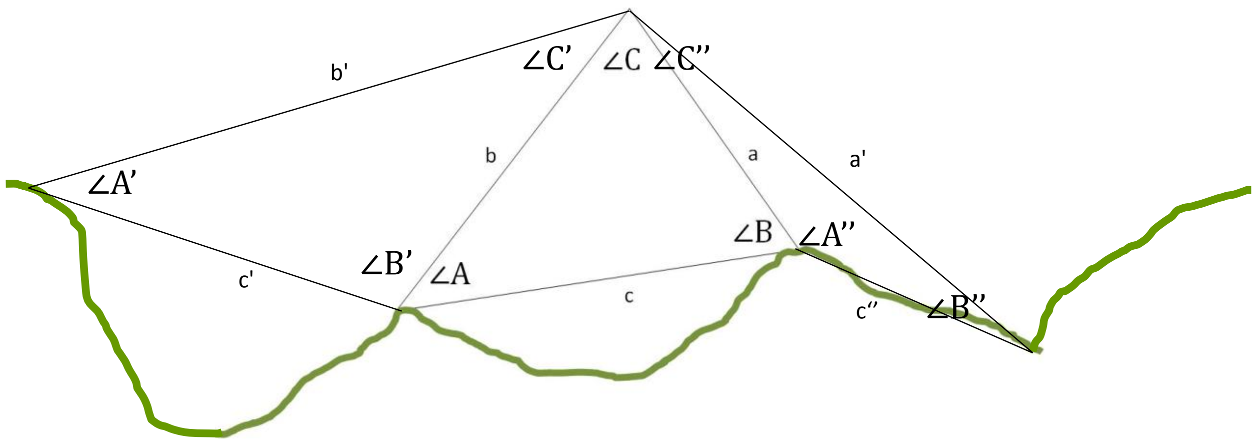


Figure 5: Solving Additional Triangles in a Coastal Survey.

D. **Class Work Exercise 2: (copy the problems and record your answers in your journal).** Solve the two additional triangles in figure 5 for all angles and sides given the following additional information:

$\angle C' = 45°$

$\angle A' = 35°$

$\angle C'' = 15°$

$\angle B'' = 15°$

E. **Mini-lesson 3: Nautical Surveying Using Tables of Trigonometric Values and Logarithms.** We will now solve the problems using our tables. Recall from our example in the previous lesson:

$$\frac{\sin 45°}{2025\ yards} = \frac{\sin 60°}{c}$$

$$c = \frac{\sin 60° \times 2025\ yards}{\sin 45°}$$

We can now use the property of logarithms, reducing complex multiplication and division into simpler addition and subtraction by reducing these values to their logarithms;

$log_{10}(\sin 60°) = 5.937530$
$log_{10}(2025) = 3.306424$
$log_{10}(\sin 45°) = 5.849485$

$(5.937530 + 3.306424) - 5.849485 = 3.394469$
$10^{3.394469} = 2480\ yards$

...which is very close to the answer in the example in the previous lesson.

F. **Class Work Exercise 3: (copy the problems and record your answers in your journal).** Solve the two additional triangles in figure 5 for all angles and sides given the following additional information. Solve using your Tables of Logarithms.

$\angle C' = 45°$

$\angle A' = 35°$

$\angle C'' = 15°$

$\angle B'' = 15°$

Name: _________________________

Date:__________________________

Advisor:__________________________

Unit Assessment

Show all work on separate sheets of paper attached to this one. Failure to follow this instruction will result in a failing grade for this assessment.

Given the following information, perform a day's work in navigation, finding the ship's latitude, longitude, distance travelled, and average speed using your tables of logarithms. Use of calculators is not permitted.

Previous Position (7 days earlier):

Declination:

First Altitude:

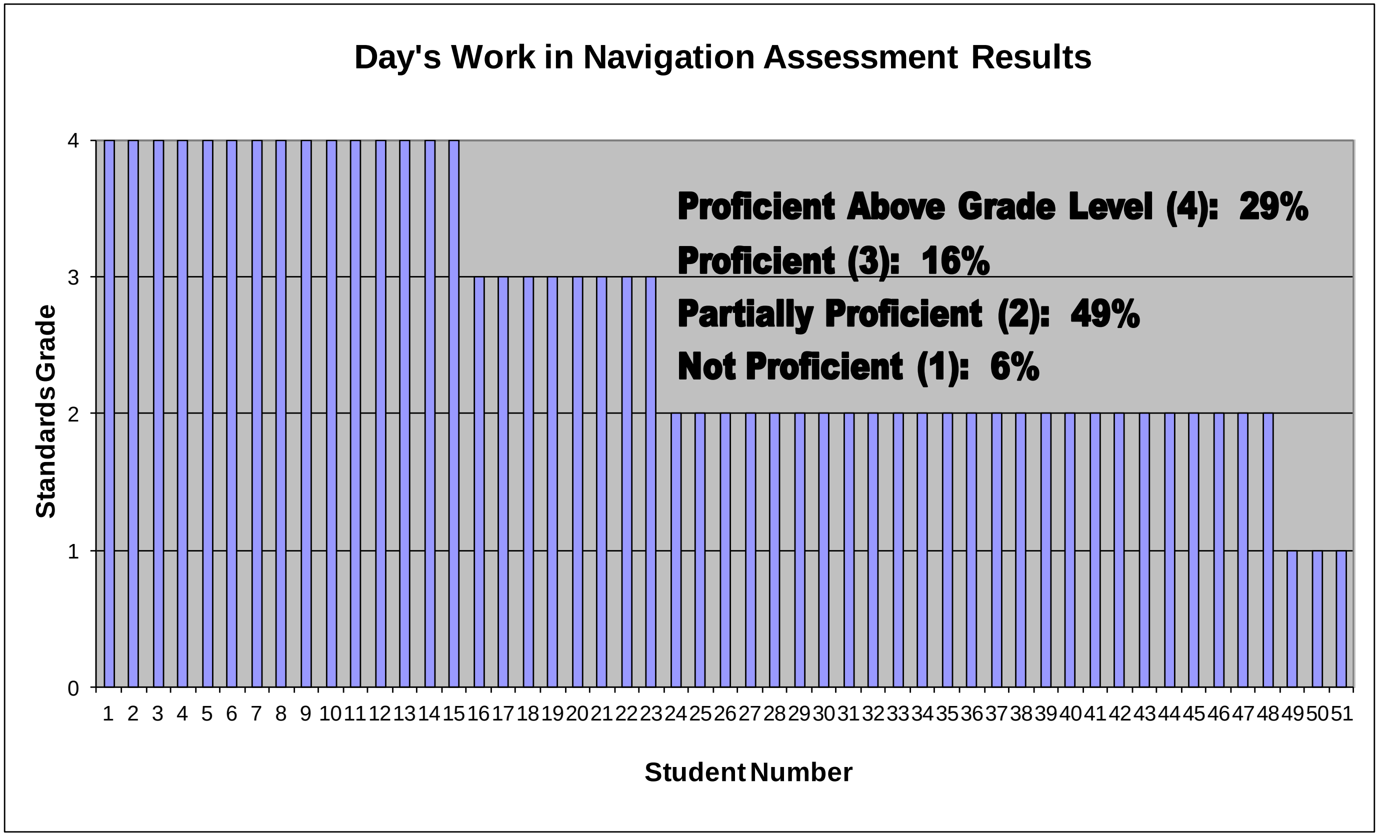
Second Altitude:

First Time:

Second Time:

GMT of LAN:

Assessment results. The following chart shows the grades received by 51 students who took the Unit 2 assessment in the 2014 – 2015 school year. Students who scored "Partially Proficient" (2) correctly solved the latitude and longitude problems using their own tables of logarithms with no calculators. They failed to solve the distance travelled and average speed problems, most often because they ran out of time. Students who scored "Proficient" (3) solved all four problems correctly. Students who scored "Proficient Above Grade Level" (4) solved all four problems correctly with a higher degree of care and accuracy in the use of logarithms than the average student.



Background. Perhaps the most famous of the 18th century voyages of exploration was that commanded by Lieutenant James Cook in 1768 – 1771. Cook was a highly skilled seaman, navigator, and surveyor who had come to the attention of the British Admiralty though his highly detailed survey of the coastal waters of Newfoundland which he made in 1765. Cook was commissioned by the Admiralty (Commissioners for Executing the Office of Lord High Admiral of Great Britain) in 1768 as Lieutenant in Command of His Majesty's Bark *Endeavour* with a mission to explore the Pacific Ocean, known then as the Great South Sea.

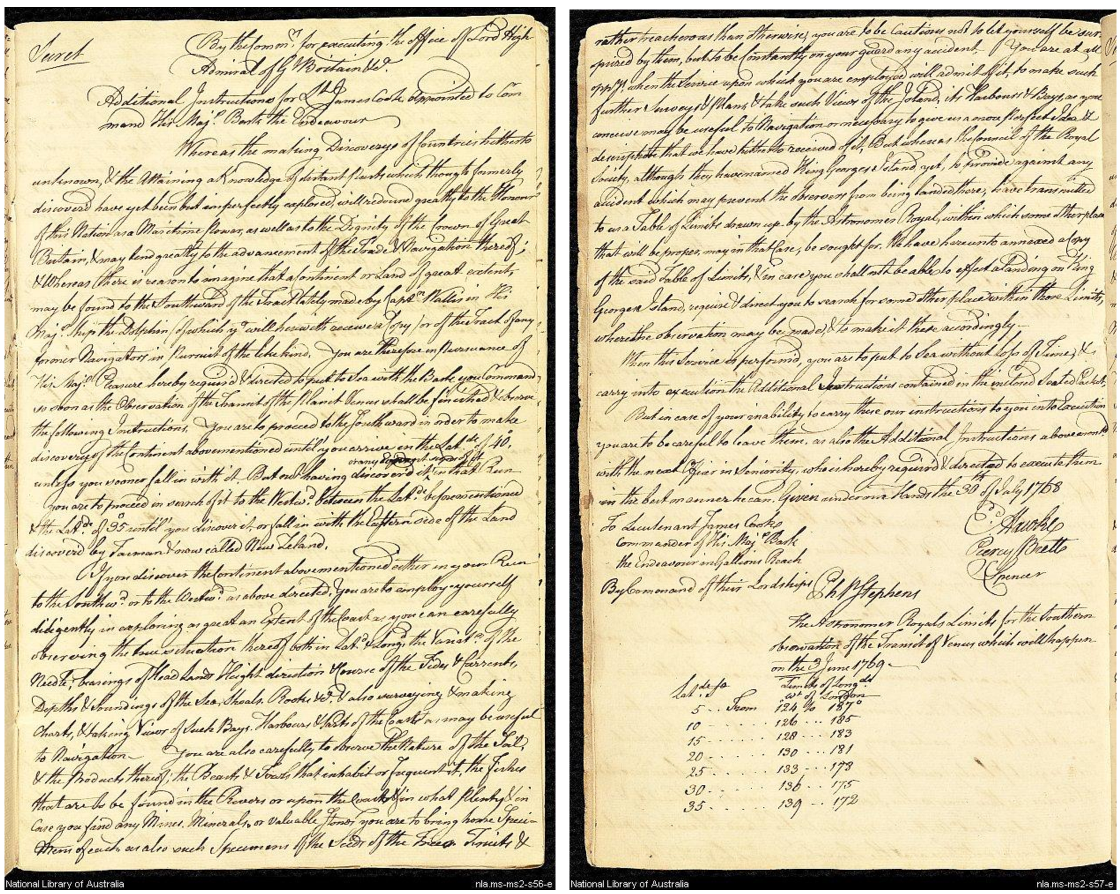


Figure 6: Instructions from the Commissioners for Executing the Office of Lord High Admiral of Great Britain to Lieutenant James Cook, 1st Lieutenant in Command of H.M. Bark *Endeavour*.[44]

During this voyage and two following, Cook enhanced his already considerable reputation as a navigator and as a surveyor by proving that Australia, while large,

[44] http://www.nla.gov.au/apps/cdview/?pi=nla.ms-ms2-s56-e.

was not the mythical Great Southern Continent, charting the entire coast of New Zealand for the first time, and locating a charting a number of Pacific Ocean islands including the Hawaiian Islands using the techniques we have learned in this unit. By turning the Pacific Ocean from a mysterious south sea into a realm of charted coasts and seas, Cook and his successors opened this vast area to settlement and exploitation by the nations of the Western World.

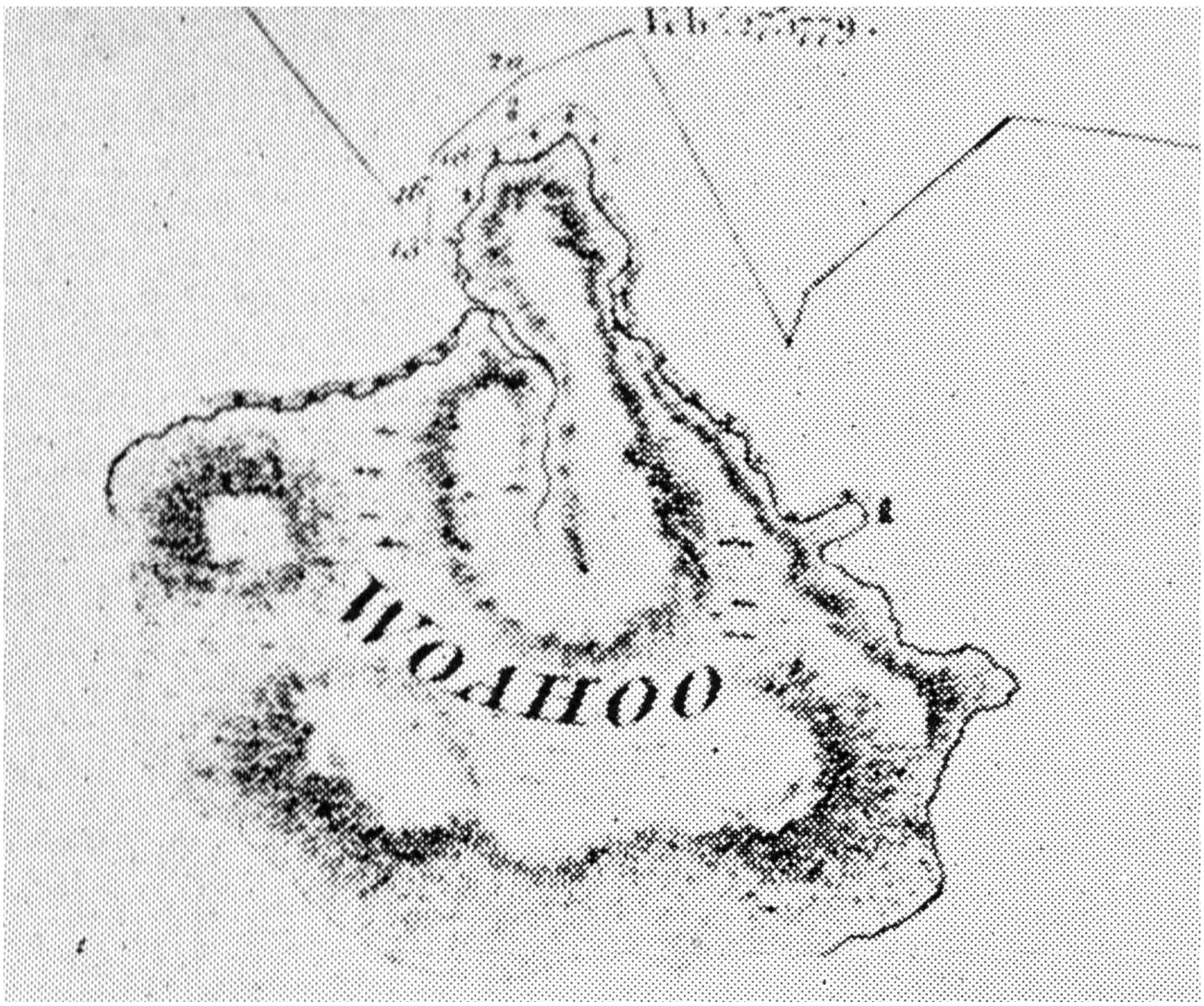
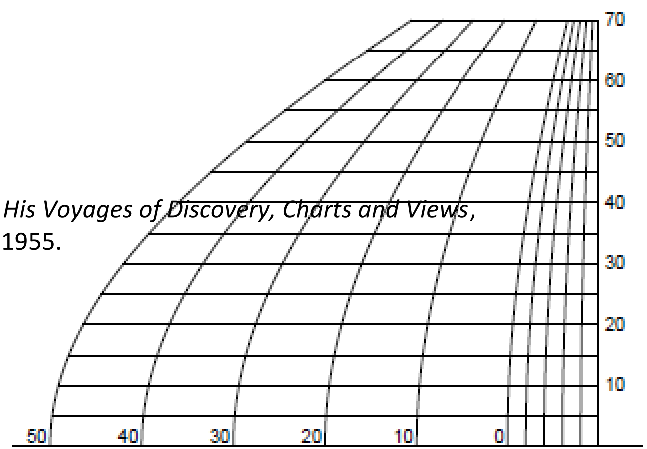


Figure 7: Early Chart of the Island of O'ahu in the Hawaiian Islands[45]

In this project, you will complete a coastal survey chart using the techniques we have learned thus far.



[45] From J.C. Beaglehole, ed., *The Journals of Captain James Cook on His Voyages of Discovery, Charts and Views*, Cambridge: Printed for the Hakluyt Society at the University Press, 1955.

Unit 2 Project – Part 2: Coastal Survey

Step 1: Preparation.

a. On the plotting sheet provided, label the longitude running down the center as 158° W.
b. On the latitude template at the bottom right of the sheet, change the numbers running along the bottom, dividing each in half (50 becomes 25, 40 becomes 20, etc.)

c. From the latitude template at the bottom right of the sheet, find the horizontal line marked "20". The larger distances between curves along this line are the distances between 5' of longitude at this latitude. The smaller distances are 1' of longitude. With a ruler and dividers, starting at the 158° W longitude line and along the perpendicular line running through the center of the compass rose, mark off 10' of longitude repeatedly until you reach the edge of the paper.

d. Label each of these markings, starting with 10', then 20', then 30'... After 50' on each side of 158° W, label the new longitudes, 159° W on the left, 157° W on the right. Continue until you reach the end of the paper on each side.

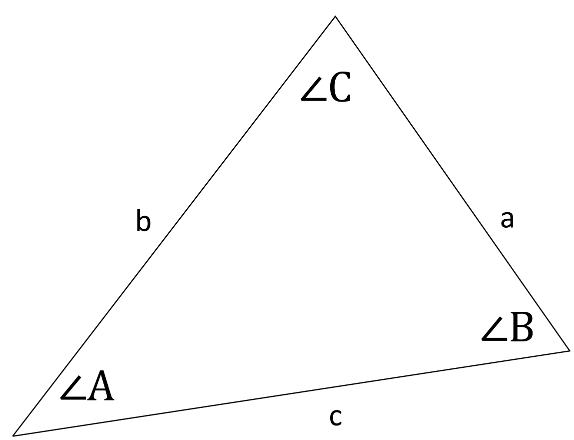
e. Label the center horizontal line 21° N latitude and label the next line north 21° 30' N, then 22° N. Label the next line south 20° 30' N, then 20° N.

f. Starting from the bottom of the sheet, along the 158° W vertical line, change the 10 to 5, 20 to 10, 30 to 15, 40 to 20, 50 to 25. After the 20° 30' N line and continuing to move up the 158° W line, change the 10 to 35, 20 to 40, 30 to 45, 40 to 50, and 50 to 55. After the 21° N line, repeat the changes made below that line.

Step 2: Solving the Triangles. Table 1 provides incomplete information on 23 triangles. Solve each triangle, finding all missing values using either the Law of Sines or Law of Cosines

a. Law of Sines. For a given triangle ΔABC,

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$



b. Law of Cosines. For a given triangle ΔABC,

$$c^2 = a^2 + b^2 - 2ab \times \cos \angle C$$
$$b^2 = a^2 + c^2 - 2ac \times \cos \angle B$$
$$a^2 = b^2 + c^2 - 2bc \times \cos \angle A$$

Step 3: Using the solved Table 1 and your plotting sheet, plot points 1 through 8, their latitude and longitude. It is very important that you plot these points precisely.

Step 4: From each of these points, use the compass rose on the plotting sheet to draw the "Initial Bearing" from each plotted point. The length of this line segment should be the first "b" listed on Table 1. For example:

- From point 1, draw a line segment in the direction of the true bearing of 204.0°T for a distance of 6.5 nm.

Step 5: From this line segment, measure an angle 30.0° counterclockwise from the point marking the 6.5 nm distance (this is the measure of angle A). The length of this new segment is the first "c" that you solved on Table 1 – 3.35 nm. Mark this distance. You now have side "c". Connect the point marked at 3.35 nm to point 1. You now have side "a".

Step 6: Side "a" becomes side "b" of the second triangle. Repeat steps 4 and 5 for each new triangle.

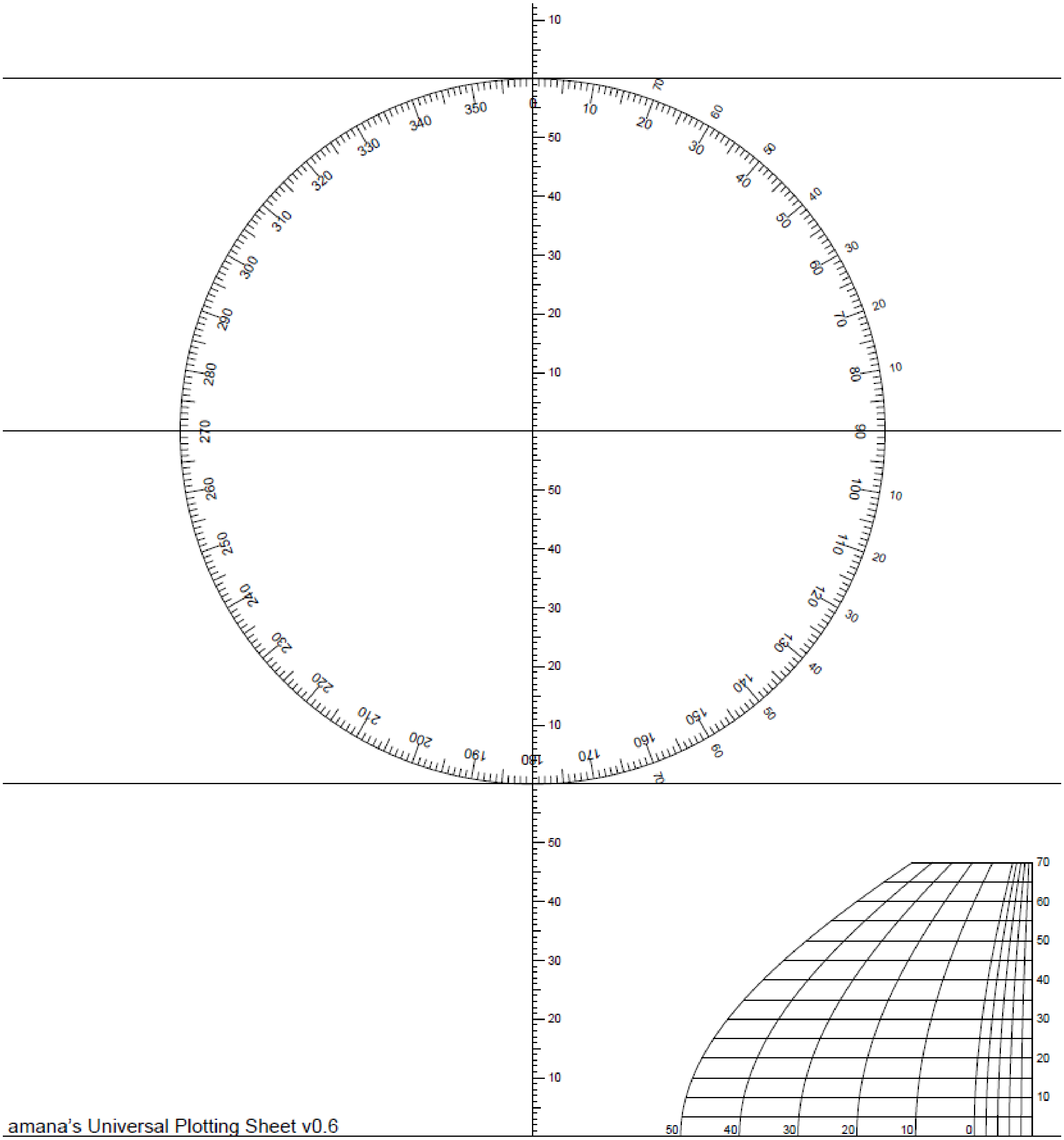


Figure 7: Plotting Sheet.

Unit 2 Project – Part 2: Coastal Survey

Point	Latitude	Longitude	Initial Bearing	Triangles	m<A	a	m<B	b	m<C	c
1	21° 46' 30" N	157° 58' 30" W	204° T	First ▲			125.00	6.50	25.00	
				Second ▲		3.97		3.97	15.00	
				Third ▲			30.00	3.97	20.00	
2	21° 39' 30" N	158° 17' 30" W	177° T	First ▲			25.00	4.50	65.00	
				Second ▲		12.50		10.50	25.00	
				Third ▲			45.00	12.50	15.00	
3	21° 30' 30" N	158° 20' 00" W	100° T	First ▲			70.00	7.00	30.00	
				Second ▲		4.91		6.00	35.00	
4	21° 27' 00" N	158° 18' 30" W	109° T	First ▲			100.00	6.75	15.00	
				Second ▲		7.07		7.83	10.00	
				Third ▲			117.00	6.00	15.00	
				Fourth ▲		5.00		5.25	15.00	
5	21° 14' 30" N	158° 09' 00" W	056° T	First ▲			50.00	3.00	30.00	
				Second ▲		5.86		3.50	20.00	
				Third ▲			45.00	6.00	20.00	
6	21° 14' 30" N	158° 04' 30" W	055° T	First ▲			45.00	6.00	110.00	
7	21° 14' 30" N	157° 58' 00" W	031° T	First ▲			85.00	4.00	10.00	
				Second ▲		4.14		4.00	15.00	
				Third ▲			85.00	3.75	10.00	
8	21° 14' 30" N	157° 54' 30" W	091° T	First ▲			90.00	4.00	45.00	
				Second ▲		3.03		3.50	30.00	
				Third ▲			40.00	3.00	25.00	
				Fourth ▲		3.77		4.00	25.00	

Table 1: Project 1 Information (all angles are measured in a counterclockwise direction)

Unit 2 Project – Part 2: Coastal Survey

Point	Latitude	Longitude	Initial Bearing	Triangles	m<A	a	m<B	b	m<C	c
1	21° 46' 30" N	157° 58' 30" W	204° T	First ▲	30.00	3.97	125.00	6.50	25.00	3.35
				Second ▲	82.50	3.97	82.50	3.97	15.00	1.04
				Third ▲	130.00	6.08	30.00	3.97	20.00	2.71
2	21° 39' 30" N	158° 17' 30" W	177° T	First ▲	90.00	10.65	25.00	4.50	65.00	9.65
				Second ▲	98.92	12.50	56.08	10.50	25.00	5.35
				Third ▲	120.00	15.31	45.00	12.50	15.00	4.58
3	21° 30' 30" N	158° 20' 00" W	100° T	First ▲	80.00	7.34	70.00	7.00	30.00	3.72
				Second ▲	55.00	4.91	90.00	6.00	35.00	3.44
4	21° 27' 00" N	158° 18' 30" W	109° T	First ▲	65.00	6.21	100.00	6.75	15.00	1.77
				Second ▲	55.00	7.07	115.00	7.83	10.00	1.50
				Third ▲	48.00	5.00	117.00	6.00	15.00	1.75
				Fourth ▲	75.00	5.00	90.00	5.25	15.00	1.36
5	21° 14' 30" N	158° 09' 00" W	056° T	First ▲	100.00	3.86	50.00	3.00	30.00	2.00
				Second ▲	135.00	5.86	25.00	3.50	20.00	2.83
				Third ▲	115.00	7.69	45.00	6.00	20.00	2.90
6	21° 14' 30" N	158° 04' 30" W	055° T	First ▲	25.00	3.60	45.00	6.00	110.00	8.00
7	21° 14' 30" N	157° 58' 00" W	031° T	First ▲	85.00	4.00	85.00	4.00	10.00	0.70
				Second ▲	90.00	4.14	75.00	4.00	15.00	1.00
				Third ▲	85.00	3.75	85.00	3.75	10.00	0.67
8	21° 14' 30" N	157° 54' 30" W	091° T	First ▲	45.00	2.83	90.00	4.00	45.00	2.83
				Second ▲	60.00	3.03	90.00	3.50	30.00	1.75
				Third ▲	115.00	4.23	40.00	3.00	25.00	1.97
				Fourth ▲	70.00	3.77	85.00	4.00	25.00	1.70

Table 2: Project 1 Information - Answers

Unit 2 Project – Part 2: Coastal Survey

Reflection

Answer these questions completely in your journal. Your journal grade will be based, in part, on how seriously you take writing reflections such as this one.

Among the skills that will bring you success in college are:

- Ability to stick to a task until it is complete.
- Ability to read and understand directions fully and completely.
- Ability to complete complex tasks correctly with very limited help from a teacher.
- Ability to work with others in study groups in order to share a very challenging work load.

1. How well were you able to understand the instructions?

2. From whom did you seek help in understanding the instructions?

3. What did you find most challenging in completing the project?

4. Imagine yourself being asked to complete this project in a small boat like the one Lieutenant William Bligh navigated after he and his men had been cast adrift. What additional challenges might you have faced?

Unit 2 Project – Part 2: Coastal Survey

Project Rubric

Name: ________________________________

Place completed rubric in your journal.

	4	**3**	**2**	**1**
Student correctly measured all distances and angles provided.				
Student created an accurate chart representing the west and south coasts of the island of O'ahu.				
Project submission was neat and demonstrated professional care in producing a quality product.				

Logarithmic and Trigonometric Tables

Base 10 Logarithms for Natural Numbers 1 - 10

N	0	1	2	3	4	5	6	7	8	9
1.00	0	0.000434	0.000868	0.001301	0.001734	0.002166	0.002598	0.003029	0.003461	0.003891
1.01	0.004321	0.004751	0.005181	0.005609	0.006038	0.006466	0.006894	0.007321	0.007748	0.008174
1.02	0.0086	0.009026	0.009451	0.009876	0.0103	0.010724	0.011147	0.01157	0.011993	0.012415
1.03	0.012837	0.013259	0.01368	0.0141	0.014521	0.01494	0.01536	0.015779	0.016197	0.016616
1.04	0.017033	0.017451	0.017868	0.018284	0.0187	0.019116	0.019532	0.019947	0.020361	0.020775
1.05	0.021189	0.021603	0.022016	0.022428	0.022841	0.023252	0.023664	0.024075	0.024486	0.024896
1.06	0.025306	0.025715	0.026125	0.026533	0.026942	0.02735	0.027757	0.028164	0.028571	0.028978
1.07	0.029384	0.029789	0.030195	0.0306	0.031004	0.031408	0.031812	0.032216	0.032619	0.033021
1.08	0.033424	0.033826	0.034227	0.034628	0.035029	0.03543	0.03583	0.03623	0.036629	0.037028
1.09	0.037426	0.037825	0.038223	0.03862	0.039017	0.039414	0.039811	0.040207	0.040602	0.040998
1.10	0.041393	0.045323	0.049218	0.053078	0.056905	0.060698	0.064458	0.068186	0.071882	0.075547
1.20	0.079181	0.082785	0.08636	0.089905	0.093422	0.09691	0.100371	0.103804	0.10721	0.11059
1.30	0.113943	0.117271	0.120574	0.123852	0.127105	0.130334	0.133539	0.136721	0.139879	0.143015
1.40	0.146128	0.149219	0.152288	0.155336	0.158362	0.161368	0.164353	0.167317	0.170262	0.173186
1.50	0.176091	0.178977	0.181844	0.184691	0.187521	0.190332	0.193125	0.1959	0.198657	0.201397
1.60	0.20412	0.206826	0.209515	0.212188	0.214844	0.217484	0.220108	0.222716	0.225309	0.227887
1.70	0.230449	0.232996	0.235528	0.238046	0.240549	0.243038	0.245513	0.247973	0.25042	0.252853
1.80	0.255273	0.257679	0.260071	0.262451	0.264818	0.267172	0.269513	0.271842	0.274158	0.276462
1.90	0.278754	0.281033	0.283301	0.285557	0.287802	0.290035	0.292256	0.294466	0.296665	0.298853
2.00	0.30103	0.303196	0.305351	0.307496	0.30963	0.311754	0.313867	0.31597	0.318063	0.320146
2.10	0.322219	0.324282	0.326336	0.32838	0.330414	0.332438	0.334454	0.33646	0.338456	0.340444
2.20	0.342423	0.344392	0.346353	0.348305	0.350248	0.352183	0.354108	0.356026	0.357935	0.359835
2.30	0.361728	0.363612	0.365488	0.367356	0.369216	0.371068	0.372912	0.374748	0.376577	0.378398
2.40	0.380211	0.382017	0.383815	0.385606	0.38739	0.389166	0.390935	0.392697	0.394452	0.396199
2.50	0.39794	0.399674	0.401401	0.403121	0.404834	0.40654	0.40824	0.409933	0.41162	0.4133
2.60	0.414973	0.416641	0.418301	0.419956	0.421604	0.423246	0.424882	0.426511	0.428135	0.429752
2.70	0.431364	0.432969	0.434569	0.436163	0.437751	0.439333	0.440909	0.44248	0.444045	0.445604
2.80	0.447158	0.448706	0.450249	0.451786	0.453318	0.454845	0.456366	0.457882	0.459392	0.460898
2.90	0.462398	0.463893	0.465383	0.466868	0.468347	0.469822	0.471292	0.472756	0.474216	0.475671
3.00	0.477121	0.478566	0.480007	0.481443	0.482874	0.4843	0.485721	0.487138	0.488551	0.489958
3.10	0.491362	0.49276	0.494155	0.495544	0.49693	0.498311	0.499687	0.501059	0.502427	0.503791
3.20	0.50515	0.506505	0.507856	0.509203	0.510545	0.511883	0.513218	0.514548	0.515874	0.517196
3.30	0.518514	0.519828	0.521138	0.522444	0.523746	0.525045	0.526339	0.52763	0.528917	0.5302
3.40	0.531479	0.532754	0.534026	0.535294	0.536558	0.537819	0.539076	0.540329	0.541579	0.542825
3.50	0.544068	0.545307	0.546543	0.547775	0.549003	0.550228	0.55145	0.552668	0.553883	0.555094
3.60	0.556303	0.557507	0.558709	0.559907	0.561101	0.562293	0.563481	0.564666	0.565848	0.567026
3.70	0.568202	0.569374	0.570543	0.571709	0.572872	0.574031	0.575188	0.576341	0.577492	0.578639
3.80	0.579784	0.580925	0.582063	0.583199	0.584331	0.585461	0.586587	0.587711	0.588832	0.58995

3.90	0.591065	0.592177	0.593286	0.594393	0.595496	0.596597	0.597695	0.598791	0.599883	0.600973
4.00	0.60206	0.603144	0.604226	0.605305	0.606381	0.607455	0.608526	0.609594	0.61066	0.611723
4.10	0.612784	0.613842	0.614897	0.61595	0.617	0.618048	0.619093	0.620136	0.621176	0.622214
4.20	0.623249	0.624282	0.625312	0.62634	0.627366	0.628389	0.62941	0.630428	0.631444	0.632457
4.30	0.633468	0.634477	0.635484	0.636488	0.63749	0.638489	0.639486	0.640481	0.641474	0.642465
4.40	0.643453	0.644439	0.645422	0.646404	0.647383	0.64836	0.649335	0.650308	0.651278	0.652246
4.50	0.653213	0.654177	0.655138	0.656098	0.657056	0.658011	0.658965	0.659916	0.660865	0.661813
4.60	0.662758	0.663701	0.664642	0.665581	0.666518	0.667453	0.668386	0.669317	0.670246	0.671173
4.70	0.672098	0.673021	0.673942	0.674861	0.675778	0.676694	0.677607	0.678518	0.679428	0.680336
4.80	0.681241	0.682145	0.683047	0.683947	0.684845	0.685742	0.686636	0.687529	0.68842	0.689309
4.90	0.690196	0.691081	0.691965	0.692847	0.693727	0.694605	0.695482	0.696356	0.697229	0.698101
5.00	0.69897	0.699838	0.700704	0.701568	0.702431	0.703291	0.704151	0.705008	0.705864	0.706718
5.10	0.70757	0.708421	0.70927	0.710117	0.710963	0.711807	0.71265	0.713491	0.71433	0.715167
5.20	0.716003	0.716838	0.717671	0.718502	0.719331	0.720159	0.720986	0.721811	0.722634	0.723456
5.30	0.724276	0.725095	0.725912	0.726727	0.727541	0.728354	0.729165	0.729974	0.730782	0.731589
5.40	0.732394	0.733197	0.733999	0.7348	0.735599	0.736397	0.737193	0.737987	0.738781	0.739572
5.50	0.740363	0.741152	0.741939	0.742725	0.74351	0.744293	0.745075	0.745855	0.746634	0.747412
5.60	0.748188	0.748963	0.749736	0.750508	0.751279	0.752048	0.752816	0.753583	0.754348	0.755112
5.70	0.755875	0.756636	0.757396	0.758155	0.758912	0.759668	0.760422	0.761176	0.761928	0.762679
5.80	0.763428	0.764176	0.764923	0.765669	0.766413	0.767156	0.767898	0.768638	0.769377	0.770115
5.90	0.770852	0.771587	0.772322	0.773055	0.773786	0.774517	0.775246	0.775974	0.776701	0.777427
6.00	0.778151	0.778874	0.779596	0.780317	0.781037	0.781755	0.782473	0.783189	0.783904	0.784617
6.10	0.78533	0.786041	0.786751	0.78746	0.788168	0.788875	0.789581	0.790285	0.790988	0.791691
6.20	0.792392	0.793092	0.79379	0.794488	0.795185	0.79588	0.796574	0.797268	0.79796	0.798651
6.30	0.799341	0.800029	0.800717	0.801404	0.802089	0.802774	0.803457	0.804139	0.804821	0.805501
6.40	0.80618	0.806858	0.807535	0.808211	0.808886	0.80956	0.810233	0.810904	0.811575	0.812245
6.50	0.812913	0.813581	0.814248	0.814913	0.815578	0.816241	0.816904	0.817565	0.818226	0.818885
6.60	0.819544	0.820201	0.820858	0.821514	0.822168	0.822822	0.823474	0.824126	0.824776	0.825426
6.70	0.826075	0.826723	0.827369	0.828015	0.82866	0.829304	0.829947	0.830589	0.83123	0.83187
6.80	0.832509	0.833147	0.833784	0.834421	0.835056	0.835691	0.836324	0.836957	0.837588	0.838219
6.90	0.838849	0.839478	0.840106	0.840733	0.841359	0.841985	0.842609	0.843233	0.843855	0.844477
7.00	0.845098	0.845718	0.846337	0.846955	0.847573	0.848189	0.848805	0.849419	0.850033	0.850646
7.10	0.851258	0.85187	0.85248	0.85309	0.853698	0.854306	0.854913	0.855519	0.856124	0.856729
7.20	0.857332	0.857935	0.858537	0.859138	0.859739	0.860338	0.860937	0.861534	0.862131	0.862728
7.30	0.863323	0.863917	0.864511	0.865104	0.865696	0.866287	0.866878	0.867467	0.868056	0.868644
7.40	0.869232	0.869818	0.870404	0.870989	0.871573	0.872156	0.872739	0.873321	0.873902	0.874482
7.50	0.875061	0.87564	0.876218	0.876795	0.877371	0.877947	0.878522	0.879096	0.879669	0.880242
7.60	0.880814	0.881385	0.881955	0.882525	0.883093	0.883661	0.884229	0.884795	0.885361	0.885926
7.70	0.886491	0.887054	0.887617	0.888179	0.888741	0.889302	0.889862	0.890421	0.89098	0.891537
7.80	0.892095	0.892651	0.893207	0.893762	0.894316	0.89487	0.895423	0.895975	0.896526	0.897077
7.90	0.897627	0.898176	0.898725	0.899273	0.899821	0.900367	0.900913	0.901458	0.902003	0.902547

Logarithmic and Trigonometric Tables

8.00	0.90309	0.903633	0.904174	0.904716	0.905256	0.905796	0.906335	0.906874	0.907411	0.907949
8.10	0.908485	0.909021	0.909556	0.910091	0.910624	0.911158	0.91169	0.912222	0.912753	0.913284
8.20	0.913814	0.914343	0.914872	0.9154	0.915927	0.916454	0.91698	0.917506	0.91803	0.918555
8.30	0.919078	0.919601	0.920123	0.920645	0.921166	0.921686	0.922206	0.922725	0.923244	0.923762
8.40	0.924279	0.924796	0.925312	0.925828	0.926342	0.926857	0.92737	0.927883	0.928396	0.928908
8.50	0.929419	0.92993	0.93044	0.930949	0.931458	0.931966	0.932474	0.932981	0.933487	0.933993
8.60	0.934498	0.935003	0.935507	0.936011	0.936514	0.937016	0.937518	0.938019	0.93852	0.93902
8.70	0.939519	0.940018	0.940516	0.941014	0.941511	0.942008	0.942504	0.943	0.943495	0.943989
8.80	0.944483	0.944976	0.945469	0.945961	0.946452	0.946943	0.947434	0.947924	0.948413	0.948902
8.90	0.94939	0.949878	0.950365	0.950851	0.951338	0.951823	0.952308	0.952792	0.953276	0.95376
9.00	0.954243	0.954725	0.955207	0.955688	0.956168	0.956649	0.957128	0.957607	0.958086	0.958564
9.10	0.959041	0.959518	0.959995	0.960471	0.960946	0.961421	0.961895	0.962369	0.962843	0.963316
9.20	0.963788	0.96426	0.964731	0.965202	0.965672	0.966142	0.966611	0.96708	0.967548	0.968016
9.30	0.968483	0.96895	0.969416	0.969882	0.970347	0.970812	0.971276	0.97174	0.972203	0.972666
9.40	0.973128	0.97359	0.974051	0.974512	0.974972	0.975432	0.975891	0.97635	0.976808	0.977266
9.50	0.977724	0.978181	0.978637	0.979093	0.979548	0.980003	0.980458	0.980912	0.981366	0.981819
9.60	0.982271	0.982723	0.983175	0.983626	0.984077	0.984527	0.984977	0.985426	0.985875	0.986324
9.70	0.986772	0.987219	0.987666	0.988113	0.988559	0.989005	0.98945	0.989895	0.990339	0.990783
9.80	0.991226	0.991669	0.992111	0.992554	0.992995	0.993436	0.993877	0.994317	0.994757	0.995196
9.90	0.995635	0.996074	0.996512	0.996949	0.997386	0.997823	0.998259	0.998695	0.999131	0.999565
10.00	1	1.000434	1.000868	1.001301	1.001734	1.002166	1.002598	1.003029	1.003461	1.003891

Table of Natural Sines

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	0	0.001745	0.003491	0.005236	0.006981	0.008727	0.010472	0.012217	0.013962	0.015707
1	0.017452	0.019197	0.020942	0.022687	0.024432	0.026177	0.027922	0.029666	0.031411	0.033155
2	0.034899	0.036644	0.038388	0.040132	0.041876	0.043619	0.045363	0.047106	0.04885	0.050593
3	0.052336	0.054079	0.055822	0.057564	0.059306	0.061049	0.062791	0.064532	0.066274	0.068015
4	0.069756	0.071497	0.073238	0.074979	0.076719	0.078459	0.080199	0.081939	0.083678	0.085417
5	0.087156	0.088894	0.090633	0.092371	0.094108	0.095846	0.097583	0.09932	0.101056	0.102793
6	0.104528	0.106264	0.107999	0.109734	0.111469	0.113203	0.114937	0.116671	0.118404	0.120137
7	0.121869	0.123601	0.125333	0.127065	0.128796	0.130526	0.132256	0.133986	0.135716	0.137445
8	0.139173	0.140901	0.142629	0.144356	0.146083	0.147809	0.149535	0.151261	0.152986	0.15471
9	0.156434	0.158158	0.159881	0.161604	0.163326	0.165048	0.166769	0.168489	0.170209	0.171929
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	0.173648	0.175367	0.177085	0.178802	0.180519	0.182236	0.183951	0.185667	0.187381	0.189095
11	0.190809	0.192522	0.194234	0.195946	0.197657	0.199368	0.201078	0.202787	0.204496	0.206204
12	0.207912	0.209619	0.211325	0.21303	0.214735	0.21644	0.218143	0.219846	0.221548	0.22325
13	0.224951	0.226651	0.228351	0.23005	0.231748	0.233445	0.235142	0.236838	0.238533	0.240228
14	0.241922	0.243615	0.245307	0.246999	0.24869	0.25038	0.252069	0.253758	0.255446	0.257133
15	0.258819	0.260505	0.262189	0.263873	0.265556	0.267238	0.26892	0.2706	0.27228	0.273959
16	0.275637	0.277315	0.278991	0.280667	0.282341	0.284015	0.285688	0.287361	0.289032	0.290702
17	0.292372	0.29404	0.295708	0.297375	0.299041	0.300706	0.30237	0.304033	0.305695	0.307357
18	0.309017	0.310676	0.312335	0.313992	0.315649	0.317305	0.318959	0.320613	0.322266	0.323917
19	0.325568	0.327218	0.328867	0.330514	0.332161	0.333807	0.335452	0.337095	0.338738	0.34038
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	0.34202	0.34366	0.345298	0.346936	0.348572	0.350207	0.351842	0.353475	0.355107	0.356738
21	0.358368	0.359997	0.361625	0.363251	0.364877	0.366501	0.368125	0.369747	0.371368	0.372988
22	0.374607	0.376224	0.377841	0.379456	0.38107	0.382683	0.384295	0.385906	0.387516	0.389124
23	0.390731	0.392337	0.393942	0.395546	0.397148	0.398749	0.400349	0.401948	0.403545	0.405142
24	0.406737	0.40833	0.409923	0.411514	0.413104	0.414693	0.416281	0.417867	0.419452	0.421036
25	0.422618	0.424199	0.425779	0.427358	0.428935	0.430511	0.432086	0.433659	0.435231	0.436802
26	0.438371	0.439939	0.441506	0.443071	0.444635	0.446198	0.447759	0.449319	0.450878	0.452435
27	0.45399	0.455545	0.457098	0.45865	0.4602	0.461749	0.463296	0.464842	0.466387	0.46793
28	0.469472	0.471012	0.472551	0.474088	0.475624	0.477159	0.478692	0.480223	0.481754	0.483282
29	0.48481	0.486335	0.48786	0.489382	0.490904	0.492424	0.493942	0.495459	0.496974	0.498488
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
30	0.5	0.501511	0.50302	0.504528	0.506034	0.507538	0.509041	0.510543	0.512043	0.513541
31	0.515038	0.516533	0.518027	0.519519	0.52101	0.522499	0.523986	0.525472	0.526956	0.528438

32	0.529919	0.531399	0.532876	0.534352	0.535827	0.5373	0.538771	0.54024	0.541708	0.543174
33	0.544639	0.546102	0.547563	0.549023	0.550481	0.551937	0.553392	0.554844	0.556296	0.557745
34	0.559193	0.560639	0.562083	0.563526	0.564967	0.566406	0.567844	0.56928	0.570714	0.572146
35	0.573576	0.575005	0.576432	0.577858	0.579281	0.580703	0.582123	0.583541	0.584958	0.586372
36	0.587785	0.589196	0.590606	0.592013	0.593419	0.594823	0.596225	0.597625	0.599024	0.60042
37	0.601815	0.603208	0.604599	0.605988	0.607376	0.608761	0.610145	0.611527	0.612907	0.614285
38	0.615661	0.617036	0.618408	0.619779	0.621148	0.622515	0.62388	0.625243	0.626604	0.627963
39	0.62932	0.630676	0.632029	0.633381	0.634731	0.636078	0.637424	0.638768	0.64011	0.64145
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	0.642788	0.644124	0.645458	0.64679	0.64812	0.649448	0.650774	0.652098	0.653421	0.654741
41	0.656059	0.657375	0.658689	0.660002	0.661312	0.66262	0.663926	0.66523	0.666532	0.667833
42	0.669131	0.670427	0.671721	0.673013	0.674302	0.67559	0.676876	0.67816	0.679441	0.680721
43	0.681998	0.683274	0.684547	0.685818	0.687088	0.688355	0.68962	0.690882	0.692143	0.693402
44	0.694658	0.695913	0.697165	0.698415	0.699663	0.700909	0.702153	0.703395	0.704634	0.705872
45	0.707107	0.70834	0.709571	0.710799	0.712026	0.71325	0.714473	0.715693	0.716911	0.718126
46	0.71934	0.720551	0.72176	0.722967	0.724172	0.725374	0.726575	0.727773	0.728969	0.730162
47	0.731354	0.732543	0.73373	0.734915	0.736097	0.737277	0.738455	0.739631	0.740805	0.741976
48	0.743145	0.744312	0.745476	0.746638	0.747798	0.748956	0.750111	0.751264	0.752415	0.753563
49	0.75471	0.755853	0.756995	0.758134	0.759271	0.760406	0.761538	0.762668	0.763796	0.764921
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	0.766044	0.767165	0.768284	0.7694	0.770513	0.771625	0.772734	0.77384	0.774944	0.776046
51	0.777146	0.778243	0.779338	0.78043	0.78152	0.782608	0.783693	0.784776	0.785857	0.786935
52	0.788011	0.789084	0.790155	0.791224	0.79229	0.793353	0.794415	0.795473	0.79653	0.797584
53	0.798636	0.799685	0.800731	0.801776	0.802817	0.803857	0.804894	0.805928	0.80696	0.80799
54	0.809017	0.810042	0.811064	0.812084	0.813101	0.814116	0.815128	0.816138	0.817145	0.81815
55	0.819152	0.820152	0.821149	0.822144	0.823136	0.824126	0.825113	0.826098	0.827081	0.82806
56	0.829038	0.830012	0.830984	0.831954	0.832921	0.833886	0.834848	0.835807	0.836764	0.837719
57	0.838671	0.83962	0.840567	0.841511	0.842452	0.843391	0.844328	0.845262	0.846193	0.847122
58	0.848048	0.848972	0.849893	0.850811	0.851727	0.85264	0.853551	0.854459	0.855364	0.856267
59	0.857167	0.858065	0.85896	0.859852	0.860742	0.861629	0.862514	0.863396	0.864275	0.865151
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	0.866025	0.866897	0.867765	0.868632	0.869495	0.870356	0.871214	0.872069	0.872922	0.873772
61	0.87462	0.875465	0.876307	0.877146	0.877983	0.878817	0.879649	0.880477	0.881303	0.882127
62	0.882948	0.883766	0.884581	0.885394	0.886204	0.887011	0.887815	0.888617	0.889416	0.890213
63	0.891007	0.891798	0.892586	0.893371	0.894154	0.894934	0.895712	0.896486	0.897258	0.898028
64	0.898794	0.899558	0.900319	0.901077	0.901833	0.902585	0.903335	0.904083	0.904827	0.905569
65	0.906308	0.907044	0.907777	0.908508	0.909236	0.909961	0.910684	0.911403	0.91212	0.912834
66	0.913545	0.914254	0.91496	0.915663	0.916363	0.91706	0.917755	0.918446	0.919135	0.919821

x	0	6'	12'	18'	24'	30'	36'	42'	48'	54'
67	0.920505	0.921185	0.921863	0.922538	0.92321	0.92388	0.924546	0.92521	0.925871	0.926529
68	0.927184	0.927836	0.928486	0.929133	0.929776	0.930418	0.931056	0.931691	0.932324	0.932954
69	0.93358	0.934204	0.934826	0.935444	0.93606	0.936672	0.937282	0.937889	0.938493	0.939094
x	0	6'	12'	18'	24'	30'	36'	42'	48'	54'
70	0.939693	0.940288	0.940881	0.941471	0.942057	0.942641	0.943223	0.943801	0.944376	0.944949
71	0.945519	0.946085	0.946649	0.94721	0.947768	0.948324	0.948876	0.949425	0.949972	0.950516
72	0.951057	0.951594	0.952129	0.952661	0.953191	0.953717	0.95424	0.954761	0.955278	0.955793
73	0.956305	0.956814	0.957319	0.957822	0.958323	0.95882	0.959314	0.959805	0.960294	0.960779
74	0.961262	0.961741	0.962218	0.962692	0.963163	0.96363	0.964095	0.964557	0.965016	0.965473
75	0.965926	0.966376	0.966823	0.967268	0.967709	0.968148	0.968583	0.969016	0.969445	0.969872
76	0.970296	0.970716	0.971134	0.971549	0.971961	0.97237	0.972776	0.973179	0.973579	0.973976
77	0.97437	0.974761	0.975149	0.975535	0.975917	0.976296	0.976672	0.977046	0.977416	0.977783
78	0.978148	0.978509	0.978867	0.979223	0.979575	0.979925	0.980271	0.980615	0.980955	0.981293
79	0.981627	0.981959	0.982287	0.982613	0.982935	0.983255	0.983571	0.983885	0.984196	0.984503
x	0	6'	12'	18'	24'	30'	36'	42'	48'	54'
80	0.984808	0.985109	0.985408	0.985703	0.985996	0.986286	0.986572	0.986856	0.987136	0.987414
81	0.987688	0.98796	0.988228	0.988494	0.988756	0.989016	0.989272	0.989526	0.989776	0.990024
82	0.990268	0.990509	0.990748	0.990983	0.991216	0.991445	0.991671	0.991894	0.992115	0.992332
83	0.992546	0.992757	0.992966	0.993171	0.993373	0.993572	0.993768	0.993961	0.994151	0.994338
84	0.994522	0.994703	0.994881	0.995056	0.995227	0.995396	0.995562	0.995725	0.995884	0.996041
85	0.996195	0.996345	0.996493	0.996637	0.996779	0.996917	0.997053	0.997185	0.997314	0.997441
86	0.997564	0.997684	0.997801	0.997916	0.998027	0.998135	0.99824	0.998342	0.998441	0.998537
87	0.99863	0.998719	0.998806	0.99889	0.998971	0.999048	0.999123	0.999194	0.999263	0.999328
88	0.999391	0.99945	0.999507	0.99956	0.99961	0.999657	0.999701	0.999743	0.999781	0.999816
89	0.999848	0.999877	0.999903	0.999925	0.999945	0.999962	0.999976	0.999986	0.999994	0.999998
90	1	0.999998	0.999994	0.999986	0.999976	0.999962	0.999945	0.999925	0.999903	0.999877

Table of Logarithms of Natural Sines

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	#NUM!	3.241877	3.542906	3.718997	3.843934	3.940842	4.020021	4.086965	4.144953	4.196102
1	4.241855	4.283243	4.321027	4.355783	4.387962	4.417919	4.445941	4.472263	4.497078	4.520551
2	4.542819	4.563999	4.584193	4.603489	4.621962	4.63968	4.656702	4.67308	4.688863	4.70409
3	4.7188	4.733027	4.746802	4.760151	4.773101	4.785675	4.797894	4.809777	4.821343	4.832607
4	4.843585	4.854291	4.864738	4.874938	4.884903	4.894643	4.904169	4.913488	4.92261	4.931544
5	4.940296	4.948874	4.957284	4.965534	4.973628	4.981573	4.989374	4.997036	5.004563	5.011962
6	5.019235	5.026386	5.033421	5.040342	5.047154	5.053859	5.06046	5.066962	5.073366	5.079676
7	5.085894	5.092024	5.098066	5.104025	5.109901	5.115698	5.121417	5.12706	5.13263	5.138128
8	5.143555	5.148915	5.154208	5.159435	5.1646	5.169702	5.174744	5.179726	5.184651	5.189519
9	5.194332	5.199091	5.203797	5.208452	5.213055	5.217609	5.222115	5.226573	5.230984	5.235349
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	5.23967	5.243947	5.248181	5.252373	5.256523	5.260633	5.264703	5.268734	5.272726	5.276681
11	5.280599	5.28448	5.288326	5.292137	5.295913	5.299655	5.303364	5.307041	5.310685	5.314297
12	5.317879	5.32143	5.32495	5.328442	5.331903	5.335337	5.338742	5.342119	5.345469	5.348792
13	5.352088	5.355358	5.358603	5.361822	5.365016	5.368185	5.37133	5.374452	5.377549	5.380624
14	5.383675	5.386704	5.389711	5.392695	5.395658	5.3986	5.40152	5.40442	5.407299	5.410157
15	5.412996	5.415815	5.418615	5.421395	5.424156	5.426899	5.429623	5.432329	5.435016	5.437686
16	5.440338	5.442973	5.44559	5.448191	5.450775	5.453342	5.455893	5.458427	5.460946	5.463448
17	5.465935	5.468407	5.470863	5.473304	5.47573	5.478142	5.480539	5.482921	5.485289	5.487643
18	5.489982	5.492308	5.494621	5.496919	5.499204	5.501476	5.503735	5.505981	5.508214	5.510434
19	5.512642	5.514837	5.51702	5.51919	5.521349	5.523495	5.52563	5.527753	5.529864	5.531963
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	5.534052	5.536129	5.538194	5.540249	5.542293	5.544325	5.546347	5.548359	5.550359	5.552349
21	5.554329	5.556299	5.558258	5.560207	5.562146	5.564075	5.565995	5.567904	5.569804	5.571695
22	5.573575	5.575447	5.577309	5.579162	5.581005	5.58284	5.584665	5.586482	5.588289	5.590088
23	5.591878	5.593659	5.595432	5.597196	5.598952	5.6007	5.602439	5.60417	5.605892	5.607607
24	5.609313	5.611012	5.612702	5.614385	5.61606	5.617727	5.619386	5.621038	5.622682	5.624319
25	5.625948	5.62757	5.629185	5.630792	5.632392	5.633984	5.63557	5.637148	5.63872	5.640284
26	5.641842	5.643393	5.644936	5.646474	5.648004	5.649527	5.651044	5.652555	5.654059	5.655556
27	5.657047	5.658531	5.660009	5.661481	5.662946	5.664406	5.665859	5.667305	5.668746	5.670181
28	5.671609	5.673032	5.674448	5.675859	5.677264	5.678663	5.680056	5.681443	5.682825	5.684201
29	5.685571	5.686936	5.688295	5.689648	5.690996	5.692339	5.693676	5.695007	5.696334	5.697654
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**

30	5.69897	5.70028	5.701585	5.702885	5.704179	5.705469	5.706753	5.708032	5.709306	5.710575
31	5.711839	5.713098	5.714352	5.715602	5.716846	5.718085	5.71932	5.720549	5.721774	5.722994
32	5.72421	5.72542	5.726626	5.727828	5.729024	5.730217	5.731404	5.732587	5.733765	5.734939
33	5.736109	5.737274	5.738434	5.73959	5.740742	5.741889	5.743033	5.744171	5.745306	5.746436
34	5.747562	5.748683	5.749801	5.750914	5.752023	5.753128	5.754229	5.755326	5.756418	5.757507
35	5.758591	5.759672	5.760748	5.761821	5.762889	5.763954	5.765015	5.766072	5.767124	5.768173
36	5.769219	5.77026	5.771298	5.772331	5.773361	5.774388	5.77541	5.776429	5.777444	5.778455
37	5.779463	5.780467	5.781468	5.782464	5.783458	5.784447	5.785433	5.786416	5.787395	5.78837
38	5.789342	5.79031	5.791275	5.792237	5.793195	5.79415	5.795101	5.796049	5.796993	5.797934
39	5.798872	5.799806	5.800737	5.801665	5.802589	5.803511	5.804428	5.805343	5.806254	5.807163
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	5.808067	5.808969	5.809868	5.810763	5.811655	5.812544	5.81343	5.814313	5.815193	5.816069
41	5.816943	5.817813	5.818681	5.819545	5.820406	5.821265	5.82212	5.822972	5.823821	5.824668
42	5.825511	5.826351	5.827189	5.828023	5.828855	5.829683	5.830509	5.831332	5.832152	5.832969
43	5.833783	5.834595	5.835403	5.836209	5.837012	5.837812	5.83861	5.839404	5.840196	5.840985
44	5.841771	5.842555	5.843336	5.844114	5.844889	5.845662	5.846432	5.847199	5.847964	5.848726
45	5.849485	5.850242	5.850996	5.851747	5.852496	5.853242	5.853986	5.854727	5.855465	5.856201
46	5.856934	5.857665	5.858393	5.859119	5.859842	5.860562	5.86128	5.861996	5.862709	5.863419
47	5.864127	5.864833	5.865536	5.866237	5.866935	5.867631	5.868324	5.869015	5.869704	5.87039
48	5.871073	5.871755	5.872434	5.87311	5.873784	5.874456	5.875126	5.875793	5.876457	5.87712
49	5.87778	5.878438	5.879093	5.879746	5.880397	5.881046	5.881692	5.882336	5.882977	5.883617
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	5.884254	5.884889	5.885522	5.886152	5.88678	5.887406	5.88803	5.888651	5.889271	5.889888
51	5.890503	5.891115	5.891726	5.892334	5.89294	5.893544	5.894146	5.894746	5.895343	5.895939
52	5.896532	5.897123	5.897712	5.898299	5.898884	5.899467	5.900047	5.900626	5.901202	5.901776
53	5.902349	5.902919	5.903487	5.904053	5.904617	5.905179	5.905739	5.906296	5.906852	5.907406
54	5.907958	5.908507	5.909055	5.909601	5.910144	5.910686	5.911226	5.911763	5.912299	5.912833
55	5.913365	5.913894	5.914422	5.914948	5.915472	5.915994	5.916514	5.917032	5.917548	5.918062
56	5.918574	5.919085	5.919593	5.920099	5.920604	5.921107	5.921607	5.922106	5.922603	5.923098
57	5.923591	5.924083	5.924572	5.92506	5.925545	5.926029	5.926511	5.926991	5.92747	5.927946
58	5.92842	5.928893	5.929364	5.929833	5.9303	5.930766	5.931229	5.931691	5.932151	5.932609
59	5.933066	5.93352	5.933973	5.934424	5.934873	5.93532	5.935766	5.93621	5.936652	5.937092
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	5.937531	5.937967	5.938402	5.938836	5.939267	5.939697	5.940125	5.940551	5.940975	5.941398
61	5.941819	5.942239	5.942656	5.943072	5.943486	5.943899	5.944309	5.944718	5.945125	5.945531
62	5.945935	5.946337	5.946738	5.947136	5.947533	5.947929	5.948323	5.948715	5.949105	5.949494
63	5.949881	5.950266	5.95065	5.951032	5.951412	5.951791	5.952168	5.952544	5.952918	5.95329

64	5.95366	5.954029	5.954396	5.954762	5.955126	5.955488	5.955849	5.956208	5.956566	5.956921
65	5.957276	5.957628	5.957979	5.958329	5.958677	5.959023	5.959368	5.959711	5.960052	5.960392
66	5.96073	5.961067	5.961402	5.961735	5.962067	5.962398	5.962727	5.963054	5.963379	5.963704
67	5.964026	5.964347	5.964666	5.964984	5.965301	5.965615	5.965929	5.96624	5.96655	5.966859
68	5.967166	5.967471	5.967775	5.968078	5.968379	5.968678	5.968976	5.969272	5.969567	5.96986
69	5.970152	5.970442	5.970731	5.971018	5.971303	5.971588	5.97187	5.972151	5.972431	5.972709

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
70	5.972986	5.973261	5.973535	5.973807	5.974077	5.974347	5.974614	5.97488	5.975145	5.975408
71	5.97567	5.97593	5.976189	5.976446	5.976702	5.976957	5.977209	5.977461	5.977711	5.977959
72	5.978206	5.978452	5.978696	5.978939	5.97918	5.97942	5.979658	5.979895	5.98013	5.980364
73	5.980596	5.980827	5.981057	5.981285	5.981512	5.981737	5.981961	5.982183	5.982404	5.982624
74	5.982842	5.983058	5.983273	5.983487	5.9837	5.983911	5.98412	5.984328	5.984535	5.98474
75	5.984944	5.985146	5.985347	5.985547	5.985745	5.985942	5.986137	5.986331	5.986523	5.986714
76	5.986904	5.987092	5.987279	5.987465	5.987649	5.987832	5.988013	5.988193	5.988371	5.988548
77	5.988724	5.988898	5.989071	5.989243	5.989413	5.989582	5.989749	5.989915	5.990079	5.990243
78	5.990404	5.990565	5.990724	5.990882	5.991038	5.991193	5.991346	5.991498	5.991649	5.991799
79	5.991947	5.992093	5.992239	5.992382	5.992525	5.992666	5.992806	5.992944	5.993081	5.993217

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
80	5.993351	5.993484	5.993616	5.993746	5.993875	5.994003	5.994129	5.994254	5.994377	5.994499
81	5.99462	5.994739	5.994857	5.994974	5.995089	5.995203	5.995316	5.995427	5.995537	5.995646
82	5.995753	5.995859	5.995963	5.996066	5.996168	5.996269	5.996368	5.996465	5.996562	5.996657
83	5.996751	5.996843	5.996934	5.997024	5.997112	5.997199	5.997285	5.997369	5.997452	5.997534
84	5.997614	5.997693	5.997771	5.997847	5.997922	5.997996	5.998068	5.998139	5.998209	5.998277
85	5.998344	5.99841	5.998474	5.998537	5.998599	5.998659	5.998718	5.998776	5.998832	5.998887
86	5.998941	5.998993	5.999044	5.999094	5.999142	5.999189	5.999235	5.999279	5.999322	5.999364
87	5.999404	5.999443	5.999481	5.999518	5.999553	5.999586	5.999619	5.99965	5.99968	5.999708
88	5.999735	5.999761	5.999786	5.999809	5.999831	5.999851	5.99987	5.999888	5.999905	5.99992
89	5.999934	5.999946	5.999958	5.999968	5.999976	5.999983	5.999989	5.999994	5.999997	5.999999
90	6									

Table of Natural Cosines

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	1	0.999998	0.999994	0.999986	0.999976	0.999962	0.999945	0.999925	0.999903	0.999877
1	0.999848	0.999816	0.999781	0.999743	0.999701	0.999657	0.99961	0.99956	0.999507	0.99945
2	0.999391	0.999328	0.999263	0.999194	0.999123	0.999048	0.998971	0.99889	0.998806	0.998719
3	0.99863	0.998537	0.998441	0.998342	0.99824	0.998135	0.998027	0.997916	0.997801	0.997684
4	0.997564	0.997441	0.997314	0.997185	0.997053	0.996917	0.996779	0.996637	0.996493	0.996345
5	0.996195	0.996041	0.995884	0.995725	0.995562	0.995396	0.995227	0.995056	0.994881	0.994703
6	0.994522	0.994338	0.994151	0.993961	0.993768	0.993572	0.993373	0.993171	0.992966	0.992757
7	0.992546	0.992332	0.992115	0.991894	0.991671	0.991445	0.991216	0.990983	0.990748	0.990509
8	0.990268	0.990024	0.989776	0.989526	0.989272	0.989016	0.988756	0.988494	0.988228	0.98796
9	0.987688	0.987414	0.987136	0.986856	0.986572	0.986286	0.985996	0.985703	0.985408	0.985109
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	0.984808	0.984503	0.984196	0.983885	0.983571	0.983255	0.982935	0.982613	0.982287	0.981959
11	0.981627	0.981293	0.980955	0.980615	0.980271	0.979925	0.979575	0.979223	0.978867	0.978509
12	0.978148	0.977783	0.977416	0.977046	0.976672	0.976296	0.975917	0.975535	0.975149	0.974761
13	0.97437	0.973976	0.973579	0.973179	0.972776	0.97237	0.971961	0.971549	0.971134	0.970716
14	0.970296	0.969872	0.969445	0.969016	0.968583	0.968148	0.967709	0.967268	0.966823	0.966376
15	0.965926	0.965473	0.965016	0.964557	0.964095	0.96363	0.963163	0.962692	0.962218	0.961741
16	0.961262	0.960779	0.960294	0.959805	0.959314	0.95882	0.958323	0.957822	0.957319	0.956814
17	0.956305	0.955793	0.955278	0.954761	0.95424	0.953717	0.953191	0.952661	0.952129	0.951594
18	0.951057	0.950516	0.949972	0.949425	0.948876	0.948324	0.947768	0.94721	0.946649	0.946085
19	0.945519	0.944949	0.944376	0.943801	0.943223	0.942641	0.942057	0.941471	0.940881	0.940288
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	0.939693	0.939094	0.938493	0.937889	0.937282	0.936672	0.93606	0.935444	0.934826	0.934204
21	0.93358	0.932954	0.932324	0.931691	0.931056	0.930418	0.929776	0.929133	0.928486	0.927836
22	0.927184	0.926529	0.925871	0.92521	0.924546	0.92388	0.92321	0.922538	0.921863	0.921185
23	0.920505	0.919821	0.919135	0.918446	0.917755	0.91706	0.916363	0.915663	0.91496	0.914254
24	0.913545	0.912834	0.91212	0.911403	0.910684	0.909961	0.909236	0.908508	0.907777	0.907044
25	0.906308	0.905569	0.904827	0.904083	0.903335	0.902585	0.901833	0.901077	0.900319	0.899558
26	0.898794	0.898028	0.897258	0.896486	0.895712	0.894934	0.894154	0.893371	0.892586	0.891798
27	0.891007	0.890213	0.889416	0.888617	0.887815	0.887011	0.886204	0.885394	0.884581	0.883766
28	0.882948	0.882127	0.881303	0.880477	0.879649	0.878817	0.877983	0.877146	0.876307	0.875465
29	0.87462	0.873772	0.872922	0.872069	0.871214	0.870356	0.869495	0.868632	0.867765	0.866897
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
30	0.866025	0.865151	0.864275	0.863396	0.862514	0.861629	0.860742	0.859852	0.85896	0.858065
31	0.857167	0.856267	0.855364	0.854459	0.853551	0.85264	0.851727	0.850811	0.849893	0.848972

32	0.848048	0.847122	0.846193	0.845262	0.844328	0.843391	0.842452	0.841511	0.840567	0.83962
33	0.838671	0.837719	0.836764	0.835807	0.834848	0.833886	0.832921	0.831954	0.830984	0.830012
34	0.829038	0.82806	0.827081	0.826098	0.825113	0.824126	0.823136	0.822144	0.821149	0.820152
35	0.819152	0.81815	0.817145	0.816138	0.815128	0.814116	0.813101	0.812084	0.811064	0.810042
36	0.809017	0.80799	0.80696	0.805928	0.804894	0.803857	0.802817	0.801776	0.800731	0.799685
37	0.798636	0.797584	0.79653	0.795473	0.794415	0.793353	0.79229	0.791224	0.790155	0.789084
38	0.788011	0.786935	0.785857	0.784776	0.783693	0.782608	0.78152	0.78043	0.779338	0.778243
39	0.777146	0.776046	0.774944	0.77384	0.772734	0.771625	0.770513	0.7694	0.768284	0.767165
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	0.766044	0.764921	0.763796	0.762668	0.761538	0.760406	0.759271	0.758134	0.756995	0.755853
41	0.75471	0.753563	0.752415	0.751264	0.750111	0.748956	0.747798	0.746638	0.745476	0.744312
42	0.743145	0.741976	0.740805	0.739631	0.738455	0.737277	0.736097	0.734915	0.73373	0.732543
43	0.731354	0.730162	0.728969	0.727773	0.726575	0.725374	0.724172	0.722967	0.72176	0.720551
44	0.71934	0.718126	0.716911	0.715693	0.714473	0.71325	0.712026	0.710799	0.709571	0.70834
45	0.707107	0.705872	0.704634	0.703395	0.702153	0.700909	0.699663	0.698415	0.697165	0.695913
46	0.694658	0.693402	0.692143	0.690882	0.68962	0.688355	0.687088	0.685818	0.684547	0.683274
47	0.681998	0.680721	0.679441	0.67816	0.676876	0.67559	0.674302	0.673013	0.671721	0.670427
48	0.669131	0.667833	0.666532	0.66523	0.663926	0.66262	0.661312	0.660002	0.658689	0.657375
49	0.656059	0.654741	0.653421	0.652098	0.650774	0.649448	0.64812	0.64679	0.645458	0.644124
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	0.642788	0.64145	0.64011	0.638768	0.637424	0.636078	0.634731	0.633381	0.632029	0.630676
51	0.62932	0.627963	0.626604	0.625243	0.62388	0.622515	0.621148	0.619779	0.618408	0.617036
52	0.615661	0.614285	0.612907	0.611527	0.610145	0.608761	0.607376	0.605988	0.604599	0.603208
53	0.601815	0.60042	0.599024	0.597625	0.596225	0.594823	0.593419	0.592013	0.590606	0.589196
54	0.587785	0.586372	0.584958	0.583541	0.582123	0.580703	0.579281	0.577858	0.576432	0.575005
55	0.573576	0.572146	0.570714	0.56928	0.567844	0.566406	0.564967	0.563526	0.562083	0.560639
56	0.559193	0.557745	0.556296	0.554844	0.553392	0.551937	0.550481	0.549023	0.547563	0.546102
57	0.544639	0.543174	0.541708	0.54024	0.538771	0.5373	0.535827	0.534352	0.532876	0.531399
58	0.529919	0.528438	0.526956	0.525472	0.523986	0.522499	0.52101	0.519519	0.518027	0.516533
59	0.515038	0.513541	0.512043	0.510543	0.509041	0.507538	0.506034	0.504528	0.50302	0.501511
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	0.5	0.498488	0.496974	0.495459	0.493942	0.492424	0.490904	0.489382	0.48786	0.486335
61	0.48481	0.483282	0.481754	0.480223	0.478692	0.477159	0.475624	0.474088	0.472551	0.471012
62	0.469472	0.46793	0.466387	0.464842	0.463296	0.461749	0.4602	0.45865	0.457098	0.455545
63	0.45399	0.452435	0.450878	0.449319	0.447759	0.446198	0.444635	0.443071	0.441506	0.439939
64	0.438371	0.436802	0.435231	0.433659	0.432086	0.430511	0.428935	0.427358	0.425779	0.424199
65	0.422618	0.421036	0.419452	0.417867	0.416281	0.414693	0.413104	0.411514	0.409923	0.40833
66	0.406737	0.405142	0.403545	0.401948	0.400349	0.398749	0.397148	0.395546	0.393942	0.392337

67	0.390731	0.389124	0.387516	0.385906	0.384295	0.382683	0.38107	0.379456	0.377841	0.376224
68	0.374607	0.372988	0.371368	0.369747	0.368125	0.366501	0.364877	0.363251	0.361625	0.359997
69	0.358368	0.356738	0.355107	0.353475	0.351842	0.350207	0.348572	0.346936	0.345298	0.34366
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	0.34202	0.34038	0.338738	0.337095	0.335452	0.333807	0.332161	0.330514	0.328867	0.327218
71	0.325568	0.323917	0.322266	0.320613	0.318959	0.317305	0.315649	0.313992	0.312335	0.310676
72	0.309017	0.307357	0.305695	0.304033	0.30237	0.300706	0.299041	0.297375	0.295708	0.29404
73	0.292372	0.290702	0.289032	0.287361	0.285688	0.284015	0.282341	0.280667	0.278991	0.277315
74	0.275637	0.273959	0.27228	0.2706	0.26892	0.267238	0.265556	0.263873	0.262189	0.260505
75	0.258819	0.257133	0.255446	0.253758	0.252069	0.25038	0.24869	0.246999	0.245307	0.243615
76	0.241922	0.240228	0.238533	0.236838	0.235142	0.233445	0.231748	0.23005	0.228351	0.226651
77	0.224951	0.22325	0.221548	0.219846	0.218143	0.21644	0.214735	0.21303	0.211325	0.209619
78	0.207912	0.206204	0.204496	0.202787	0.201078	0.199368	0.197657	0.195946	0.194234	0.192522
79	0.190809	0.189095	0.187381	0.185667	0.183951	0.182236	0.180519	0.178802	0.177085	0.175367
x	**0**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	0.173648	0.171929	0.170209	0.168489	0.166769	0.165048	0.163326	0.161604	0.159881	0.158158
81	0.156434	0.15471	0.152986	0.151261	0.149535	0.147809	0.146083	0.144356	0.142629	0.140901
82	0.139173	0.137445	0.135716	0.133986	0.132256	0.130526	0.128796	0.127065	0.125333	0.123601
83	0.121869	0.120137	0.118404	0.116671	0.114937	0.113203	0.111469	0.109734	0.107999	0.106264
84	0.104528	0.102793	0.101056	0.09932	0.097583	0.095846	0.094108	0.092371	0.090633	0.088894
85	0.087156	0.085417	0.083678	0.081939	0.080199	0.078459	0.076719	0.074979	0.073238	0.071497
86	0.069756	0.068015	0.066274	0.064532	0.062791	0.061049	0.059306	0.057564	0.055822	0.054079
87	0.052336	0.050593	0.04885	0.047106	0.045363	0.043619	0.041876	0.040132	0.038388	0.036644
88	0.034899	0.033155	0.031411	0.029666	0.027922	0.026177	0.024432	0.022687	0.020942	0.019197
89	0.017452	0.015707	0.013962	0.012217	0.010472	0.008727	0.006981	0.005236	0.003491	0.001745
90	6.13E-17									

Table of Logarithms of Natural Cosines

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	6	5.999999	5.999997	5.999994	5.999989	5.999983	5.999976	5.999968	5.999958	5.999946
1	5.999934	5.99992	5.999905	5.999888	5.99987	5.999851	5.999831	5.999809	5.999786	5.999761
2	5.999735	5.999708	5.99968	5.99965	5.999619	5.999586	5.999553	5.999518	5.999481	5.999443
3	5.999404	5.999364	5.999322	5.999279	5.999235	5.999189	5.999142	5.999094	5.999044	5.998993
4	5.998941	5.998887	5.998832	5.998776	5.998718	5.998659	5.998599	5.998537	5.998474	5.99841
5	5.998344	5.998277	5.998209	5.998139	5.998068	5.997996	5.997922	5.997847	5.997771	5.997693
6	5.997614	5.997534	5.997452	5.997369	5.997285	5.997199	5.997112	5.997024	5.996934	5.996843
7	5.996751	5.996657	5.996562	5.996465	5.996368	5.996269	5.996168	5.996066	5.995963	5.995859
8	5.995753	5.995646	5.995537	5.995427	5.995316	5.995203	5.995089	5.994974	5.994857	5.994739
9	5.99462	5.994499	5.994377	5.994254	5.994129	5.994003	5.993875	5.993746	5.993616	5.993484
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	5.993351	5.993217	5.993081	5.992944	5.992806	5.992666	5.992525	5.992382	5.992239	5.992093
11	5.991947	5.991799	5.991649	5.991498	5.991346	5.991193	5.991038	5.990882	5.990724	5.990565
12	5.990404	5.990243	5.990079	5.989915	5.989749	5.989582	5.989413	5.989243	5.989071	5.988898
13	5.988724	5.988548	5.988371	5.988193	5.988013	5.987832	5.987649	5.987465	5.987279	5.987092
14	5.986904	5.986714	5.986523	5.986331	5.986137	5.985942	5.985745	5.985547	5.985347	5.985146
15	5.984944	5.98474	5.984535	5.984328	5.98412	5.983911	5.9837	5.983487	5.983273	5.983058
16	5.982842	5.982624	5.982404	5.982183	5.981961	5.981737	5.981512	5.981285	5.981057	5.980827
17	5.980596	5.980364	5.98013	5.979895	5.979658	5.97942	5.97918	5.978939	5.978696	5.978452
18	5.978206	5.977959	5.977711	5.977461	5.977209	5.976957	5.976702	5.976446	5.976189	5.97593
19	5.97567	5.975408	5.975145	5.97488	5.974614	5.974347	5.974077	5.973807	5.973535	5.973261
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	5.972986	5.972709	5.972431	5.972151	5.97187	5.971588	5.971303	5.971018	5.970731	5.970442
21	5.970152	5.96986	5.969567	5.969272	5.968976	5.968678	5.968379	5.968078	5.967775	5.967471
22	5.967166	5.966859	5.96655	5.96624	5.965929	5.965615	5.965301	5.964984	5.964666	5.964347
23	5.964026	5.963704	5.963379	5.963054	5.962727	5.962398	5.962067	5.961735	5.961402	5.961067
24	5.96073	5.960392	5.960052	5.959711	5.959368	5.959023	5.958677	5.958329	5.957979	5.957628
25	5.957276	5.956921	5.956566	5.956208	5.955849	5.955488	5.955126	5.954762	5.954396	5.954029
26	5.95366	5.95329	5.952918	5.952544	5.952168	5.951791	5.951412	5.951032	5.95065	5.950266
27	5.949881	5.949494	5.949105	5.948715	5.948323	5.947929	5.947533	5.947136	5.946738	5.946337
28	5.945935	5.945531	5.945125	5.944718	5.944309	5.943899	5.943486	5.943072	5.942656	5.942239
29	5.941819	5.941398	5.940975	5.940551	5.940125	5.939697	5.939267	5.938836	5.938402	5.937967
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**

30	5.937531	5.937092	5.936652	5.93621	5.935766	5.93532	5.934873	5.934424	5.933973	5.93352
31	5.933066	5.932609	5.932151	5.931691	5.931229	5.930766	5.9303	5.929833	5.929364	5.928893
32	5.92842	5.927946	5.92747	5.926991	5.926511	5.926029	5.925545	5.92506	5.924572	5.924083
33	5.923591	5.923098	5.922603	5.922106	5.921607	5.921107	5.920604	5.920099	5.919593	5.919085
34	5.918574	5.918062	5.917548	5.917032	5.916514	5.915994	5.915472	5.914948	5.914422	5.913894
35	5.913365	5.912833	5.912299	5.911763	5.911226	5.910686	5.910144	5.909601	5.909055	5.908507
36	5.907958	5.907406	5.906852	5.906296	5.905739	5.905179	5.904617	5.904053	5.903487	5.902919
37	5.902349	5.901776	5.901202	5.900626	5.900047	5.899467	5.898884	5.898299	5.897712	5.897123
38	5.896532	5.895939	5.895343	5.894746	5.894146	5.893544	5.89294	5.892334	5.891726	5.891115
39	5.890503	5.889888	5.889271	5.888651	5.88803	5.887406	5.88678	5.886152	5.885522	5.884889

x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	5.884254	5.883617	5.882977	5.882336	5.881692	5.881046	5.880397	5.879746	5.879093	5.878438
41	5.87778	5.87712	5.876457	5.875793	5.875126	5.874456	5.873784	5.87311	5.872434	5.871755
42	5.871073	5.87039	5.869704	5.869015	5.868324	5.867631	5.866935	5.866237	5.865536	5.864833
43	5.864127	5.863419	5.862709	5.861996	5.86128	5.860562	5.859842	5.859119	5.858393	5.857665
44	5.856934	5.856201	5.855465	5.854727	5.853986	5.853242	5.852496	5.851747	5.850996	5.850242
45	5.849485	5.848726	5.847964	5.847199	5.846432	5.845662	5.844889	5.844114	5.843336	5.842555
46	5.841771	5.840985	5.840196	5.839404	5.83861	5.837812	5.837012	5.836209	5.835403	5.834595
47	5.833783	5.832969	5.832152	5.831332	5.830509	5.829683	5.828855	5.828023	5.827189	5.826351
48	5.825511	5.824668	5.823821	5.822972	5.82212	5.821265	5.820406	5.819545	5.818681	5.817813
49	5.816943	5.816069	5.815193	5.814313	5.81343	5.812544	5.811655	5.810763	5.809868	5.808969

x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	5.808067	5.807163	5.806254	5.805343	5.804428	5.803511	5.802589	5.801665	5.800737	5.799806
51	5.798872	5.797934	5.796993	5.796049	5.795101	5.79415	5.793195	5.792237	5.791275	5.79031
52	5.789342	5.78837	5.787395	5.786416	5.785433	5.784447	5.783458	5.782464	5.781468	5.780467
53	5.779463	5.778455	5.777444	5.776429	5.77541	5.774388	5.773361	5.772331	5.771298	5.77026
54	5.769219	5.768173	5.767124	5.766072	5.765015	5.763954	5.762889	5.761821	5.760748	5.759672
55	5.758591	5.757507	5.756418	5.755326	5.754229	5.753128	5.752023	5.750914	5.749801	5.748683
56	5.747562	5.746436	5.745306	5.744171	5.743033	5.741889	5.740742	5.73959	5.738434	5.737274
57	5.736109	5.734939	5.733765	5.732587	5.731404	5.730217	5.729024	5.727828	5.726626	5.72542
58	5.72421	5.722994	5.721774	5.720549	5.71932	5.718085	5.716846	5.715602	5.714352	5.713098
59	5.711839	5.710575	5.709306	5.708032	5.706753	5.705469	5.704179	5.702885	5.701585	5.70028

x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	5.69897	5.697654	5.696334	5.695007	5.693676	5.692339	5.690996	5.689648	5.688295	5.686936
61	5.685571	5.684201	5.682825	5.681443	5.680056	5.678663	5.677264	5.675859	5.674448	5.673032
62	5.671609	5.670181	5.668746	5.667305	5.665859	5.664406	5.662946	5.661481	5.660009	5.658531
63	5.657047	5.655556	5.654059	5.652555	5.651044	5.649527	5.648004	5.646474	5.644936	5.643393

64	5.641842	5.640284	5.63872	5.637148	5.63557	5.633984	5.632392	5.630792	5.629185	5.62757
65	5.625948	5.624319	5.622682	5.621038	5.619386	5.617727	5.61606	5.614385	5.612702	5.611012
66	5.609313	5.607607	5.605892	5.60417	5.602439	5.6007	5.598952	5.597196	5.595432	5.593659
67	5.591878	5.590088	5.588289	5.586482	5.584665	5.58284	5.581005	5.579162	5.577309	5.575447
68	5.573575	5.571695	5.569804	5.567904	5.565995	5.564075	5.562146	5.560207	5.558258	5.556299
69	5.554329	5.552349	5.550359	5.548359	5.546347	5.544325	5.542293	5.540249	5.538194	5.536129
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	5.534052	5.531963	5.529864	5.527753	5.52563	5.523495	5.521349	5.51919	5.51702	5.514837
71	5.512642	5.510434	5.508214	5.505981	5.503735	5.501476	5.499204	5.496919	5.494621	5.492308
72	5.489982	5.487643	5.485289	5.482921	5.480539	5.478142	5.47573	5.473304	5.470863	5.468407
73	5.465935	5.463448	5.460946	5.458427	5.455893	5.453342	5.450775	5.448191	5.44559	5.442973
74	5.440338	5.437686	5.435016	5.432329	5.429623	5.426899	5.424156	5.421395	5.418615	5.415815
75	5.412996	5.410157	5.407299	5.40442	5.40152	5.3986	5.395658	5.392695	5.389711	5.386704
76	5.383675	5.380624	5.377549	5.374452	5.37133	5.368185	5.365016	5.361822	5.358603	5.355358
77	5.352088	5.348792	5.345469	5.342119	5.338742	5.335337	5.331903	5.328442	5.32495	5.32143
78	5.317879	5.314297	5.310685	5.307041	5.303364	5.299655	5.295913	5.292137	5.288326	5.28448
79	5.280599	5.276681	5.272726	5.268734	5.264703	5.260633	5.256523	5.252373	5.248181	5.243947
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	5.23967	5.235349	5.230984	5.226573	5.222115	5.217609	5.213055	5.208452	5.203797	5.199091
81	5.194332	5.189519	5.184651	5.179726	5.174744	5.169702	5.1646	5.159435	5.154208	5.148915
82	5.143555	5.138128	5.13263	5.12706	5.121417	5.115698	5.109901	5.104025	5.098066	5.092024
83	5.085894	5.079676	5.073366	5.066962	5.06046	5.053859	5.047154	5.040342	5.033421	5.026386
84	5.019235	5.011962	5.004563	4.997036	4.989374	4.981573	4.973628	4.965534	4.957284	4.948874
85	4.940296	4.931544	4.92261	4.913488	4.904169	4.894643	4.884903	4.874938	4.864738	4.854291
86	4.843585	4.832607	4.821343	4.809777	4.797894	4.785675	4.773101	4.760151	4.746802	4.733027
87	4.7188	4.70409	4.688863	4.67308	4.656702	4.63968	4.621962	4.603489	4.584193	4.563999
88	4.542819	4.520551	4.497078	4.472263	4.445941	4.417919	4.387962	4.355783	4.321027	4.283243
89	4.241855	4.196102	4.144953	4.086965	4.020021	3.940842	3.843934	3.718997	3.542906	3.241877
90	#NUM!									

Table of Natural Tangents

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	0	0.001745	0.003491	0.005236	0.006981	0.008727	0.010472	0.012218	0.013964	0.015709
1	0.017455	0.019201	0.020947	0.022693	0.024439	0.026186	0.027933	0.029679	0.031426	0.033173
2	0.034921	0.036668	0.038416	0.040164	0.041912	0.043661	0.04541	0.047159	0.048908	0.050658
3	0.052408	0.054158	0.055909	0.05766	0.059411	0.061163	0.062915	0.064667	0.06642	0.068173
4	0.069927	0.071681	0.073435	0.07519	0.076946	0.078702	0.080458	0.082215	0.083972	0.08573
5	0.087489	0.089248	0.091007	0.092767	0.094528	0.096289	0.098051	0.099813	0.101576	0.10334
6	0.105104	0.106869	0.108635	0.110401	0.112168	0.113936	0.115704	0.117473	0.119243	0.121013
7	0.122785	0.124557	0.126329	0.128103	0.129877	0.131652	0.133428	0.135205	0.136983	0.138761
8	0.140541	0.142321	0.144102	0.145884	0.147667	0.149451	0.151236	0.153022	0.154808	0.156596
9	0.158384	0.160174	0.161965	0.163756	0.165549	0.167343	0.169137	0.170933	0.17273	0.174528
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	0.176327	0.178127	0.179928	0.181731	0.183534	0.185339	0.187145	0.188952	0.19076	0.19257
11	0.19438	0.196192	0.198005	0.19982	0.201635	0.203452	0.205271	0.20709	0.208911	0.210733
12	0.212557	0.214381	0.216208	0.218035	0.219864	0.221695	0.223526	0.22536	0.227194	0.229031
13	0.230868	0.232707	0.234548	0.23639	0.238234	0.240079	0.241925	0.243774	0.245624	0.247475
14	0.249328	0.251183	0.253039	0.254897	0.256756	0.258618	0.26048	0.262345	0.264211	0.266079
15	0.267949	0.269821	0.271694	0.273569	0.275446	0.277325	0.279205	0.281087	0.282971	0.284857
16	0.286745	0.288635	0.290527	0.29242	0.294316	0.296213	0.298113	0.300014	0.301918	0.303823
17	0.305731	0.30764	0.309552	0.311465	0.313381	0.315299	0.317219	0.319141	0.321065	0.322991
18	0.32492	0.32685	0.328783	0.330718	0.332656	0.334595	0.336537	0.338481	0.340428	0.342377
19	0.344328	0.346281	0.348237	0.350195	0.352156	0.354119	0.356084	0.358052	0.360022	0.361995
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	0.36397	0.365948	0.367928	0.369911	0.371897	0.373885	0.375875	0.377869	0.379864	0.381863
21	0.383864	0.385868	0.387874	0.389884	0.391896	0.39391	0.395928	0.397948	0.399971	0.401997
22	0.404026	0.406058	0.408092	0.41013	0.41217	0.414214	0.41626	0.418309	0.420361	0.422417
23	0.424475	0.426536	0.428601	0.430668	0.432739	0.434812	0.436889	0.438969	0.441053	0.443139
24	0.445229	0.447322	0.449418	0.451517	0.45362	0.455726	0.457836	0.459949	0.462065	0.464185
25	0.466308	0.468434	0.470564	0.472698	0.474835	0.476976	0.47912	0.481267	0.483419	0.485574
26	0.487733	0.489895	0.492061	0.494231	0.496404	0.498582	0.500763	0.502948	0.505136	0.507329
27	0.509525	0.511726	0.51393	0.516138	0.518351	0.520567	0.522787	0.525012	0.52724	0.529473
28	0.531709	0.53395	0.536195	0.538445	0.540698	0.542956	0.545218	0.547484	0.549755	0.55203
29	0.554309	0.556593	0.558881	0.561174	0.563471	0.565773	0.568079	0.57039	0.572705	0.575026
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**

30	0.57735	0.57968	0.582014	0.584353	0.586697	0.589045	0.591398	0.593757	0.59612	0.598488
31	0.600861	0.603239	0.605622	0.60801	0.610403	0.612801	0.615204	0.617613	0.620026	0.622445
32	0.624869	0.627299	0.629734	0.632174	0.634619	0.63707	0.639527	0.641989	0.644456	0.646929
33	0.649408	0.651892	0.654382	0.656877	0.659379	0.661886	0.664398	0.666917	0.669442	0.671972
34	0.674509	0.677051	0.679599	0.682154	0.684714	0.687281	0.689854	0.692433	0.695018	0.69761
35	0.700208	0.702812	0.705422	0.708039	0.710663	0.713293	0.71593	0.718573	0.721223	0.723879
36	0.726543	0.729213	0.731889	0.734573	0.737264	0.739961	0.742666	0.745377	0.748096	0.750821
37	0.753554	0.756294	0.759041	0.761796	0.764558	0.767327	0.770104	0.772888	0.77568	0.778479
38	0.781286	0.7841	0.786922	0.789752	0.79259	0.795436	0.79829	0.801151	0.804021	0.806898
39	0.809784	0.812678	0.81558	0.818491	0.821409	0.824336	0.827272	0.830216	0.833169	0.83613

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
40	0.8391	0.842078	0.845066	0.848062	0.851067	0.854081	0.857104	0.860136	0.863177	0.866227
41	0.869287	0.872356	0.875434	0.878521	0.881619	0.884725	0.887842	0.890967	0.894103	0.897249
42	0.900404	0.903569	0.906745	0.90993	0.913125	0.916331	0.919547	0.922773	0.92601	0.929257
43	0.932515	0.935783	0.939063	0.942352	0.945653	0.948965	0.952287	0.955621	0.958966	0.962322
44	0.965689	0.969067	0.972458	0.975859	0.979272	0.982697	0.986134	0.989582	0.993043	0.996515
45	1	1.003497	1.007006	1.010527	1.014061	1.017607	1.021166	1.024738	1.028323	1.03192
46	1.03553	1.039154	1.04279	1.04644	1.050103	1.05378	1.05747	1.061174	1.064892	1.068623
47	1.072369	1.076128	1.079902	1.08369	1.087492	1.091309	1.09514	1.098986	1.102846	1.106722
48	1.110613	1.114518	1.118439	1.122375	1.126327	1.130294	1.134277	1.138276	1.142291	1.146322
49	1.150368	1.154432	1.158511	1.162607	1.16672	1.17085	1.174996	1.17916	1.18334	1.187538

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
50	1.191754	1.195987	1.200237	1.204506	1.208792	1.213097	1.21742	1.221761	1.226121	1.2305
51	1.234897	1.239314	1.243749	1.248204	1.252678	1.257172	1.261686	1.26622	1.270773	1.275347
52	1.279942	1.284557	1.289192	1.293849	1.298526	1.303225	1.307946	1.312688	1.317451	1.322237
53	1.327045	1.331875	1.336728	1.341603	1.346501	1.351422	1.356367	1.361335	1.366327	1.371342
54	1.376382	1.381446	1.386534	1.391647	1.396785	1.401948	1.407137	1.412351	1.41759	1.422856
55	1.428148	1.433466	1.438811	1.444183	1.449583	1.455009	1.460463	1.465945	1.471455	1.476994
56	1.482561	1.488157	1.493782	1.499437	1.505121	1.510835	1.51658	1.522355	1.52816	1.533997
57	1.539865	1.545765	1.551696	1.55766	1.563656	1.569686	1.575748	1.581844	1.587973	1.594137
58	1.600335	1.606567	1.612835	1.619138	1.625477	1.631852	1.638263	1.644711	1.651196	1.657719
59	1.664279	1.670878	1.677516	1.684192	1.690908	1.697663	1.704459	1.711295	1.718172	1.725091

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
60	1.732051	1.739053	1.746098	1.753187	1.760318	1.767494	1.774714	1.781979	1.789289	1.796645
61	1.804048	1.811497	1.818993	1.826537	1.83413	1.841771	1.849461	1.857202	1.864992	1.872834
62	1.880726	1.888671	1.896669	1.904719	1.912824	1.920982	1.929196	1.937465	1.94579	1.954171
63	1.962611	1.971108	1.979664	1.988279	1.996954	2.00569	2.014487	2.023346	2.032268	2.041254

64	2.050304	2.059419	2.068599	2.077847	2.087161	2.096544	2.105995	2.115516	2.125108	2.134771
65	2.144507	2.154316	2.164198	2.174156	2.184189	2.1943	2.204488	2.214754	2.225101	2.235528
66	2.246037	2.256628	2.267304	2.278064	2.28891	2.299843	2.310864	2.321974	2.333175	2.344467
67	2.355852	2.367332	2.378906	2.390577	2.402346	2.414214	2.426182	2.438252	2.450425	2.462703
68	2.475087	2.487578	2.500178	2.512889	2.525712	2.538648	2.551699	2.564867	2.578154	2.591561
69	2.605089	2.618741	2.632519	2.646423	2.660457	2.674621	2.688919	2.703351	2.71792	2.732628
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	2.747477	2.76247	2.777607	2.792892	2.808326	2.823913	2.839654	2.855552	2.871609	2.887828
71	2.904211	2.920761	2.937481	2.954373	2.97144	2.988685	3.006111	3.023721	3.041517	3.059504
72	3.077684	3.09606	3.114635	3.133414	3.152399	3.171595	3.191004	3.21063	3.230478	3.250551
73	3.270853	3.291388	3.31216	3.333174	3.354433	3.375943	3.397709	3.419733	3.442023	3.464581
74	3.487414	3.510527	3.533925	3.557613	3.581598	3.605884	3.630477	3.655384	3.680611	3.706165
75	3.732051	3.758276	3.784848	3.811773	3.839059	3.866713	3.894743	3.923156	3.951962	3.981167
76	4.010781	4.040813	4.071271	4.102165	4.133505	4.1653	4.197561	4.230298	4.263522	4.297244
77	4.331476	4.366229	4.401516	4.43735	4.473743	4.510709	4.548261	4.586414	4.625183	4.664583
78	4.70463	4.74534	4.78673	4.828817	4.87162	4.915157	4.959447	5.004511	5.050369	5.097043
79	5.144554	5.192926	5.242184	5.29235	5.343453	5.395517	5.448572	5.502645	5.557766	5.613968
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	5.671282	5.729742	5.789383	5.850241	5.912355	5.975764	6.04051	6.106636	6.174186	6.243209
81	6.313752	6.385866	6.459607	6.535029	6.612192	6.691156	6.771987	6.854751	6.939519	7.026366
82	7.11537	7.206612	7.300178	7.39616	7.494651	7.595754	7.699574	7.806221	7.915815	8.02848
83	8.144346	8.263555	8.386252	8.512594	8.642747	8.776887	8.915201	9.057887	9.205156	9.357236
84	9.514364	9.6768	9.844817	10.01871	10.19879	10.3854	10.57889	10.77967	10.98815	11.20478
85	11.43005	11.6645	11.90868	12.16324	12.42883	12.7062	12.99616	13.29957	13.61741	13.95072
86	14.30067	14.66853	15.05572	15.46381	15.89454	16.34986	16.83191	17.34315	17.88631	18.46447
87	19.08114	19.74029	20.44649	21.20495	22.02171	22.90377	23.85928	24.89783	26.03074	27.27149
88	28.63625	30.14462	31.82052	33.69351	35.80055	38.18846	40.91741	44.06611	47.7395	52.08067
89	57.28996	63.65674	71.61507	81.84704	95.48948	114.5887	143.2371	190.9842	286.4777	572.9572
90	1.63E+16									

Table of Logarithms of Natural Tangents

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	#NUM!	3.241878	3.542909	3.719003	3.843944	3.940858	4.020044	4.086997	4.144996	4.196156
1	4.241921	4.283323	4.321122	4.355895	4.388092	4.418068	4.44611	4.472454	4.497293	4.52079
2	4.543084	4.564291	4.584514	4.603839	4.622343	4.640093	4.657149	4.673563	4.689381	4.704646
3	4.719396	4.733663	4.747479	4.760872	4.773866	4.786486	4.798752	4.810683	4.822298	4.833613

4	4.844644	4.855403	4.865906	4.876162	4.886185	4.895984	4.90557	4.914951	4.924136	4.933134
5	4.941952	4.950597	4.959075	4.967394	4.97556	4.983577	4.991451	4.999188	5.006792	5.014268
6	5.02162	5.028852	5.035969	5.042973	5.049869	5.056659	5.063348	5.069938	5.076432	5.082833
7	5.089144	5.095367	5.101504	5.107559	5.113533	5.119429	5.125249	5.130994	5.136667	5.142269
8	5.147803	5.153269	5.158671	5.164008	5.169284	5.174499	5.179655	5.184752	5.189794	5.19478
9	5.199713	5.204592	5.20942	5.214198	5.218926	5.223607	5.228239	5.232826	5.237368	5.241865
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	5.246319	5.25073	5.2551	5.259429	5.263717	5.267967	5.272178	5.276351	5.280488	5.284588
11	5.288652	5.292682	5.296677	5.300638	5.304567	5.308463	5.312327	5.316159	5.319961	5.323733
12	5.327475	5.331187	5.334871	5.338527	5.342155	5.345755	5.349329	5.352876	5.356398	5.359893
13	5.363364	5.36681	5.370232	5.373629	5.377003	5.380354	5.383682	5.386987	5.39027	5.393531
14	5.396771	5.39999	5.403187	5.406364	5.409521	5.412658	5.415775	5.418873	5.421952	5.425011
15	5.428052	5.431075	5.43408	5.437067	5.440036	5.442988	5.445923	5.448841	5.451743	5.454628
16	5.457496	5.460349	5.463186	5.466008	5.468814	5.471605	5.474381	5.477142	5.479889	5.482621
17	5.485339	5.488043	5.490733	5.49341	5.496073	5.498722	5.501359	5.503982	5.506593	5.509191
18	5.511776	5.514349	5.51691	5.519458	5.521995	5.52452	5.527033	5.529535	5.532025	5.534504
19	5.536972	5.539429	5.541875	5.54431	5.546735	5.549149	5.551552	5.553946	5.556329	5.558703
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	5.561066	5.563419	5.565763	5.568098	5.570422	5.572738	5.575044	5.577341	5.579629	5.581907
21	5.584177	5.586439	5.588691	5.590935	5.593171	5.595398	5.597616	5.599827	5.602029	5.604223
22	5.60641	5.608588	5.610759	5.612921	5.615077	5.617224	5.619364	5.621497	5.623623	5.625741
23	5.627852	5.629956	5.632053	5.634143	5.636226	5.638302	5.640371	5.642434	5.64449	5.64654
24	5.648583	5.65062	5.65265	5.654674	5.656692	5.658704	5.66071	5.662709	5.664703	5.666691
25	5.668673	5.670649	5.672619	5.674584	5.676543	5.678496	5.680444	5.682387	5.684324	5.686255
26	5.688182	5.690103	5.692019	5.69393	5.695836	5.697736	5.699632	5.701523	5.703409	5.70529
27	5.707166	5.709037	5.710904	5.712766	5.714624	5.716477	5.718325	5.720169	5.722009	5.723844
28	5.725674	5.727501	5.729323	5.731141	5.732955	5.734764	5.73657	5.738371	5.740169	5.741962
29	5.743752	5.745538	5.747319	5.749097	5.750872	5.752642	5.754409	5.756172	5.757931	5.759687
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
30	5.761439	5.763188	5.764933	5.766675	5.768414	5.770148	5.77188	5.773608	5.775333	5.777055
31	5.778774	5.780489	5.782201	5.78391	5.785616	5.787319	5.789019	5.790716	5.79241	5.794101
32	5.795789	5.797474	5.799157	5.800836	5.802513	5.804187	5.805859	5.807527	5.809193	5.810857
33	5.812517	5.814176	5.815831	5.817484	5.819135	5.820783	5.822429	5.824072	5.825713	5.827351
34	5.828987	5.830621	5.832253	5.833882	5.835509	5.837134	5.838757	5.840378	5.841996	5.843612
35	5.845227	5.846839	5.848449	5.850057	5.851664	5.853268	5.85487	5.856471	5.858069	5.859666
36	5.861261	5.862854	5.864445	5.866035	5.867623	5.869209	5.870793	5.872376	5.873957	5.875537
37	5.877114	5.878691	5.880265	5.881839	5.88341	5.88498	5.886549	5.888116	5.889682	5.891247

38	5.89281	5.894372	5.895932	5.897491	5.899049	5.900605	5.90216	5.903714	5.905267	5.906819
39	5.908369	5.909918	5.911467	5.913014	5.91456	5.916104	5.917648	5.919191	5.920733	5.922274
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	5.923814	5.925352	5.92689	5.928427	5.929964	5.931499	5.933033	5.934567	5.9361	5.937632
41	5.939163	5.940694	5.942223	5.943752	5.945281	5.946808	5.948335	5.949862	5.951388	5.952913
42	5.954437	5.955961	5.957485	5.959008	5.96053	5.962052	5.963574	5.965095	5.966616	5.968136
43	5.969656	5.971175	5.972695	5.974213	5.975732	5.97725	5.978768	5.980286	5.981803	5.98332
44	5.984837	5.986354	5.987871	5.989387	5.990903	5.99242	5.993936	5.995452	5.996968	5.998484
45	6	6.001516	6.003032	6.004548	6.006064	6.00758	6.009097	6.010613	6.012129	6.013646
46	6.015163	6.01668	6.018197	6.019714	6.021232	6.02275	6.024268	6.025787	6.027305	6.028825
47	6.030344	6.031864	6.033384	6.034905	6.036426	6.037948	6.03947	6.040992	6.042515	6.044039
48	6.045563	6.047087	6.048612	6.050138	6.051665	6.053192	6.054719	6.056248	6.057777	6.059306
49	6.060837	6.062368	6.0639	6.065433	6.066967	6.068501	6.070036	6.071573	6.07311	6.074648
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	6.076186	6.077726	6.079267	6.080809	6.082352	6.083896	6.08544	6.086986	6.088533	6.090082
51	6.091631	6.093181	6.094733	6.096286	6.09784	6.099395	6.100951	6.102509	6.104068	6.105628
52	6.10719	6.108753	6.110318	6.111884	6.113451	6.11502	6.11659	6.118161	6.119735	6.121309
53	6.122886	6.124463	6.126043	6.127624	6.129207	6.130791	6.132377	6.133965	6.135555	6.137146
54	6.138739	6.140334	6.141931	6.143529	6.14513	6.146732	6.148336	6.149943	6.151551	6.153161
55	6.154773	6.156388	6.158004	6.159622	6.161243	6.162866	6.164491	6.166118	6.167747	6.169379
56	6.171013	6.172649	6.174287	6.175928	6.177571	6.179217	6.180865	6.182516	6.184169	6.185824
57	6.187483	6.189143	6.190807	6.192473	6.194141	6.195813	6.197487	6.199164	6.200843	6.202526
58	6.204211	6.205899	6.20759	6.209284	6.210981	6.212681	6.214384	6.21609	6.217799	6.219511
59	6.221226	6.222945	6.224667	6.226392	6.22812	6.229852	6.231586	6.233325	6.235067	6.236812
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	6.238561	6.240313	6.242069	6.243828	6.245591	6.247358	6.249128	6.250903	6.252681	6.254462
61	6.256248	6.258038	6.259831	6.261629	6.26343	6.265236	6.267045	6.268859	6.270677	6.272499
62	6.274326	6.276156	6.277991	6.279831	6.281675	6.283523	6.285376	6.287234	6.289096	6.290963
63	6.292834	6.29471	6.296591	6.298477	6.300368	6.302264	6.304164	6.30607	6.307981	6.309897
64	6.311818	6.313745	6.315676	6.317613	6.319556	6.321504	6.323457	6.325416	6.327381	6.329351
65	6.331327	6.333309	6.335297	6.337291	6.33929	6.341296	6.343308	6.345326	6.34735	6.34938
66	6.351417	6.35346	6.35551	6.357566	6.359629	6.361698	6.363774	6.365857	6.367947	6.370044
67	6.372148	6.374259	6.376377	6.378503	6.380636	6.382776	6.384923	6.387079	6.389241	6.391412
68	6.39359	6.395777	6.397971	6.400173	6.402384	6.404602	6.406829	6.409065	6.411309	6.413561
69	6.415823	6.418093	6.420371	6.422659	6.424956	6.427262	6.429578	6.431902	6.434237	6.436581
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	6.438934	6.441297	6.443671	6.446054	6.448448	6.450851	6.453265	6.45569	6.458125	6.460571

71	6.463028	6.465496	6.467975	6.470465	6.472967	6.47548	6.478005	6.480542	6.48309	6.485651
72	6.488224	6.490809	6.493407	6.496018	6.498641	6.501278	6.503927	6.50659	6.509267	6.511957
73	6.514661	6.517379	6.520111	6.522858	6.525619	6.528395	6.531186	6.533992	6.536814	6.539651
74	6.542504	6.545372	6.548257	6.551159	6.554077	6.557012	6.559964	6.562933	6.56592	6.568925
75	6.571948	6.574989	6.578048	6.581127	6.584225	6.587342	6.590479	6.593636	6.596813	6.60001
76	6.603229	6.606469	6.60973	6.613013	6.616318	6.619646	6.622997	6.626371	6.629768	6.63319
77	6.636636	6.640107	6.643602	6.647124	6.650671	6.654245	6.657845	6.661473	6.665129	6.668813
78	6.672525	6.676267	6.680039	6.683841	6.687673	6.691537	6.695433	6.699362	6.703323	6.707318
79	6.711348	6.715412	6.719512	6.723649	6.727822	6.732033	6.736283	6.740571	6.7449	6.74927
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	6.753681	6.758135	6.762632	6.767174	6.771761	6.776393	6.781074	6.785802	6.79058	6.795408
81	6.800287	6.80522	6.810206	6.815248	6.820345	6.825501	6.830716	6.835992	6.841329	6.846731
82	6.852197	6.857731	6.863333	6.869006	6.874751	6.880571	6.886467	6.892441	6.898496	6.904633
83	6.910856	6.917167	6.923568	6.930062	6.936652	6.943341	6.950131	6.957027	6.964031	6.971148
84	6.97838	6.985732	6.993208	7.000812	7.008549	7.016423	7.02444	7.032606	7.040925	7.049403
85	7.058048	7.066866	7.075864	7.085049	7.09443	7.104016	7.113815	7.123838	7.134094	7.144597
86	7.155356	7.166387	7.177702	7.189317	7.201248	7.213514	7.226134	7.239128	7.252521	7.266337
87	7.280604	7.295354	7.310619	7.326437	7.342851	7.359907	7.377657	7.396161	7.415486	7.435709
88	7.456916	7.47921	7.502707	7.527546	7.55389	7.581932	7.611908	7.644105	7.678878	7.716677
89	7.758079	7.803844	7.855004	7.913003	7.979956	8.059142	8.156056	8.280997	8.457091	8.758122
90	22.21284									

Logarithmic and Trigonometric Tables

Table of Natural Cotangents

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	#DIV/0!	572.9572	286.4777	190.9842	143.2371	114.5887	95.48948	81.84704	71.61507	63.65674
1	57.28996	52.08067	47.7395	44.06611	40.91741	38.18846	35.80055	33.69351	31.82052	30.14462
2	28.63625	27.27149	26.03074	24.89783	23.85928	22.90377	22.02171	21.20495	20.44649	19.74029
3	19.08114	18.46447	17.88631	17.34315	16.83191	16.34986	15.89454	15.46381	15.05572	14.66853
4	14.30067	13.95072	13.61741	13.29957	12.99616	12.7062	12.42883	12.16324	11.90868	11.6645
5	11.43005	11.20478	10.98815	10.77967	10.57889	10.3854	10.19879	10.01871	9.844817	9.6768
6	9.514364	9.357236	9.205156	9.057887	8.915201	8.776887	8.642747	8.512594	8.386252	8.263555
7	8.144346	8.02848	7.915815	7.806221	7.699574	7.595754	7.494651	7.39616	7.300178	7.206612
8	7.11537	7.026366	6.939519	6.854751	6.771987	6.691156	6.612192	6.535029	6.459607	6.385866
9	6.313752	6.243209	6.174186	6.106636	6.04051	5.975764	5.912355	5.850241	5.789383	5.729742
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	5.671282	5.613968	5.557766	5.502645	5.448572	5.395517	5.343453	5.29235	5.242184	5.192926
11	5.144554	5.097043	5.050369	5.004511	4.959447	4.915157	4.87162	4.828817	4.78673	4.74534
12	4.70463	4.664583	4.625183	4.586414	4.548261	4.510709	4.473743	4.43735	4.401516	4.366229
13	4.331476	4.297244	4.263522	4.230298	4.197561	4.1653	4.133505	4.102165	4.071271	4.040813
14	4.010781	3.981167	3.951962	3.923156	3.894743	3.866713	3.839059	3.811773	3.784848	3.758276
15	3.732051	3.706165	3.680611	3.655384	3.630477	3.605884	3.581598	3.557613	3.533925	3.510527
16	3.487414	3.464581	3.442023	3.419733	3.397709	3.375943	3.354433	3.333174	3.31216	3.291388
17	3.270853	3.250551	3.230478	3.21063	3.191004	3.171595	3.152399	3.133414	3.114635	3.09606
18	3.077684	3.059504	3.041517	3.023721	3.006111	2.988685	2.97144	2.954373	2.937481	2.920761
19	2.904211	2.887828	2.871609	2.855552	2.839654	2.823913	2.808326	2.792892	2.777607	2.76247
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	2.747477	2.732628	2.71792	2.703351	2.688919	2.674621	2.660457	2.646423	2.632519	2.618741
21	2.605089	2.591561	2.578154	2.564867	2.551699	2.538648	2.525712	2.512889	2.500178	2.487578
22	2.475087	2.462703	2.450425	2.438252	2.426182	2.414214	2.402346	2.390577	2.378906	2.367332
23	2.355852	2.344467	2.333175	2.321974	2.310864	2.299843	2.28891	2.278064	2.267304	2.256628
24	2.246037	2.235528	2.225101	2.214754	2.204488	2.1943	2.184189	2.174156	2.164198	2.154316
25	2.144507	2.134771	2.125108	2.115516	2.105995	2.096544	2.087161	2.077847	2.068599	2.059419
26	2.050304	2.041254	2.032268	2.023346	2.014487	2.00569	1.996954	1.988279	1.979664	1.971108
27	1.962611	1.954171	1.94579	1.937465	1.929196	1.920982	1.912824	1.904719	1.896669	1.888671
28	1.880726	1.872834	1.864992	1.857202	1.849461	1.841771	1.83413	1.826537	1.818993	1.811497
29	1.804048	1.796645	1.789289	1.781979	1.774714	1.767494	1.760318	1.753187	1.746098	1.739053
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**

30	1.732051	1.725091	1.718172	1.711295	1.704459	1.697663	1.690908	1.684192	1.677516	1.670878
31	1.664279	1.657719	1.651196	1.644711	1.638263	1.631852	1.625477	1.619138	1.612835	1.606567
32	1.600335	1.594137	1.587973	1.581844	1.575748	1.569686	1.563656	1.55766	1.551696	1.545765
33	1.539865	1.533997	1.52816	1.522355	1.51658	1.510835	1.505121	1.499437	1.493782	1.488157
34	1.482561	1.476994	1.471455	1.465945	1.460463	1.455009	1.449583	1.444183	1.438811	1.433466
35	1.428148	1.422856	1.41759	1.412351	1.407137	1.401948	1.396785	1.391647	1.386534	1.381446
36	1.376382	1.371342	1.366327	1.361335	1.356367	1.351422	1.346501	1.341603	1.336728	1.331875
37	1.327045	1.322237	1.317451	1.312688	1.307946	1.303225	1.298526	1.293849	1.289192	1.284557
38	1.279942	1.275347	1.270773	1.26622	1.261686	1.257172	1.252678	1.248204	1.243749	1.239314
39	1.234897	1.2305	1.226121	1.221761	1.21742	1.213097	1.208792	1.204506	1.200237	1.195987

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
40	1.191754	1.187538	1.18334	1.17916	1.174996	1.17085	1.16672	1.162607	1.158511	1.154432
41	1.150368	1.146322	1.142291	1.138276	1.134277	1.130294	1.126327	1.122375	1.118439	1.114518
42	1.110613	1.106722	1.102846	1.098986	1.09514	1.091309	1.087492	1.08369	1.079902	1.076128
43	1.072369	1.068623	1.064892	1.061174	1.05747	1.05378	1.050103	1.04644	1.04279	1.039154
44	1.03553	1.03192	1.028323	1.024738	1.021166	1.017607	1.014061	1.010527	1.007006	1.003497
45	1	0.996515	0.993043	0.989582	0.986134	0.982697	0.979272	0.975859	0.972458	0.969067
46	0.965689	0.962322	0.958966	0.955621	0.952287	0.948965	0.945653	0.942352	0.939063	0.935783
47	0.932515	0.929257	0.92601	0.922773	0.919547	0.916331	0.913125	0.90993	0.906745	0.903569
48	0.900404	0.897249	0.894103	0.890967	0.887842	0.884725	0.881619	0.878521	0.875434	0.872356
49	0.869287	0.866227	0.863177	0.860136	0.857104	0.854081	0.851067	0.848062	0.845066	0.842078

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
50	0.8391	0.83613	0.833169	0.830216	0.827272	0.824336	0.821409	0.818491	0.81558	0.812678
51	0.809784	0.806898	0.804021	0.801151	0.79829	0.795436	0.79259	0.789752	0.786922	0.7841
52	0.781286	0.778479	0.77568	0.772888	0.770104	0.767327	0.764558	0.761796	0.759041	0.756294
53	0.753554	0.750821	0.748096	0.745377	0.742666	0.739961	0.737264	0.734573	0.731889	0.729213
54	0.726543	0.723879	0.721223	0.718573	0.71593	0.713293	0.710663	0.708039	0.705422	0.702812
55	0.700208	0.69761	0.695018	0.692433	0.689854	0.687281	0.684714	0.682154	0.679599	0.677051
56	0.674509	0.671972	0.669442	0.666917	0.664398	0.661886	0.659379	0.656877	0.654382	0.651892
57	0.649408	0.646929	0.644456	0.641989	0.639527	0.63707	0.634619	0.632174	0.629734	0.627299
58	0.624869	0.622445	0.620026	0.617613	0.615204	0.612801	0.610403	0.60801	0.605622	0.603239
59	0.600861	0.598488	0.59612	0.593757	0.591398	0.589045	0.586697	0.584353	0.582014	0.57968

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
60	0.57735	0.575026	0.572705	0.57039	0.568079	0.565773	0.563471	0.561174	0.558881	0.556593
61	0.554309	0.55203	0.549755	0.547484	0.545218	0.542956	0.540698	0.538445	0.536195	0.53395
62	0.531709	0.529473	0.52724	0.525012	0.522787	0.520567	0.518351	0.516138	0.51393	0.511726
63	0.509525	0.507329	0.505136	0.502948	0.500763	0.498582	0.496404	0.494231	0.492061	0.489895

64	0.487733	0.485574	0.483419	0.481267	0.47912	0.476976	0.474835	0.472698	0.470564	0.468434
65	0.466308	0.464185	0.462065	0.459949	0.457836	0.455726	0.45362	0.451517	0.449418	0.447322
66	0.445229	0.443139	0.441053	0.438969	0.436889	0.434812	0.432739	0.430668	0.428601	0.426536
67	0.424475	0.422417	0.420361	0.418309	0.41626	0.414214	0.41217	0.41013	0.408092	0.406058
68	0.404026	0.401997	0.399971	0.397948	0.395928	0.39391	0.391896	0.389884	0.387874	0.385868
69	0.383864	0.381863	0.379864	0.377869	0.375875	0.373885	0.371897	0.369911	0.367928	0.365948
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	0.36397	0.361995	0.360022	0.358052	0.356084	0.354119	0.352156	0.350195	0.348237	0.346281
71	0.344328	0.342377	0.340428	0.338481	0.336537	0.334595	0.332656	0.330718	0.328783	0.32685
72	0.32492	0.322991	0.321065	0.319141	0.317219	0.315299	0.313381	0.311465	0.309552	0.30764
73	0.305731	0.303823	0.301918	0.300014	0.298113	0.296213	0.294316	0.29242	0.290527	0.288635
74	0.286745	0.284857	0.282971	0.281087	0.279205	0.277325	0.275446	0.273569	0.271694	0.269821
75	0.267949	0.266079	0.264211	0.262345	0.26048	0.258618	0.256756	0.254897	0.253039	0.251183
76	0.249328	0.247475	0.245624	0.243774	0.241925	0.240079	0.238234	0.23639	0.234548	0.232707
77	0.230868	0.229031	0.227194	0.22536	0.223526	0.221695	0.219864	0.218035	0.216208	0.214381
78	0.212557	0.210733	0.208911	0.20709	0.205271	0.203452	0.201635	0.19982	0.198005	0.196192
79	0.19438	0.19257	0.19076	0.188952	0.187145	0.185339	0.183534	0.181731	0.179928	0.178127
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	0.176327	0.174528	0.17273	0.170933	0.169137	0.167343	0.165549	0.163756	0.161965	0.160174
81	0.158384	0.156596	0.154808	0.153022	0.151236	0.149451	0.147667	0.145884	0.144102	0.142321
82	0.140541	0.138761	0.136983	0.135205	0.133428	0.131652	0.129877	0.128103	0.126329	0.124557
83	0.122785	0.121013	0.119243	0.117473	0.115704	0.113936	0.112168	0.110401	0.108635	0.106869
84	0.105104	0.10334	0.101576	0.099813	0.098051	0.096289	0.094528	0.092767	0.091007	0.089248
85	0.087489	0.08573	0.083972	0.082215	0.080458	0.078702	0.076946	0.07519	0.073435	0.071681
86	0.069927	0.068173	0.06642	0.064667	0.062915	0.061163	0.059411	0.05766	0.055909	0.054158
87	0.052408	0.050658	0.048908	0.047159	0.04541	0.043661	0.041912	0.040164	0.038416	0.036668
88	0.034921	0.033173	0.031426	0.029679	0.027933	0.026186	0.024439	0.022693	0.020947	0.019201
89	0.017455	0.015709	0.013964	0.012218	0.010472	0.008727	0.006981	0.005236	0.003491	0.001745
90	6.13E-17									

Table of Logarithms of Natural Cotangents

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	#DIV/0!	8.758122	8.457091	8.280997	8.156056	8.059142	7.979956	7.913003	7.855004	7.803844
1	7.758079	7.716677	7.678878	7.644105	7.611908	7.581932	7.55389	7.527546	7.502707	7.47921
2	7.456916	7.435709	7.415486	7.396161	7.377657	7.359907	7.342851	7.326437	7.310619	7.295354
3	7.280604	7.266337	7.252521	7.239128	7.226134	7.213514	7.201248	7.189317	7.177702	7.166387

4	7.155356	7.144597	7.134094	7.123838	7.113815	7.104016	7.09443	7.085049	7.075864	7.066866
5	7.058048	7.049403	7.040925	7.032606	7.02444	7.016423	7.008549	7.000812	6.993208	6.985732
6	6.97838	6.971148	6.964031	6.957027	6.950131	6.943341	6.936652	6.930062	6.923568	6.917167
7	6.910856	6.904633	6.898496	6.892441	6.886467	6.880571	6.874751	6.869006	6.863333	6.857731
8	6.852197	6.846731	6.841329	6.835992	6.830716	6.825501	6.820345	6.815248	6.810206	6.80522
9	6.800287	6.795408	6.79058	6.785802	6.781074	6.776393	6.771761	6.767174	6.762632	6.758135

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
10	6.753681	6.74927	6.7449	6.740571	6.736283	6.732033	6.727822	6.723649	6.719512	6.715412
11	6.711348	6.707318	6.703323	6.699362	6.695433	6.691537	6.687673	6.683841	6.680039	6.676267
12	6.672525	6.668813	6.665129	6.661473	6.657845	6.654245	6.650671	6.647124	6.643602	6.640107
13	6.636636	6.63319	6.629768	6.626371	6.622997	6.619646	6.616318	6.613013	6.60973	6.606469
14	6.603229	6.60001	6.596813	6.593636	6.590479	6.587342	6.584225	6.581127	6.578048	6.574989
15	6.571948	6.568925	6.56592	6.562933	6.559964	6.557012	6.554077	6.551159	6.548257	6.545372
16	6.542504	6.539651	6.536814	6.533992	6.531186	6.528395	6.525619	6.522858	6.520111	6.517379
17	6.514661	6.511957	6.509267	6.50659	6.503927	6.501278	6.498641	6.496018	6.493407	6.490809
18	6.488224	6.485651	6.48309	6.480542	6.478005	6.47548	6.472967	6.470465	6.467975	6.465496
19	6.463028	6.460571	6.458125	6.45569	6.453265	6.450851	6.448448	6.446054	6.443671	6.441297

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
20	6.438934	6.436581	6.434237	6.431902	6.429578	6.427262	6.424956	6.422659	6.420371	6.418093
21	6.415823	6.413561	6.411309	6.409065	6.406829	6.404602	6.402384	6.400173	6.397971	6.395777
22	6.39359	6.391412	6.389241	6.387079	6.384923	6.382776	6.380636	6.378503	6.376377	6.374259
23	6.372148	6.370044	6.367947	6.365857	6.363774	6.361698	6.359629	6.357566	6.35551	6.35346
24	6.351417	6.34938	6.34735	6.345326	6.343308	6.341296	6.33929	6.337291	6.335297	6.333309
25	6.331327	6.329351	6.327381	6.325416	6.323457	6.321504	6.319556	6.317613	6.315676	6.313745
26	6.311818	6.309897	6.307981	6.30607	6.304164	6.302264	6.300368	6.298477	6.296591	6.29471
27	6.292834	6.290963	6.289096	6.287234	6.285376	6.283523	6.281675	6.279831	6.277991	6.276156
28	6.274326	6.272499	6.270677	6.268859	6.267045	6.265236	6.26343	6.261629	6.259831	6.258038
29	6.256248	6.254462	6.252681	6.250903	6.249128	6.247358	6.245591	6.243828	6.242069	6.240313

x	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
30	6.238561	6.236812	6.235067	6.233325	6.231586	6.229852	6.22812	6.226392	6.224667	6.222945
31	6.221226	6.219511	6.217799	6.21609	6.214384	6.212681	6.210981	6.209284	6.20759	6.205899
32	6.204211	6.202526	6.200843	6.199164	6.197487	6.195813	6.194141	6.192473	6.190807	6.189143
33	6.187483	6.185824	6.184169	6.182516	6.180865	6.179217	6.177571	6.175928	6.174287	6.172649
34	6.171013	6.169379	6.167747	6.166118	6.164491	6.162866	6.161243	6.159622	6.158004	6.156388
35	6.154773	6.153161	6.151551	6.149943	6.148336	6.146732	6.14513	6.143529	6.141931	6.140334
36	6.138739	6.137146	6.135555	6.133965	6.132377	6.130791	6.129207	6.127624	6.126043	6.124463
37	6.122886	6.121309	6.119735	6.118161	6.11659	6.11502	6.113451	6.111884	6.110318	6.108753

38	6.10719	6.105628	6.104068	6.102509	6.100951	6.099395	6.09784	6.096286	6.094733	6.093181
39	6.091631	6.090082	6.088533	6.086986	6.08544	6.083896	6.082352	6.080809	6.079267	6.077726
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	6.076186	6.074648	6.07311	6.071573	6.070036	6.068501	6.066967	6.065433	6.0639	6.062368
41	6.060837	6.059306	6.057777	6.056248	6.054719	6.053192	6.051665	6.050138	6.048612	6.047087
42	6.045563	6.044039	6.042515	6.040992	6.03947	6.037948	6.036426	6.034905	6.033384	6.031864
43	6.030344	6.028825	6.027305	6.025787	6.024268	6.02275	6.021232	6.019714	6.018197	6.01668
44	6.015163	6.013646	6.012129	6.010613	6.009097	6.00758	6.006064	6.004548	6.003032	6.001516
45	6	5.998484	5.996968	5.995452	5.993936	5.99242	5.990903	5.989387	5.987871	5.986354
46	5.984837	5.98332	5.981803	5.980286	5.978768	5.97725	5.975732	5.974213	5.972695	5.971175
47	5.969656	5.968136	5.966616	5.965095	5.963574	5.962052	5.96053	5.959008	5.957485	5.955961
48	5.954437	5.952913	5.951388	5.949862	5.948335	5.946808	5.945281	5.943752	5.942223	5.940694
49	5.939163	5.937632	5.9361	5.934567	5.933033	5.931499	5.929964	5.928427	5.92689	5.925352
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	5.923814	5.922274	5.920733	5.919191	5.917648	5.916104	5.91456	5.913014	5.911467	5.909918
51	5.908369	5.906819	5.905267	5.903714	5.90216	5.900605	5.899049	5.897491	5.895932	5.894372
52	5.89281	5.891247	5.889682	5.888116	5.886549	5.88498	5.88341	5.881839	5.880265	5.878691
53	5.877114	5.875537	5.873957	5.872376	5.870793	5.869209	5.867623	5.866035	5.864445	5.862854
54	5.861261	5.859666	5.858069	5.856471	5.85487	5.853268	5.851664	5.850057	5.848449	5.846839
55	5.845227	5.843612	5.841996	5.840378	5.838757	5.837134	5.835509	5.833882	5.832253	5.830621
56	5.828987	5.827351	5.825713	5.824072	5.822429	5.820783	5.819135	5.817484	5.815831	5.814176
57	5.812517	5.810857	5.809193	5.807527	5.805859	5.804187	5.802513	5.800836	5.799157	5.797474
58	5.795789	5.794101	5.79241	5.790716	5.789019	5.787319	5.785616	5.78391	5.782201	5.780489
59	5.778774	5.777055	5.775333	5.773608	5.77188	5.770148	5.768414	5.766675	5.764933	5.763188
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	5.761439	5.759687	5.757931	5.756172	5.754409	5.752642	5.750872	5.749097	5.747319	5.745538
61	5.743752	5.741962	5.740169	5.738371	5.73657	5.734764	5.732955	5.731141	5.729323	5.727501
62	5.725674	5.723844	5.722009	5.720169	5.718325	5.716477	5.714624	5.712766	5.710904	5.709037
63	5.707166	5.70529	5.703409	5.701523	5.699632	5.697736	5.695836	5.69393	5.692019	5.690103
64	5.688182	5.686255	5.684324	5.682387	5.680444	5.678496	5.676543	5.674584	5.672619	5.670649
65	5.668673	5.666691	5.664703	5.662709	5.66071	5.658704	5.656692	5.654674	5.65265	5.65062
66	5.648583	5.64654	5.64449	5.642434	5.640371	5.638302	5.636226	5.634143	5.632053	5.629956
67	5.627852	5.625741	5.623623	5.621497	5.619364	5.617224	5.615077	5.612921	5.610759	5.608588
68	5.60641	5.604223	5.602029	5.599827	5.597616	5.595398	5.593171	5.590935	5.588691	5.586439
69	5.584177	5.581907	5.579629	5.577341	5.575044	5.572738	5.570422	5.568098	5.565763	5.563419
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	5.561066	5.558703	5.556329	5.553946	5.551552	5.549149	5.546735	5.54431	5.541875	5.539429

71	5.536972	5.534504	5.532025	5.529535	5.527033	5.52452	5.521995	5.519458	5.51691	5.514349
72	5.511776	5.509191	5.506593	5.503982	5.501359	5.498722	5.496073	5.49341	5.490733	5.488043
73	5.485339	5.482621	5.479889	5.477142	5.474381	5.471605	5.468814	5.466008	5.463186	5.460349
74	5.457496	5.454628	5.451743	5.448841	5.445923	5.442988	5.440036	5.437067	5.43408	5.431075
75	5.428052	5.425011	5.421952	5.418873	5.415775	5.412658	5.409521	5.406364	5.403187	5.39999
76	5.396771	5.393531	5.39027	5.386987	5.383682	5.380354	5.377003	5.373629	5.370232	5.36681
77	5.363364	5.359893	5.356398	5.352876	5.349329	5.345755	5.342155	5.338527	5.334871	5.331187
78	5.327475	5.323733	5.319961	5.316159	5.312327	5.308463	5.304567	5.300638	5.296677	5.292682
79	5.288652	5.284588	5.280488	5.276351	5.272178	5.267967	5.263717	5.259429	5.2551	5.25073
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	5.246319	5.241865	5.237368	5.232826	5.228239	5.223607	5.218926	5.214198	5.20942	5.204592
81	5.199713	5.19478	5.189794	5.184752	5.179655	5.174499	5.169284	5.164008	5.158671	5.153269
82	5.147803	5.142269	5.136667	5.130994	5.125249	5.119429	5.113533	5.107559	5.101504	5.095367
83	5.089144	5.082833	5.076432	5.069938	5.063348	5.056659	5.049869	5.042973	5.035969	5.028852
84	5.02162	5.014268	5.006792	4.999188	4.991451	4.983577	4.97556	4.967394	4.959075	4.950597
85	4.941952	4.933134	4.924136	4.914951	4.90557	4.895984	4.886185	4.876162	4.865906	4.855403
86	4.844644	4.833613	4.822298	4.810683	4.798752	4.786486	4.773866	4.760872	4.747479	4.733663
87	4.719396	4.704646	4.689381	4.673563	4.657149	4.640093	4.622343	4.603839	4.584514	4.564291
88	4.543084	4.52079	4.497293	4.472454	4.44611	4.418068	4.388092	4.355895	4.321122	4.283323
89	4.241921	4.196156	4.144996	4.086997	4.020044	3.940858	3.843944	3.719003	3.542909	3.241878
90	-10.2128									

Table of Natural Secants

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	1	1.000002	1.000006	1.000014	1.000024	1.000038	1.000055	1.000075	1.000097	1.000123
1	1.000152	1.000184	1.000219	1.000257	1.000299	1.000343	1.00039	1.00044	1.000494	1.00055
2	1.00061	1.000672	1.000738	1.000806	1.000878	1.000953	1.00103	1.001111	1.001195	1.001282
3	1.001372	1.001465	1.001562	1.001661	1.001763	1.001869	1.001977	1.002089	1.002203	1.002321
4	1.002442	1.002566	1.002693	1.002823	1.002956	1.003092	1.003232	1.003374	1.003519	1.003668
5	1.00382	1.003975	1.004133	1.004294	1.004458	1.004625	1.004795	1.004969	1.005146	1.005325
6	1.005508	1.005694	1.005883	1.006076	1.006271	1.00647	1.006671	1.006876	1.007084	1.007295
7	1.00751	1.007727	1.007948	1.008172	1.008399	1.008629	1.008862	1.009099	1.009339	1.009581
8	1.009828	1.010077	1.010329	1.010585	1.010844	1.011106	1.011371	1.01164	1.011912	1.012187
9	1.012465	1.012747	1.013031	1.013319	1.013611	1.013905	1.014203	1.014504	1.014808	1.015116
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	1.015427	1.015741	1.016058	1.016379	1.016703	1.01703	1.017361	1.017695	1.018032	1.018373
11	1.018717	1.019064	1.019415	1.019769	1.020126	1.020487	1.020851	1.021218	1.021589	1.021963
12	1.022341	1.022722	1.023106	1.023494	1.023885	1.02428	1.024678	1.025079	1.025484	1.025892
13	1.026304	1.026719	1.027138	1.02756	1.027986	1.028415	1.028848	1.029284	1.029724	1.030167
14	1.030614	1.031064	1.031518	1.031975	1.032436	1.0329	1.033368	1.03384	1.034315	1.034794
15	1.035276	1.035762	1.036252	1.036745	1.037242	1.037742	1.038246	1.038754	1.039266	1.039781
16	1.040299	1.040822	1.041348	1.041878	1.042412	1.042949	1.04349	1.044035	1.044583	1.045136
17	1.045692	1.046252	1.046815	1.047383	1.047954	1.048529	1.049108	1.049691	1.050277	1.050868
18	1.051462	1.05206	1.052663	1.053269	1.053878	1.054492	1.05511	1.055732	1.056357	1.056987
19	1.057621	1.058258	1.0589	1.059545	1.060195	1.060849	1.061506	1.062168	1.062834	1.063504
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	1.064178	1.064856	1.065538	1.066224	1.066915	1.067609	1.068308	1.069011	1.069718	1.070429
21	1.071145	1.071865	1.072589	1.073317	1.074049	1.074786	1.075527	1.076273	1.077022	1.077776
22	1.078535	1.079297	1.080065	1.080836	1.081612	1.082392	1.083177	1.083966	1.08476	1.085558
23	1.08636	1.087167	1.087979	1.088795	1.089616	1.090441	1.091271	1.092105	1.092944	1.093788
24	1.094636	1.095489	1.096347	1.097209	1.098076	1.098948	1.099824	1.100706	1.101592	1.102482
25	1.103378	1.104278	1.105184	1.106094	1.107009	1.107929	1.108853	1.109783	1.110718	1.111657
26	1.112602	1.113552	1.114506	1.115466	1.116431	1.1174	1.118375	1.119355	1.12034	1.121331
27	1.122326	1.123327	1.124333	1.125344	1.12636	1.127382	1.128409	1.129441	1.130479	1.131522
28	1.13257	1.133624	1.134683	1.135748	1.136818	1.137893	1.138974	1.140061	1.141153	1.142251
29	1.143354	1.144463	1.145578	1.146698	1.147824	1.148956	1.150093	1.151236	1.152385	1.15354
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**

30	1.154701	1.155867	1.157039	1.158218	1.159402	1.160592	1.161788	1.16299	1.164199	1.165413
31	1.166633	1.16786	1.169093	1.170331	1.171576	1.172828	1.174085	1.175349	1.176619	1.177896
32	1.179178	1.180468	1.181763	1.183065	1.184374	1.185689	1.187011	1.188339	1.189674	1.191015
33	1.192363	1.193718	1.19508	1.196448	1.197823	1.199205	1.200594	1.201989	1.203392	1.204801
34	1.206218	1.207641	1.209072	1.21051	1.211954	1.213406	1.214866	1.216332	1.217805	1.219286
35	1.220775	1.22227	1.223773	1.225284	1.226801	1.228327	1.22986	1.2314	1.232949	1.234504
36	1.236068	1.237639	1.239218	1.240805	1.2424	1.244003	1.245613	1.247232	1.248858	1.250493
37	1.252136	1.253787	1.255446	1.257113	1.258789	1.260472	1.262165	1.263865	1.265574	1.267292
38	1.269018	1.270753	1.272496	1.274248	1.276009	1.277779	1.279557	1.281344	1.28314	1.284945
39	1.28676	1.288583	1.290415	1.292256	1.294107	1.295967	1.297836	1.299715	1.301603	1.3035
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	1.305407	1.307324	1.30925	1.311186	1.313132	1.315087	1.317052	1.319027	1.321013	1.323008
41	1.325013	1.327028	1.329054	1.33109	1.333136	1.335192	1.337259	1.339337	1.341425	1.343523
42	1.345633	1.347753	1.349884	1.352025	1.354178	1.356342	1.358516	1.360702	1.362899	1.365108
43	1.367327	1.369559	1.371801	1.374055	1.376321	1.378598	1.380888	1.383189	1.385502	1.387827
44	1.390164	1.392513	1.394874	1.397248	1.399634	1.402032	1.404443	1.406867	1.409303	1.411752
45	1.414214	1.416688	1.419176	1.421677	1.424191	1.426718	1.429259	1.431813	1.43438	1.436962
46	1.439557	1.442165	1.444788	1.447424	1.450075	1.45274	1.455419	1.458112	1.46082	1.463542
47	1.466279	1.469031	1.471797	1.474579	1.477376	1.480187	1.483014	1.485856	1.488714	1.491588
48	1.494477	1.497381	1.500302	1.503239	1.506191	1.50916	1.512146	1.515148	1.518166	1.521201
49	1.524253	1.527322	1.530408	1.533511	1.536631	1.539769	1.542924	1.546097	1.549288	1.552497
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	1.555724	1.558969	1.562232	1.565514	1.568815	1.572134	1.575472	1.578829	1.582205	1.585601
51	1.589016	1.59245	1.595905	1.599379	1.602873	1.606388	1.609923	1.613478	1.617054	1.620651
52	1.624269	1.627908	1.631569	1.635251	1.638954	1.64268	1.646427	1.650197	1.653989	1.657803
53	1.66164	1.6655	1.669383	1.67329	1.67722	1.681173	1.68515	1.689152	1.693177	1.697227
54	1.701302	1.705401	1.709525	1.713675	1.71785	1.722051	1.726277	1.73053	1.734809	1.739115
55	1.743447	1.747806	1.752192	1.756606	1.761048	1.765517	1.770015	1.774541	1.779095	1.783679
56	1.788292	1.792934	1.797605	1.802307	1.807039	1.811801	1.816594	1.821418	1.826273	1.83116
57	1.836078	1.841029	1.846012	1.851028	1.856077	1.861159	1.866275	1.871424	1.876608	1.881827
58	1.88708	1.892368	1.897692	1.903052	1.908448	1.913881	1.91935	1.924857	1.930401	1.935983
59	1.941604	1.947263	1.952961	1.958699	1.964477	1.970294	1.976153	1.982052	1.987993	1.993975
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	2	2.006067	2.012178	2.018332	2.02453	2.030772	2.037059	2.043392	2.04977	2.056194
61	2.062665	2.069184	2.07575	2.082364	2.089027	2.095739	2.1025	2.109312	2.116175	2.123089
62	2.130054	2.137073	2.144144	2.151268	2.158447	2.165681	2.172969	2.180314	2.187715	2.195173
63	2.202689	2.210264	2.217897	2.22559	2.233344	2.241158	2.249035	2.256974	2.264976	2.273042

64	2.281172	2.289368	2.29763	2.305959	2.314355	2.32282	2.331355	2.339959	2.348635	2.357382
65	2.366202	2.375095	2.384063	2.393106	2.402225	2.411421	2.420695	2.430049	2.439482	2.448997
66	2.458593	2.468273	2.478037	2.487885	2.49782	2.507843	2.517954	2.528154	2.538445	2.548828
67	2.559305	2.569875	2.580541	2.591304	2.602165	2.613126	2.624187	2.635351	2.646617	2.657989
68	2.669467	2.681053	2.692748	2.704554	2.716472	2.728504	2.740651	2.752916	2.765299	2.777802
69	2.790428	2.803178	2.816053	2.829056	2.842188	2.855451	2.868847	2.882379	2.896048	2.909855
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	2.923804	2.937897	2.952135	2.966521	2.981056	2.995744	3.010587	3.025587	3.040746	3.056068
71	3.071553	3.087207	3.10303	3.119025	3.135196	3.151545	3.168076	3.18479	3.201691	3.218783
72	3.236068	3.25355	3.271231	3.289116	3.307208	3.32551	3.344025	3.362759	3.381714	3.400894
73	3.420304	3.439947	3.459827	3.479949	3.500318	3.520937	3.541811	3.562945	3.584344	3.606012
74	3.627955	3.650178	3.672687	3.695485	3.71858	3.741978	3.765682	3.789701	3.81404	3.838705
75	3.863703	3.889041	3.914725	3.940763	3.967162	3.993929	4.021072	4.048599	4.076518	4.104837
76	4.133565	4.162711	4.192284	4.222293	4.252747	4.283658	4.315034	4.346886	4.379226	4.412064
77	4.445411	4.479281	4.513684	4.548634	4.584144	4.620226	4.656896	4.694166	4.732052	4.77057
78	4.809734	4.849562	4.89007	4.931275	4.973196	5.015852	5.059261	5.103443	5.14842	5.194212
79	5.240843	5.288335	5.336711	5.385998	5.43622	5.487404	5.539579	5.592772	5.647014	5.702336
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	5.75877	5.816351	5.875113	5.935092	5.996327	6.058858	6.122725	6.187972	6.254645	6.322788
81	6.392453	6.46369	6.536553	6.611097	6.687382	6.765469	6.845422	6.927309	7.0112	7.09717
82	7.185297	7.275662	7.368351	7.463456	7.561071	7.661298	7.764241	7.870012	7.97873	8.090518
83	8.205509	8.323841	8.445663	8.57113	8.700407	8.833671	8.97111	9.11292	9.259314	9.410518
84	9.566772	9.728333	9.895474	10.06849	10.2477	10.43343	10.62605	10.82596	11.03356	11.24932
85	11.47371	11.70728	11.95059	12.20427	12.469	12.74549	13.03458	13.33712	13.65408	13.98651
86	14.33559	14.70258	15.0889	15.49611	15.92597	16.38041	16.86159	17.37196	17.91424	18.49153
87	19.10732	19.7656	20.47093	21.22852	22.0444	22.92559	23.88022	24.9179	26.04994	27.28981
88	28.65371	30.1612	31.83623	33.70835	35.81452	38.20155	40.92963	44.07746	47.74997	52.09027
89	57.29869	63.6646	71.62205	81.85315	95.49471	114.593	143.2406	190.9868	286.4795	572.9581
90	#DIV/0!									

Table of Logarithms of Natural Secants

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	6	6.000001	6.000003	6.000006	6.000011	6.000017	6.000024	6.000032	6.000042	6.000054
1	6.000066	6.00008	6.000095	6.000112	6.00013	6.000149	6.000169	6.000191	6.000214	6.000239
2	6.000265	6.000292	6.00032	6.00035	6.000381	6.000414	6.000447	6.000482	6.000519	6.000557
3	6.000596	6.000636	6.000678	6.000721	6.000765	6.000811	6.000858	6.000906	6.000956	6.001007

4	6.001059	6.001113	6.001168	6.001224	6.001282	6.001341	6.001401	6.001463	6.001526	6.00159
5	6.001656	6.001723	6.001791	6.001861	6.001932	6.002004	6.002078	6.002153	6.002229	6.002307
6	6.002386	6.002466	6.002548	6.002631	6.002715	6.002801	6.002888	6.002976	6.003066	6.003157
7	6.003249	6.003343	6.003438	6.003535	6.003632	6.003731	6.003832	6.003934	6.004037	6.004141
8	6.004247	6.004354	6.004463	6.004573	6.004684	6.004797	6.004911	6.005026	6.005143	6.005261
9	6.00538	6.005501	6.005623	6.005746	6.005871	6.005997	6.006125	6.006254	6.006384	6.006516
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	6.006649	6.006783	6.006919	6.007056	6.007194	6.007334	6.007475	6.007618	6.007761	6.007907
11	6.008053	6.008201	6.008351	6.008502	6.008654	6.008807	6.008962	6.009118	6.009276	6.009435
12	6.009596	6.009757	6.009921	6.010085	6.010251	6.010418	6.010587	6.010757	6.010929	6.011102
13	6.011276	6.011452	6.011629	6.011807	6.011987	6.012168	6.012351	6.012535	6.012721	6.012908
14	6.013096	6.013286	6.013477	6.013669	6.013863	6.014058	6.014255	6.014453	6.014653	6.014854
15	6.015056	6.01526	6.015465	6.015672	6.01588	6.016089	6.0163	6.016513	6.016727	6.016942
16	6.017158	6.017376	6.017596	6.017817	6.018039	6.018263	6.018488	6.018715	6.018943	6.019173
17	6.019404	6.019636	6.01987	6.020105	6.020342	6.02058	6.02082	6.021061	6.021304	6.021548
18	6.021794	6.022041	6.022289	6.022539	6.022791	6.023043	6.023298	6.023554	6.023811	6.02407
19	6.02433	6.024592	6.024855	6.02512	6.025386	6.025653	6.025923	6.026193	6.026465	6.026739
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	6.027014	6.027291	6.027569	6.027849	6.02813	6.028412	6.028697	6.028982	6.029269	6.029558
21	6.029848	6.03014	6.030433	6.030728	6.031024	6.031322	6.031621	6.031922	6.032225	6.032529
22	6.032834	6.033141	6.03345	6.03376	6.034071	6.034385	6.034699	6.035016	6.035334	6.035653
23	6.035974	6.036296	6.036621	6.036946	6.037273	6.037602	6.037933	6.038265	6.038598	6.038933
24	6.03927	6.039608	6.039948	6.040289	6.040632	6.040977	6.041323	6.041671	6.042021	6.042372
25	6.042724	6.043079	6.043434	6.043792	6.044151	6.044512	6.044874	6.045238	6.045604	6.045971
26	6.04634	6.04671	6.047082	6.047456	6.047832	6.048209	6.048588	6.048968	6.04935	6.049734
27	6.050119	6.050506	6.050895	6.051285	6.051677	6.052071	6.052467	6.052864	6.053262	6.053663
28	6.054065	6.054469	6.054875	6.055282	6.055691	6.056101	6.056514	6.056928	6.057344	6.057761
29	6.058181	6.058602	6.059025	6.059449	6.059875	6.060303	6.060733	6.061164	6.061598	6.062033
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
30	6.062469	6.062908	6.063348	6.06379	6.064234	6.06468	6.065127	6.065576	6.066027	6.06648
31	6.066934	6.067391	6.067849	6.068309	6.068771	6.069234	6.0697	6.070167	6.070636	6.071107
32	6.07158	6.072054	6.07253	6.073009	6.073489	6.073971	6.074455	6.07494	6.075428	6.075917
33	6.076409	6.076902	6.077397	6.077894	6.078393	6.078893	6.079396	6.079901	6.080407	6.080915
34	6.081426	6.081938	6.082452	6.082968	6.083486	6.084006	6.084528	6.085052	6.085578	6.086106
35	6.086635	6.087167	6.087701	6.088237	6.088774	6.089314	6.089856	6.090399	6.090945	6.091493
36	6.092042	6.092594	6.093148	6.093704	6.094261	6.094821	6.095383	6.095947	6.096513	6.097081
37	6.097651	6.098224	6.098798	6.099374	6.099953	6.100533	6.101116	6.101701	6.102288	6.102877

38	6.103468	6.104061	6.104657	6.105254	6.105854	6.106456	6.10706	6.107666	6.108274	6.108885
39	6.109497	6.110112	6.110729	6.111349	6.11197	6.112594	6.11322	6.113848	6.114478	6.115111
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	6.115746	6.116383	6.117023	6.117664	6.118308	6.118954	6.119603	6.120254	6.120907	6.121562
41	6.12222	6.12288	6.123543	6.124207	6.124874	6.125544	6.126216	6.12689	6.127566	6.128245
42	6.128927	6.12961	6.130296	6.130985	6.131676	6.132369	6.133065	6.133763	6.134464	6.135167
43	6.135873	6.136581	6.137291	6.138004	6.13872	6.139438	6.140158	6.140881	6.141607	6.142335
44	6.143066	6.143799	6.144535	6.145273	6.146014	6.146758	6.147504	6.148253	6.149004	6.149758
45	6.150515	6.151274	6.152036	6.152801	6.153568	6.154338	6.155111	6.155886	6.156664	6.157445
46	6.158229	6.159015	6.159804	6.160596	6.16139	6.162188	6.162988	6.163791	6.164597	6.165405
47	6.166217	6.167031	6.167848	6.168668	6.169491	6.170317	6.171145	6.171977	6.172811	6.173649
48	6.174489	6.175332	6.176179	6.177028	6.17788	6.178735	6.179594	6.180455	6.181319	6.182187
49	6.183057	6.183931	6.184807	6.185687	6.18657	6.187456	6.188345	6.189237	6.190132	6.191031
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	6.191933	6.192837	6.193746	6.194657	6.195572	6.196489	6.197411	6.198335	6.199263	6.200194
51	6.201128	6.202066	6.203007	6.203951	6.204899	6.20585	6.206805	6.207763	6.208725	6.20969
52	6.210658	6.21163	6.212605	6.213584	6.214567	6.215553	6.216542	6.217536	6.218532	6.219533
53	6.220537	6.221545	6.222556	6.223571	6.22459	6.225612	6.226639	6.227669	6.228702	6.22974
54	6.230781	6.231827	6.232876	6.233928	6.234985	6.236046	6.237111	6.238179	6.239252	6.240328
55	6.241409	6.242493	6.243582	6.244674	6.245771	6.246872	6.247977	6.249086	6.250199	6.251317
56	6.252438	6.253564	6.254694	6.255829	6.256967	6.258111	6.259258	6.26041	6.261566	6.262726
57	6.263891	6.265061	6.266235	6.267413	6.268596	6.269783	6.270976	6.272172	6.273374	6.27458
58	6.27579	6.277006	6.278226	6.279451	6.28068	6.281915	6.283154	6.284398	6.285648	6.286902
59	6.288161	6.289425	6.290694	6.291968	6.293247	6.294531	6.295821	6.297115	6.298415	6.29972
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	6.30103	6.302346	6.303666	6.304993	6.306324	6.307661	6.309004	6.310352	6.311705	6.313064
61	6.314429	6.315799	6.317175	6.318557	6.319944	6.321337	6.322736	6.324141	6.325552	6.326968
62	6.328391	6.329819	6.331254	6.332695	6.334141	6.335594	6.337054	6.338519	6.339991	6.341469
63	6.342953	6.344444	6.345941	6.347445	6.348956	6.350473	6.351996	6.353526	6.355064	6.356607
64	6.358158	6.359716	6.36128	6.362852	6.36443	6.366016	6.367608	6.369208	6.370815	6.37243
65	6.374052	6.375681	6.377318	6.378962	6.380614	6.382273	6.38394	6.385615	6.387298	6.388988
66	6.390687	6.392393	6.394108	6.39583	6.397561	6.3993	6.401048	6.402804	6.404568	6.406341
67	6.408122	6.409912	6.411711	6.413518	6.415335	6.41716	6.418995	6.420838	6.422691	6.424553
68	6.426425	6.428305	6.430196	6.432096	6.434005	6.435925	6.437854	6.439793	6.441742	6.443701
69	6.445671	6.447651	6.449641	6.451641	6.453653	6.455675	6.457707	6.459751	6.461806	6.463871
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	6.465948	6.468037	6.470136	6.472247	6.47437	6.476505	6.478651	6.48081	6.48298	6.485163

71	6.487358	6.489566	6.491786	6.494019	6.496265	6.498524	6.500796	6.503081	6.505379	6.507692
72	6.510018	6.512357	6.514711	6.517079	6.519461	6.521858	6.52427	6.526696	6.529137	6.531593
73	6.534065	6.536552	6.539054	6.541573	6.544107	6.546658	6.549225	6.551809	6.55441	6.557027
74	6.559662	6.562314	6.564984	6.567671	6.570377	6.573101	6.575844	6.578605	6.581385	6.584185
75	6.587004	6.589843	6.592701	6.59558	6.59848	6.6014	6.604342	6.607305	6.610289	6.613296
76	6.616325	6.619376	6.622451	6.625548	6.62867	6.631815	6.634984	6.638178	6.641397	6.644642
77	6.647912	6.651208	6.654531	6.657881	6.661258	6.664663	6.668097	6.671558	6.67505	6.67857
78	6.682121	6.685703	6.689315	6.692959	6.696636	6.700345	6.704087	6.707863	6.711674	6.71552
79	6.719401	6.723319	6.727274	6.731266	6.735297	6.739367	6.743477	6.747627	6.751819	6.756053
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	6.76033	6.764651	6.769016	6.773427	6.777885	6.782391	6.786945	6.791548	6.796203	6.800909
81	6.805668	6.810481	6.815349	6.820274	6.825256	6.830298	6.8354	6.840565	6.845792	6.851085
82	6.856445	6.861872	6.86737	6.87294	6.878583	6.884302	6.890099	6.895975	6.901934	6.907976
83	6.914106	6.920324	6.926634	6.933038	6.93954	6.946141	6.952846	6.959658	6.966579	6.973614
84	6.980765	6.988038	6.995437	7.002964	7.010626	7.018427	7.026372	7.034466	7.042716	7.051126
85	7.059704	7.068456	7.07739	7.086512	7.095831	7.105357	7.115097	7.125062	7.135262	7.145709
86	7.156415	7.167393	7.178657	7.190223	7.202106	7.214325	7.226899	7.239849	7.253198	7.266973
87	7.2812	7.29591	7.311137	7.32692	7.343298	7.36032	7.378038	7.396511	7.415807	7.436001
88	7.457181	7.479449	7.502922	7.527737	7.554059	7.582081	7.612038	7.644217	7.678973	7.716757
89	7.758145	7.803898	7.855047	7.913035	7.979979	8.059158	8.156066	8.281003	8.457094	8.758123
90	#DIV/0!									

Table of Natural Cosecants

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	#DIV/0!	572.9581	286.4795	190.9868	143.2406	114.593	95.49471	81.85315	71.62205	63.6646
1	57.29869	52.09027	47.74997	44.07746	40.92963	38.20155	35.81452	33.70835	31.83623	30.1612
2	28.65371	27.28981	26.04994	24.9179	23.88022	22.92559	22.0444	21.22852	20.47093	19.7656
3	19.10732	18.49153	17.91424	17.37196	16.86159	16.38041	15.92597	15.49611	15.0889	14.70258
4	14.33559	13.98651	13.65408	13.33712	13.03458	12.74549	12.469	12.20427	11.95059	11.70728
5	11.47371	11.24932	11.03356	10.82596	10.62605	10.43343	10.2477	10.06849	9.895474	9.728333
6	9.566772	9.410518	9.259314	9.11292	8.97111	8.833671	8.700407	8.57113	8.445663	8.323841
7	8.205509	8.090518	7.97873	7.870012	7.764241	7.661298	7.561071	7.463456	7.368351	7.275662
8	7.185297	7.09717	7.0112	6.927309	6.845422	6.765469	6.687382	6.611097	6.536553	6.46369
9	6.392453	6.322788	6.254645	6.187972	6.122725	6.058858	5.996327	5.935092	5.875113	5.816351
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	5.75877	5.702336	5.647014	5.592772	5.539579	5.487404	5.43622	5.385998	5.336711	5.288335
11	5.240843	5.194212	5.14842	5.103443	5.059261	5.015852	4.973196	4.931275	4.89007	4.849562
12	4.809734	4.77057	4.732052	4.694166	4.656896	4.620226	4.584144	4.548634	4.513684	4.479281
13	4.445411	4.412064	4.379226	4.346886	4.315034	4.283658	4.252747	4.222293	4.192284	4.162711
14	4.133565	4.104837	4.076518	4.048599	4.021072	3.993929	3.967162	3.940763	3.914725	3.889041
15	3.863703	3.838705	3.81404	3.789701	3.765682	3.741978	3.71858	3.695485	3.672687	3.650178
16	3.627955	3.606012	3.584344	3.562945	3.541811	3.520937	3.500318	3.479949	3.459827	3.439947
17	3.420304	3.400894	3.381714	3.362759	3.344025	3.32551	3.307208	3.289116	3.271231	3.25355
18	3.236068	3.218783	3.201691	3.18479	3.168076	3.151545	3.135196	3.119025	3.10303	3.087207
19	3.071553	3.056068	3.040746	3.025587	3.010587	2.995744	2.981056	2.966521	2.952135	2.937897
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	2.923804	2.909855	2.896048	2.882379	2.868847	2.855451	2.842188	2.829056	2.816053	2.803178
21	2.790428	2.777802	2.765299	2.752916	2.740651	2.728504	2.716472	2.704554	2.692748	2.681053
22	2.669467	2.657989	2.646617	2.635351	2.624187	2.613126	2.602165	2.591304	2.580541	2.569875
23	2.559305	2.548828	2.538445	2.528154	2.517954	2.507843	2.49782	2.487885	2.478037	2.468273
24	2.458593	2.448997	2.439482	2.430049	2.420695	2.411421	2.402225	2.393106	2.384063	2.375095
25	2.366202	2.357382	2.348635	2.339959	2.331355	2.32282	2.314355	2.305959	2.29763	2.289368
26	2.281172	2.273042	2.264976	2.256974	2.249035	2.241158	2.233344	2.22559	2.217897	2.210264
27	2.202689	2.195173	2.187715	2.180314	2.172969	2.165681	2.158447	2.151268	2.144144	2.137073
28	2.130054	2.123089	2.116175	2.109312	2.1025	2.095739	2.089027	2.082364	2.07575	2.069184
29	2.062665	2.056194	2.04977	2.043392	2.037059	2.030772	2.02453	2.018332	2.012178	2.006067
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**

30	2	1.993975	1.987993	1.982052	1.976153	1.970294	1.964477	1.958699	1.952961	1.947263
31	1.941604	1.935983	1.930401	1.924857	1.91935	1.913881	1.908448	1.903052	1.897692	1.892368
32	1.88708	1.881827	1.876608	1.871424	1.866275	1.861159	1.856077	1.851028	1.846012	1.841029
33	1.836078	1.83116	1.826273	1.821418	1.816594	1.811801	1.807039	1.802307	1.797605	1.792934
34	1.788292	1.783679	1.779095	1.774541	1.770015	1.765517	1.761048	1.756606	1.752192	1.747806
35	1.743447	1.739115	1.734809	1.73053	1.726277	1.722051	1.71785	1.713675	1.709525	1.705401
36	1.701302	1.697227	1.693177	1.689152	1.68515	1.681173	1.67722	1.67329	1.669383	1.6655
37	1.66164	1.657803	1.653989	1.650197	1.646427	1.64268	1.638954	1.635251	1.631569	1.627908
38	1.624269	1.620651	1.617054	1.613478	1.609923	1.606388	1.602873	1.599379	1.595905	1.59245
39	1.589016	1.585601	1.582205	1.578829	1.575472	1.572134	1.568815	1.565514	1.562232	1.558969
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	1.555724	1.552497	1.549288	1.546097	1.542924	1.539769	1.536631	1.533511	1.530408	1.527322
41	1.524253	1.521201	1.518166	1.515148	1.512146	1.50916	1.506191	1.503239	1.500302	1.497381
42	1.494477	1.491588	1.488714	1.485856	1.483014	1.480187	1.477376	1.474579	1.471797	1.469031
43	1.466279	1.463542	1.46082	1.458112	1.455419	1.45274	1.450075	1.447424	1.444788	1.442165
44	1.439557	1.436962	1.43438	1.431813	1.429259	1.426718	1.424191	1.421677	1.419176	1.416688
45	1.414214	1.411752	1.409303	1.406867	1.404443	1.402032	1.399634	1.397248	1.394874	1.392513
46	1.390164	1.387827	1.385502	1.383189	1.380888	1.378598	1.376321	1.374055	1.371801	1.369559
47	1.367327	1.365108	1.362899	1.360702	1.358516	1.356342	1.354178	1.352025	1.349884	1.347753
48	1.345633	1.343523	1.341425	1.339337	1.337259	1.335192	1.333136	1.33109	1.329054	1.327028
49	1.325013	1.323008	1.321013	1.319027	1.317052	1.315087	1.313132	1.311186	1.30925	1.307324
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	1.305407	1.3035	1.301603	1.299715	1.297836	1.295967	1.294107	1.292256	1.290415	1.288583
51	1.28676	1.284945	1.28314	1.281344	1.279557	1.277779	1.276009	1.274248	1.272496	1.270753
52	1.269018	1.267292	1.265574	1.263865	1.262165	1.260472	1.258789	1.257113	1.255446	1.253787
53	1.252136	1.250493	1.248858	1.247232	1.245613	1.244003	1.2424	1.240805	1.239218	1.237639
54	1.236068	1.234504	1.232949	1.2314	1.22986	1.228327	1.226801	1.225284	1.223773	1.22227
55	1.220775	1.219286	1.217805	1.216332	1.214866	1.213406	1.211954	1.21051	1.209072	1.207641
56	1.206218	1.204801	1.203392	1.201989	1.200594	1.199205	1.197823	1.196448	1.19508	1.193718
57	1.192363	1.191015	1.189674	1.188339	1.187011	1.185689	1.184374	1.183065	1.181763	1.180468
58	1.179178	1.177896	1.176619	1.175349	1.174085	1.172828	1.171576	1.170331	1.169093	1.16786
59	1.166633	1.165413	1.164199	1.16299	1.161788	1.160592	1.159402	1.158218	1.157039	1.155867
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	1.154701	1.15354	1.152385	1.151236	1.150093	1.148956	1.147824	1.146698	1.145578	1.144463
61	1.143354	1.142251	1.141153	1.140061	1.138974	1.137893	1.136818	1.135748	1.134683	1.133624
62	1.13257	1.131522	1.130479	1.129441	1.128409	1.127382	1.12636	1.125344	1.124333	1.123327
63	1.122326	1.121331	1.12034	1.119355	1.118375	1.1174	1.116431	1.115466	1.114506	1.113552

64	1.112602	1.111657	1.110718	1.109783	1.108853	1.107929	1.107009	1.106094	1.105184	1.104278
65	1.103378	1.102482	1.101592	1.100706	1.099824	1.098948	1.098076	1.097209	1.096347	1.095489
66	1.094636	1.093788	1.092944	1.092105	1.091271	1.090441	1.089616	1.088795	1.087979	1.087167
67	1.08636	1.085558	1.08476	1.083966	1.083177	1.082392	1.081612	1.080836	1.080065	1.079297
68	1.078535	1.077776	1.077022	1.076273	1.075527	1.074786	1.074049	1.073317	1.072589	1.071865
69	1.071145	1.070429	1.069718	1.069011	1.068308	1.067609	1.066915	1.066224	1.065538	1.064856
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	1.064178	1.063504	1.062834	1.062168	1.061506	1.060849	1.060195	1.059545	1.0589	1.058258
71	1.057621	1.056987	1.056357	1.055732	1.05511	1.054492	1.053878	1.053269	1.052663	1.05206
72	1.051462	1.050868	1.050277	1.049691	1.049108	1.048529	1.047954	1.047383	1.046815	1.046252
73	1.045692	1.045136	1.044583	1.044035	1.04349	1.042949	1.042412	1.041878	1.041348	1.040822
74	1.040299	1.039781	1.039266	1.038754	1.038246	1.037742	1.037242	1.036745	1.036252	1.035762
75	1.035276	1.034794	1.034315	1.03384	1.033368	1.0329	1.032436	1.031975	1.031518	1.031064
76	1.030614	1.030167	1.029724	1.029284	1.028848	1.028415	1.027986	1.02756	1.027138	1.026719
77	1.026304	1.025892	1.025484	1.025079	1.024678	1.02428	1.023885	1.023494	1.023106	1.022722
78	1.022341	1.021963	1.021589	1.021218	1.020851	1.020487	1.020126	1.019769	1.019415	1.019064
79	1.018717	1.018373	1.018032	1.017695	1.017361	1.01703	1.016703	1.016379	1.016058	1.015741
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	1.015427	1.015116	1.014808	1.014504	1.014203	1.013905	1.013611	1.013319	1.013031	1.012747
81	1.012465	1.012187	1.011912	1.01164	1.011371	1.011106	1.010844	1.010585	1.010329	1.010077
82	1.009828	1.009581	1.009339	1.009099	1.008862	1.008629	1.008399	1.008172	1.007948	1.007727
83	1.00751	1.007295	1.007084	1.006876	1.006671	1.00647	1.006271	1.006076	1.005883	1.005694
84	1.005508	1.005325	1.005146	1.004969	1.004795	1.004625	1.004458	1.004294	1.004133	1.003975
85	1.00382	1.003668	1.003519	1.003374	1.003232	1.003092	1.002956	1.002823	1.002693	1.002566
86	1.002442	1.002321	1.002203	1.002089	1.001977	1.001869	1.001763	1.001661	1.001562	1.001465
87	1.001372	1.001282	1.001195	1.001111	1.00103	1.000953	1.000878	1.000806	1.000738	1.000672
88	1.00061	1.00055	1.000494	1.00044	1.00039	1.000343	1.000299	1.000257	1.000219	1.000184
89	1.000152	1.000123	1.000097	1.000075	1.000055	1.000038	1.000024	1.000014	1.000006	1.000002
90	1									

Table of Logarithms of Natural Cosecants

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
0	#DIV/0!	8.758123	8.457094	8.281003	8.156066	8.059158	7.979979	7.913035	7.855047	7.803898
1	7.758145	7.716757	7.678973	7.644217	7.612038	7.582081	7.554059	7.527737	7.502922	7.479449
2	7.457181	7.436001	7.415807	7.396511	7.378038	7.36032	7.343298	7.32692	7.311137	7.29591
3	7.2812	7.266973	7.253198	7.239849	7.226899	7.214325	7.202106	7.190223	7.178657	7.167393

4	7.156415	7.145709	7.135262	7.125062	7.115097	7.105357	7.095831	7.086512	7.07739	7.068456
5	7.059704	7.051126	7.042716	7.034466	7.026372	7.018427	7.010626	7.002964	6.995437	6.988038
6	6.980765	6.973614	6.966579	6.959658	6.952846	6.946141	6.93954	6.933038	6.926634	6.920324
7	6.914106	6.907976	6.901934	6.895975	6.890099	6.884302	6.878583	6.87294	6.86737	6.861872
8	6.856445	6.851085	6.845792	6.840565	6.8354	6.830298	6.825256	6.820274	6.815349	6.810481
9	6.805668	6.800909	6.796203	6.791548	6.786945	6.782391	6.777885	6.773427	6.769016	6.764651
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
10	6.76033	6.756053	6.751819	6.747627	6.743477	6.739367	6.735297	6.731266	6.727274	6.723319
11	6.719401	6.71552	6.711674	6.707863	6.704087	6.700345	6.696636	6.692959	6.689315	6.685703
12	6.682121	6.67857	6.67505	6.671558	6.668097	6.664663	6.661258	6.657881	6.654531	6.651208
13	6.647912	6.644642	6.641397	6.638178	6.634984	6.631815	6.62867	6.625548	6.622451	6.619376
14	6.616325	6.613296	6.610289	6.607305	6.604342	6.6014	6.59848	6.59558	6.592701	6.589843
15	6.587004	6.584185	6.581385	6.578605	6.575844	6.573101	6.570377	6.567671	6.564984	6.562314
16	6.559662	6.557027	6.55441	6.551809	6.549225	6.546658	6.544107	6.541573	6.539054	6.536552
17	6.534065	6.531593	6.529137	6.526696	6.52427	6.521858	6.519461	6.517079	6.514711	6.512357
18	6.510018	6.507692	6.505379	6.503081	6.500796	6.498524	6.496265	6.494019	6.491786	6.489566
19	6.487358	6.485163	6.48298	6.48081	6.478651	6.476505	6.47437	6.472247	6.470136	6.468037
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
20	6.465948	6.463871	6.461806	6.459751	6.457707	6.455675	6.453653	6.451641	6.449641	6.447651
21	6.445671	6.443701	6.441742	6.439793	6.437854	6.435925	6.434005	6.432096	6.430196	6.428305
22	6.426425	6.424553	6.422691	6.420838	6.418995	6.41716	6.415335	6.413518	6.411711	6.409912
23	6.408122	6.406341	6.404568	6.402804	6.401048	6.3993	6.397561	6.39583	6.394108	6.392393
24	6.390687	6.388988	6.387298	6.385615	6.38394	6.382273	6.380614	6.378962	6.377318	6.375681
25	6.374052	6.37243	6.370815	6.369208	6.367608	6.366016	6.36443	6.362852	6.36128	6.359716
26	6.358158	6.356607	6.355064	6.353526	6.351996	6.350473	6.348956	6.347445	6.345941	6.344444
27	6.342953	6.341469	6.339991	6.338519	6.337054	6.335594	6.334141	6.332695	6.331254	6.329819
28	6.328391	6.326968	6.325552	6.324141	6.322736	6.321337	6.319944	6.318557	6.317175	6.315799
29	6.314429	6.313064	6.311705	6.310352	6.309004	6.307661	6.306324	6.304993	6.303666	6.302346
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
30	6.30103	6.29972	6.298415	6.297115	6.295821	6.294531	6.293247	6.291968	6.290694	6.289425
31	6.288161	6.286902	6.285648	6.284398	6.283154	6.281915	6.28068	6.279451	6.278226	6.277006
32	6.27579	6.27458	6.273374	6.272172	6.270976	6.269783	6.268596	6.267413	6.266235	6.265061
33	6.263891	6.262726	6.261566	6.26041	6.259258	6.258111	6.256967	6.255829	6.254694	6.253564
34	6.252438	6.251317	6.250199	6.249086	6.247977	6.246872	6.245771	6.244674	6.243582	6.242493
35	6.241409	6.240328	6.239252	6.238179	6.237111	6.236046	6.234985	6.233928	6.232876	6.231827
36	6.230781	6.22974	6.228702	6.227669	6.226639	6.225612	6.22459	6.223571	6.222556	6.221545
37	6.220537	6.219533	6.218532	6.217536	6.216542	6.215553	6.214567	6.213584	6.212605	6.21163

38	6.210658	6.20969	6.208725	6.207763	6.206805	6.20585	6.204899	6.203951	6.203007	6.202066
39	6.201128	6.200194	6.199263	6.198335	6.197411	6.196489	6.195572	6.194657	6.193746	6.192837
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
40	6.191933	6.191031	6.190132	6.189237	6.188345	6.187456	6.18657	6.185687	6.184807	6.183931
41	6.183057	6.182187	6.181319	6.180455	6.179594	6.178735	6.17788	6.177028	6.176179	6.175332
42	6.174489	6.173649	6.172811	6.171977	6.171145	6.170317	6.169491	6.168668	6.167848	6.167031
43	6.166217	6.165405	6.164597	6.163791	6.162988	6.162188	6.16139	6.160596	6.159804	6.159015
44	6.158229	6.157445	6.156664	6.155886	6.155111	6.154338	6.153568	6.152801	6.152036	6.151274
45	6.150515	6.149758	6.149004	6.148253	6.147504	6.146758	6.146014	6.145273	6.144535	6.143799
46	6.143066	6.142335	6.141607	6.140881	6.140158	6.139438	6.13872	6.138004	6.137291	6.136581
47	6.135873	6.135167	6.134464	6.133763	6.133065	6.132369	6.131676	6.130985	6.130296	6.12961
48	6.128927	6.128245	6.127566	6.12689	6.126216	6.125544	6.124874	6.124207	6.123543	6.12288
49	6.12222	6.121562	6.120907	6.120254	6.119603	6.118954	6.118308	6.117664	6.117023	6.116383
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
50	6.115746	6.115111	6.114478	6.113848	6.11322	6.112594	6.11197	6.111349	6.110729	6.110112
51	6.109497	6.108885	6.108274	6.107666	6.10706	6.106456	6.105854	6.105254	6.104657	6.104061
52	6.103468	6.102877	6.102288	6.101701	6.101116	6.100533	6.099953	6.099374	6.098798	6.098224
53	6.097651	6.097081	6.096513	6.095947	6.095383	6.094821	6.094261	6.093704	6.093148	6.092594
54	6.092042	6.091493	6.090945	6.090399	6.089856	6.089314	6.088774	6.088237	6.087701	6.087167
55	6.086635	6.086106	6.085578	6.085052	6.084528	6.084006	6.083486	6.082968	6.082452	6.081938
56	6.081426	6.080915	6.080407	6.079901	6.079396	6.078893	6.078393	6.077894	6.077397	6.076902
57	6.076409	6.075917	6.075428	6.07494	6.074455	6.073971	6.073489	6.073009	6.07253	6.072054
58	6.07158	6.071107	6.070636	6.070167	6.0697	6.069234	6.068771	6.068309	6.067849	6.067391
59	6.066934	6.06648	6.066027	6.065576	6.065127	6.06468	6.064234	6.06379	6.063348	6.062908
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
60	6.062469	6.062033	6.061598	6.061164	6.060733	6.060303	6.059875	6.059449	6.059025	6.058602
61	6.058181	6.057761	6.057344	6.056928	6.056514	6.056101	6.055691	6.055282	6.054875	6.054469
62	6.054065	6.053663	6.053262	6.052864	6.052467	6.052071	6.051677	6.051285	6.050895	6.050506
63	6.050119	6.049734	6.04935	6.048968	6.048588	6.048209	6.047832	6.047456	6.047082	6.04671
64	6.04634	6.045971	6.045604	6.045238	6.044874	6.044512	6.044151	6.043792	6.043434	6.043079
65	6.042724	6.042372	6.042021	6.041671	6.041323	6.040977	6.040632	6.040289	6.039948	6.039608
66	6.03927	6.038933	6.038598	6.038265	6.037933	6.037602	6.037273	6.036946	6.036621	6.036296
67	6.035974	6.035653	6.035334	6.035016	6.034699	6.034385	6.034071	6.03376	6.03345	6.033141
68	6.032834	6.032529	6.032225	6.031922	6.031621	6.031322	6.031024	6.030728	6.030433	6.03014
69	6.029848	6.029558	6.029269	6.028982	6.028697	6.028412	6.02813	6.027849	6.027569	6.027291
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
70	6.027014	6.026739	6.026465	6.026193	6.025923	6.025653	6.025386	6.02512	6.024855	6.024592

71	6.02433	6.02407	6.023811	6.023554	6.023298	6.023043	6.022791	6.022539	6.022289	6.022041
72	6.021794	6.021548	6.021304	6.021061	6.02082	6.02058	6.020342	6.020105	6.01987	6.019636
73	6.019404	6.019173	6.018943	6.018715	6.018488	6.018263	6.018039	6.017817	6.017596	6.017376
74	6.017158	6.016942	6.016727	6.016513	6.0163	6.016089	6.01588	6.015672	6.015465	6.01526
75	6.015056	6.014854	6.014653	6.014453	6.014255	6.014058	6.013863	6.013669	6.013477	6.013286
76	6.013096	6.012908	6.012721	6.012535	6.012351	6.012168	6.011987	6.011807	6.011629	6.011452
77	6.011276	6.011102	6.010929	6.010757	6.010587	6.010418	6.010251	6.010085	6.009921	6.009757
78	6.009596	6.009435	6.009276	6.009118	6.008962	6.008807	6.008654	6.008502	6.008351	6.008201
79	6.008053	6.007907	6.007761	6.007618	6.007475	6.007334	6.007194	6.007056	6.006919	6.006783
x	**0'**	**6'**	**12'**	**18'**	**24'**	**30'**	**36'**	**42'**	**48'**	**54'**
80	6.006649	6.006516	6.006384	6.006254	6.006125	6.005997	6.005871	6.005746	6.005623	6.005501
81	6.00538	6.005261	6.005143	6.005026	6.004911	6.004797	6.004684	6.004573	6.004463	6.004354
82	6.004247	6.004141	6.004037	6.003934	6.003832	6.003731	6.003632	6.003535	6.003438	6.003343
83	6.003249	6.003157	6.003066	6.002976	6.002888	6.002801	6.002715	6.002631	6.002548	6.002466
84	6.002386	6.002307	6.002229	6.002153	6.002078	6.002004	6.001932	6.001861	6.001791	6.001723
85	6.001656	6.00159	6.001526	6.001463	6.001401	6.001341	6.001282	6.001224	6.001168	6.001113
86	6.001059	6.001007	6.000956	6.000906	6.000858	6.000811	6.000765	6.000721	6.000678	6.000636
87	6.000596	6.000557	6.000519	6.000482	6.000447	6.000414	6.000381	6.00035	6.00032	6.000292
88	6.000265	6.000239	6.000214	6.000191	6.000169	6.000149	6.00013	6.000112	6.000095	6.00008
89	6.000066	6.000054	6.000042	6.000032	6.000024	6.000017	6.000011	6.000006	6.000003	6.000001
90	6									

Student Work Examples

Advisor: Bean

Unit Assessment

Show all work on separate sheets of paper attached to this one. Failure to follow this instruction will result in a failing grade for this assessment.

Given the following information, perform a day's work in navigation, finding the ship's latitude, longitude, distance travelled, and average speed.

Previous Position (7 days earlier): 37.8 N 122.W

Declination: 16.3 N

First Altitude: 22.4

Second Altitude: 60.9

First Time: 0900

Second Time: 1000

GMT of LAN: 20:08

47.7°N N lat.

122°W W long.

594 nm

3.53 knots

95

2014 – 2015 School Year
Original

Student Work Examples

Given	Find
Decl 16.3°N	pos
1stAlt 22.4	47.7°N/122°W
2ndAlt 60.9	distance
1sttime 0900	594nm
2ndtime 1000	speed
GMTofLAN 20:08	3.53knots
Lastpos 37.8°N/122°W	

Latitude

$\log_{10}(\sec 16.3) =$ 6.017816

$60.9 + 22.4 = 83.3$ $\quad 83.30 \div 2 = 41.65$ $\quad \log_{10}(\cos 41.6) =$ 5.873784

$60.9 - 22.4 = 38.5$ $\quad 38.50 \div 2 = 19.25$ $\quad \log_{10}(\sin 19.2) =$ 5.517019

$1200 - 900 = 3°$ $\quad 1200 - 1000 = 2°$

.017816 $\quad \frac{3+2}{2}\ \frac{5}{2}\ 2.5 \quad \log_{10}(\csc 2.5) = 7.360320$

.873784

.517019 $\quad \frac{3-2}{2}\ \frac{1}{2}\ 0.5 \quad \log_{10}(\csc 0.5) \quad 8.059158$

.360320

.059158 $\quad 47.7°N = 5.828023\,1$

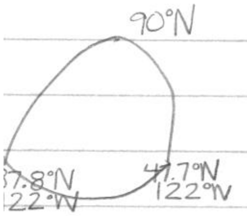
1.828097

Student Work Examples

Longitude

$$\begin{array}{r} 20{:}08 \\ -\ 12{:}00 \\ \hline 8{:}08 \end{array} \qquad \begin{array}{r} 15 \\ \times\ 8 \\ \hline 120 \end{array} \qquad \frac{8}{4} = 2 \qquad 120 + 2 = 122°W$$

Distance



$$\begin{array}{r} 90.0 \\ -\ 37.8 \\ \hline 52.2 \end{array} \qquad \begin{array}{r} 90.0 \\ -\ 47.7 \\ \hline 42.3 \end{array} \qquad \begin{array}{r} 122 \\ -122 \\ \hline 0 \end{array}$$

$\log_{10}(\cos 52.2)$ = 5.787394

$\log_{10}(\cos 42.3)$ 5.869015

11.656409 $\quad \frac{4.53}{10}$ .453

$\log_{10}(\sin 52.2)$ 5.897712

$\log_{10}(\sin 42.3)$ 5.828023 $\quad \frac{5.32}{10}$ .532

$\log_{10}(\cos 0)$ 6. $\quad$ .985

17.725735

$\cos^{-1}(.985)$

9.9

$$\begin{array}{r} 60 \\ \times\ 9.9 \\ \hline 540 \\ 5400 \\ \hline 594.00 \end{array}$$ 594nm

Speed

$$\begin{array}{r} 84.85 \\ 7\overline{)594.00} \\ -56 \\ \hline 34 \\ -28 \\ \hline 60 \\ -56 \\ \hline 40 \\ -35 \\ \hline 5 \end{array} \qquad \begin{array}{r} 3.53 \\ 24\overline{)84.85} \\ -72 \\ \hline 128 \\ -120 \\ \hline 85 \\ -72 \\ \hline 13 \end{array}$$

Student Work Examples

Unit Assessment

Show all work on separate sheets of paper attached to this one. Failure to follow this instruction will result in a failing grade for this assessment.

Given the following information, perform a day's work in navigation, finding the ship's latitude, longitude, distance travelled, and average speed.

Previous Position (7 days earlier): 37.8 N 122 W

Declination: 16.3 N

First Altitude: 22.4

Second Altitude: 60.9

First Time: 0900

Second Time: 1000

GMT of LAN: 20:08

47° 42'N ⇒ 47.7°N ______ N lat.

122°W ______ W long.

588 nm ______ nm

3.50 nm/hr ______ knots

Trigonometry of Navigation
and Surveying **WORK**

Givens

Previous Position (7days ago) 37.8N 122W

Declination: 16.3N

First Altitude: 22.4

Second Altitude: 60.9

First time = 0900

second: 1000

GMT of LAN: 20:08

Longitude:

Step 1 20:08

$$\begin{array}{r} 20{:}08 \\ -12{:}00 \\ \hline 8{:}08 \end{array}$$

$$8\text{hrs} \times \frac{15^\circ}{1\text{hr}} = 120^\circ$$

Step 2. $08\text{min} \times \frac{1^\circ}{4} = 2^\circ$

$$\boxed{\text{Longitude} = 122^\circ W}$$

Latitude -

1. ~~[illegible]~~ Log(sec(16.3)) = .017816

2. $\begin{array}{r} 60.9 \\ +\ 22.4 \\ \hline 83.3 \end{array}$ $\frac{83.3}{2} = 41.65$

log(cos(41.65)) = .873784

3. $\begin{array}{r} 60.9 \\ -\ 22.4 \\ \hline 38.5 \end{array}$ $\frac{38.5}{2} = 19.25$

log(sin(19.25)) = .517019

4. $\begin{array}{r} 1200 \\ -0900 \\ \hline 3\text{hrs} \end{array}$ $\begin{array}{r} 1200 \\ -1000 \\ \hline 2\text{hrs} \end{array}$

3° 2°

$\frac{(3^\circ + 2^\circ)}{2} = 2.5$

log(csc(2.5)) = .360320

$\frac{(3^\circ - 2^\circ)}{2} = .5$

log(csc(.5)) = .059158

Wrong

$\begin{array}{r} .017816 \\ +.873784 \\ \hline .891600 \end{array}$ $\begin{array}{r} .891600 \\ +.877339 \\ \hline 1.768939 \\ +.059158 \\ \hline 1.828097 \end{array}$

$\begin{array}{r} .517019 \\ +.360320 \\ \hline .877339 \end{array}$

Addition

$\left.\begin{array}{r} .017816 \\ .873784 \end{array}\right\} .891600$

$\left.\begin{array}{r} .517019 \\ .360320 \end{array}\right\} .877339$

$\begin{array}{r} .059158 \\ \hline \boxed{.828097} \end{array}$

log(cos(.828097) → 47° 42' N

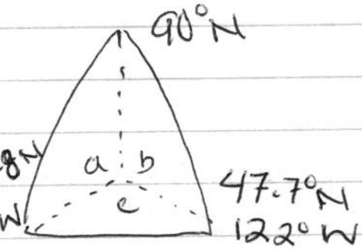
47.7°

Distance:

m∠a = 52.2

m∠b = 42.3

m∠c = 0°



log(cos(52.2)) = .78739462

log(cos(42.3)) = .86901516

1.656409 → $\frac{4.53}{10}$ = .453

log(sin(52.2)) = .8977123

log(sin(42.3)) = .8280231

7.705154 → $\frac{5.07}{10}$

.507

log(cos(0°)) = 6

.453

+ .507

.960

18.01

× 60

$\cos^{-1}(.960) \times 60 =$

~~18.01 × 60 =~~

~~1080~~

16.2 × 60 = 972 588nm

Avg. speed

$\frac{588\text{nm}}{7} = 84$

$\frac{84}{24\text{ days}} = 3.50\frac{\text{nm}}{\text{h}}$

60.0

× 16.2

120.0

360.00

600

.00

90.10

−37.8

53.2

90.10

−21.3

59.7

90.10

−21.3

68.7

90.10

−37.8

52.2

90.10

−47.7

42.3

.787394

+.869015

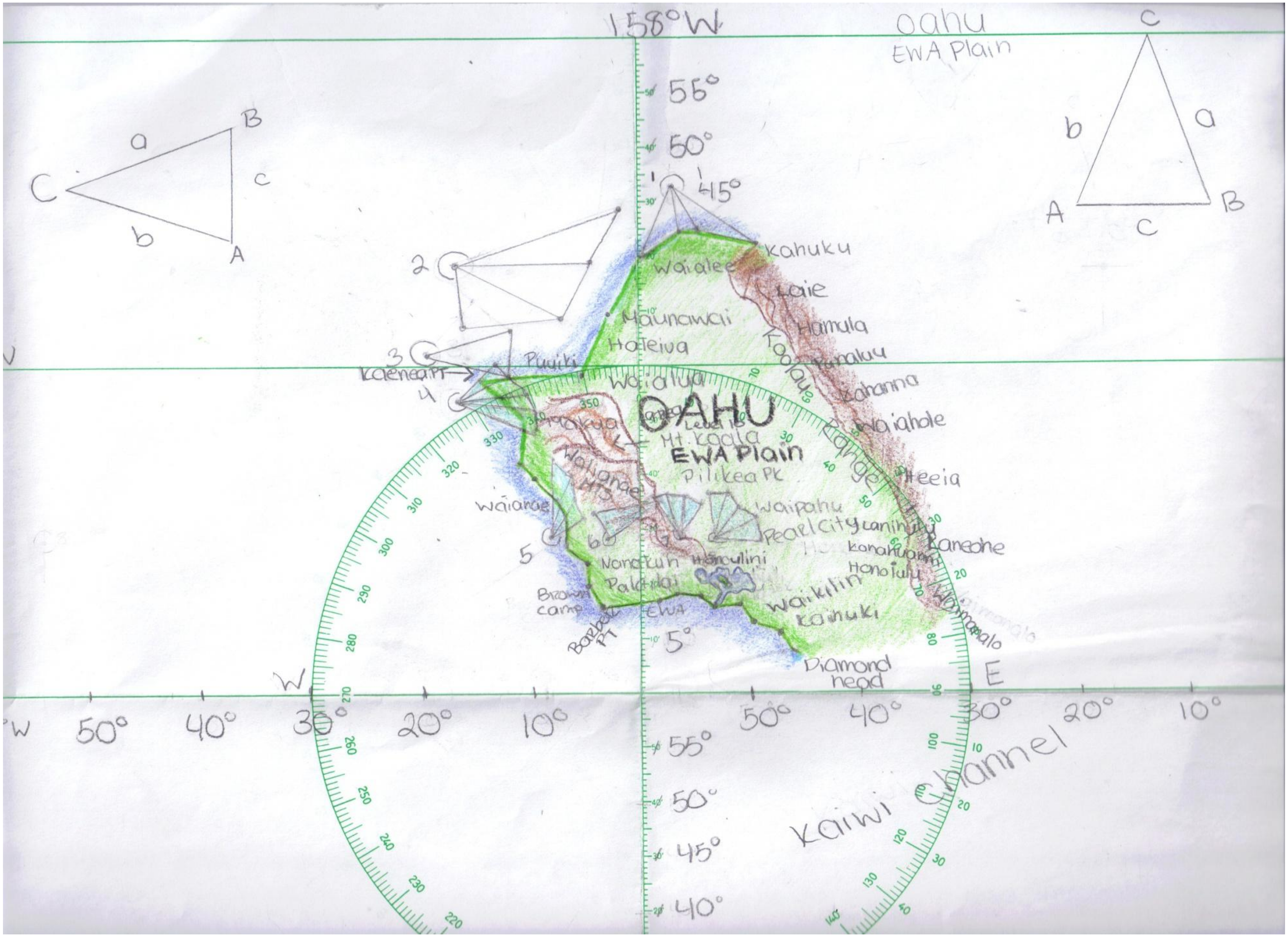
1.656409

.877123

+.828031

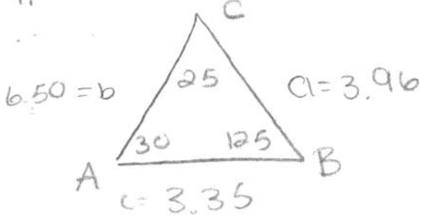
1.705154

Student Work Examples



Student Work Examples

1.



$180 - 125 - 25 = 30$

$\angle A = 30$

$c = \dfrac{\sin 35}{0.126023} = 3.35$

1st Initial Bearing 204°T

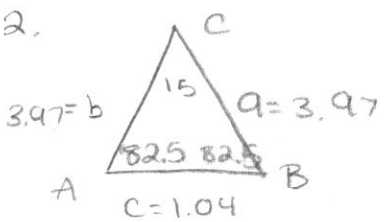
$\dfrac{\sin 125}{6.50} = \dfrac{\sin 30A}{a}$

$0.12602 = \dfrac{\sin 30A}{a}$

$\dfrac{a \times \cancel{0.126023}}{\cancel{0.126023}} = \dfrac{\sin 30A}{0.126023}$

$a = \dfrac{\sin 30}{0.126023} = 3.96$

2.



$c^2 = b^2 + a^2 - 2ab \cos\angle C$

$c^2 = 3.97^2 + 3.97^2 - 2(3.97)(3.97) \times \cos 15$

$15.76 + 15.76 - 31.52 \times \cos 15$

$c^2 = 31.52 - 30.44 = 1.08$

$\sqrt{1.08} = 1.039$

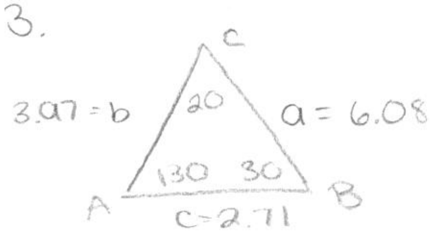
$\dfrac{\sin 15\,C}{1.04} = \dfrac{0.258}{1.04} = \dfrac{\sin\angle B}{3.97}$

$\dfrac{3.97 \times 0.258}{1.04} = \sin\angle B$
$= .9848$

$\sin^{-1}(.9848) = 82.50$

$180 - 82.50 - 15 =$
82.50

3.



$180 - 50 = 130$

$a = \dfrac{\sin 130}{0.125944} = 6.08$

$\dfrac{\sin 30B}{3.97} = \dfrac{\sin 20C}{c}$

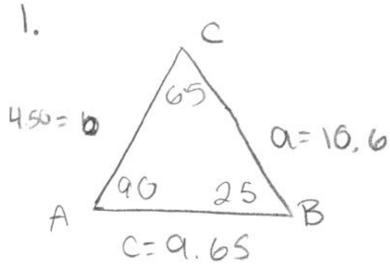
$0.125944 = \dfrac{\sin 20C}{c}$

$\dfrac{c \times 0.125944}{0.125944} = \dfrac{\sin 20C}{0.125944}$

$c = \dfrac{\sin 20C}{0.125944} = 2.71$

Student Work Examples

1.



$180-90=90$

2nd Initial Bearing 177°T

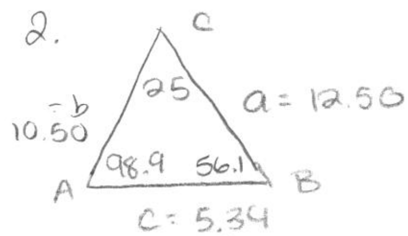
$$\frac{\sin 25B}{4.50} = \frac{\sin 65C}{c}$$

$$0.093915 = \frac{\sin 65C}{c}$$

$$\frac{c \times \cancel{0.093915}}{\cancel{0.093915}} = \frac{\sin 65}{0.093915} = 9.65$$

$$a = \frac{\sin 90}{0.093915} = 10.6$$

2.



$$c^2 = b^2 + a^2 - 2ab\cos\angle C$$

$$c^2 = 10.50^2 + 12.50^2 - 2(10.50)(12.50)\cos 25$$

$$110.25 + 156.25 - 262.5\cos 25$$

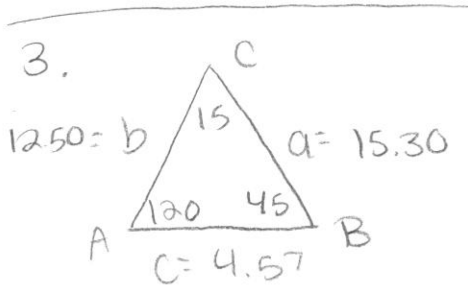
$$c^2 = 266.5 - 237.9 = 28.6$$

$$\sqrt{28.6} = 5.34$$

$$\frac{\sin 25}{5.34} = \frac{0.423}{5.34} = \frac{\sin\angle B}{10.50}$$

$$\frac{10.50 \times 0.423}{5.34} = .83 \qquad \sin^{-1}(.83) = 56.09$$

3.



$180-60=120$

$$\frac{\sin 45B}{12.50} = \frac{\sin 15C}{c}$$

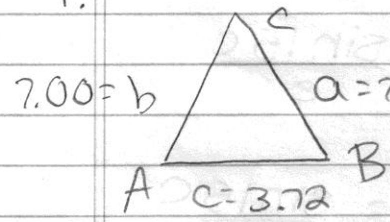
$$\frac{c \times 0.056568}{0.056568} = \frac{\sin 15C}{0.056568} = 4.57$$

$$a = \frac{\sin 120A}{0.056568} = 15.30$$

Student Work Examples

3rd Initial Bearing 100°T

1.



$$\frac{\sin 70B}{7.00} = \frac{\sin 80A}{a}$$

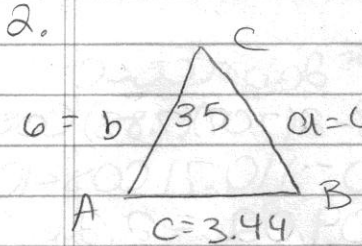
$$0.134241 = \frac{\sin 80A}{a}$$

$$\frac{a \times 0.134241}{0.134241} = \frac{\sin 80A}{0.134241}$$

$$C = \frac{\sin 30C}{0.134241} = 3.72$$

$$a = \frac{\sin 80}{0.134241} = 7.33$$

2.



$4.91^2 + 6^2 - 2(4.91)(6)\cos 35$

$24.1 + 36 - 58.92 \times \cos 35$

$c^2 = 60.1 - 48.26 = 11.84$

$\sqrt{11.84} = 3.44$

$$\frac{\sin 35}{3.44} = \frac{0.57}{3.44} = \frac{\sin \angle A}{4.91}$$

$$\frac{4.91 \times 0.57}{3.44} = \sin \angle A = .81357$$

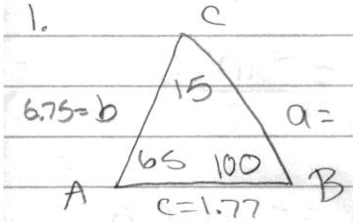
$\sin^{-1}(.81357) = 54.4$

$180 - 55.4 - 35 = 89.6$

Student Work Examples

4th Initial Bearing 109° T

1.



$180 - 115 = 65$

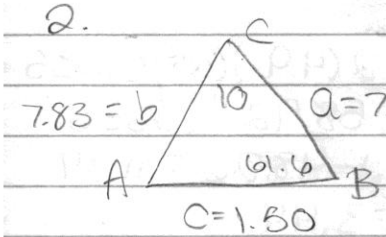
$$\frac{\sin 100 B}{6.75} = \frac{\sin 15 C}{c}$$

$$\frac{c \times 0.145897}{0.145897} = \frac{\sin 15 C}{0.145897}$$

$$c = \frac{\sin 15}{0.145897} = 1.77$$

$$a = \frac{\sin 65}{0.145897} = 6.21$$

2.



$$c^2 = b^2 + a^2 - 2ab \times \cos\angle C$$

$$7.07^2 + 7.83^2 - 2(7.07)(7.83) \cos 10$$

$$49.98 + 61.30 - 110.71 \cos\angle 10$$

$$111.28 - 109.02 = 2.26$$

$$\sqrt{2.26} = 1.50$$

$$\frac{\sin 10 C}{1.50} = \frac{0.17}{1.50} = \frac{\sin\angle B}{7.83}$$

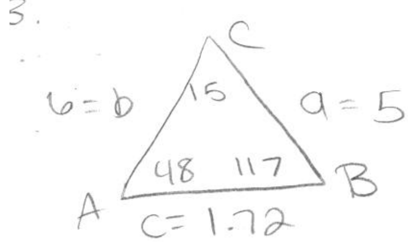
$$\frac{0.17 \times 7.83}{1.50} = \sin\angle B = 0.8874$$

$$\sin^{-1}(.88) = 53.6$$

$$180 - 61.6 - 10 = 116.91$$

Student Work Examples

3.



4th Initial Bearing 204°T

$$\frac{\sin 117B}{6.00} = \frac{\sin 15C}{c}$$

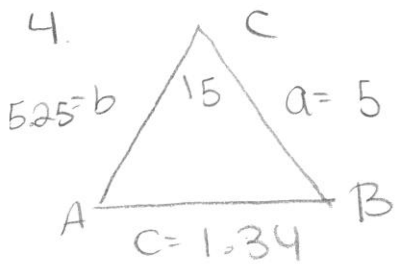
$$\frac{c \times 0.148501}{0.148501} = \frac{\sin 15}{0.148501}$$

$$c = \frac{\sin 15C}{0.148501} = 1.74$$

$$a = \frac{\sin 48}{0.148501} = 5$$

$180-15-117=48$

4.



$5^2 + 5.25^2 - 2(5)(5.25) \times \cos 15$

$25 + 27.56 - 52.5 \times \cos 15$

$c^2 = 52.56 - 50.76$

$= 1.8 \quad \sqrt{1.8} = 1.34$

$180-15-75.9$

$= 89.1$

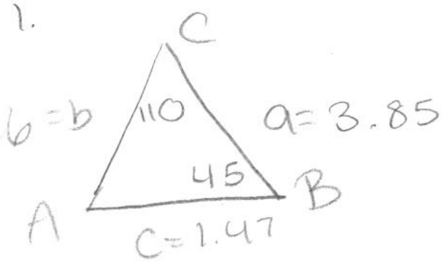
$$\frac{\sin 15}{1.34} = \frac{.25}{1.34} = \frac{\sin \angle B}{5.25}$$

$$\frac{5.25 \times .25}{1.34} = \sin \angle B$$

$= 0.97$

$\sin^{-1}(.97) = 76.9$

1.



6th Initial Bearing 55°T

$$\frac{\sin 45B}{6.00} = \frac{\sin 110C}{c}$$

$$\frac{c \times .117851}{.117851} = \frac{\sin 110}{.117851}$$

$$c = \frac{\sin 110}{.117851} = 7.97$$

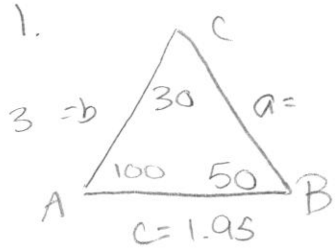
$180-25-110$

$=$

$$a = \frac{\sin 25A}{.117851} = 3.58$$

Student Work Examples

1.



$180-80=100$

5th Initial Bearing 56°

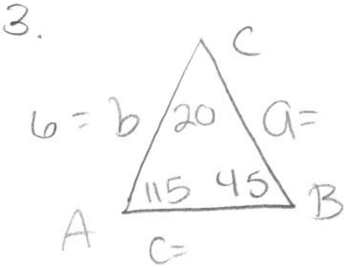
$$\frac{\sin 50B}{3.00} = \frac{\sin 30c}{c}$$

$$cx\ \frac{.255348}{.255348} = \frac{\sin 30c}{c}$$

$$c = \frac{\sin 30C}{0.255348} = 1.95$$

$$a = \frac{\sin 100A}{0.255348} = 3.86$$

3.



$180-45-20$
$=115$

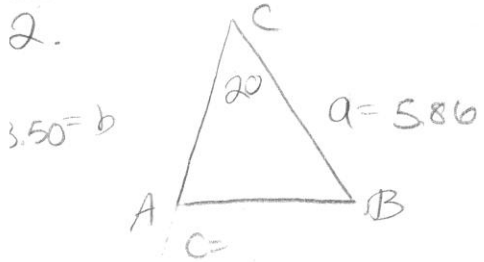
$$\frac{\sin 45B}{6.00} = \frac{\sin 20c}{c}$$

$$cx\ .117851 = \frac{\sin 30C}{c}$$

$$c = \frac{\sin 20c}{.117851} = 2.90$$

$$a = \frac{\sin 115A}{.117851} = 7.69$$

2.



$\sin^{-1}(.42) = 24.8$

$180-24.8-20$
$=135.2$

$5.86^2 + 3.50^2 - 2(3.50)(5.86)\cos 20$

$34.3 + 12.25 - 41.02\cos 20$

$46.55 - 38.54$

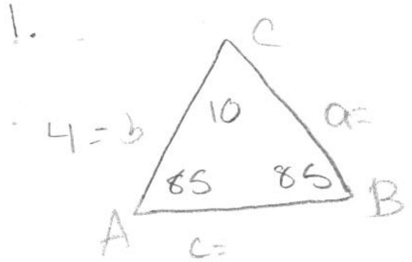
$= 8.01 \quad \sqrt{8.01} = 2.83$

$$\frac{\sin 20}{2.83} = \frac{0.34}{2.83} = \frac{\sin cB}{3.50}$$

$$\frac{0.34 \times 3.50}{2.83} = .42$$

Student Work Examples

1.



180-85-10=

$a = \frac{\sin 85\ A}{0.249048} = 4$

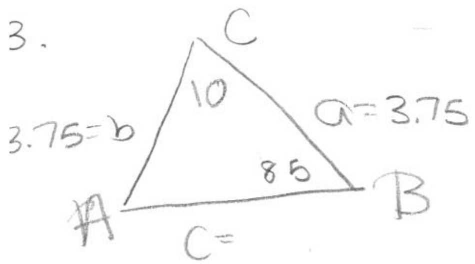
7th Initial Bearing 31°

$\frac{\sin 85 B}{4.00} = \frac{\sin 10 C}{c}$

$c \times \underline{0.249048} = \frac{\sin 10 C}{c}$

$c = \frac{\sin 10 C}{0.249048} = 0.69$

3.



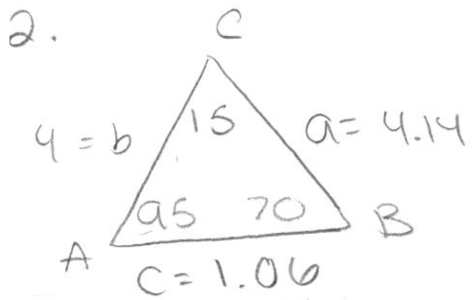
180-85-10=85

$\frac{\sin 85 B}{3.75} = \frac{\sin 10 C}{c}$

$c \times \frac{0.265651}{.265651} = \frac{\sin 10 C}{c}$

$c = \frac{\sin 10 C}{.265651} = .65$

2.



180-70-15=95

$4.14^2 + 4^2 - 2(4)(4.14) \times \cos 15 C$

$16 + 17.13 - 33.12 \times \cos 15$

$33.13 - 31.99 = 1.14$

$\sqrt{1.14} = 1.06$

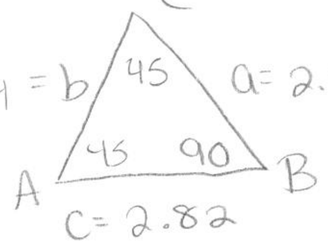
$\frac{\sin 15}{1.06} = \frac{.25}{1.06} = \frac{\sin \angle B}{4.00}$

$\frac{4.00 \times .25}{1.06} = \sin \angle B = .94339$

$\sin^{-1}(.94) = 70.0$

Student Work Examples

1.



$180-90-45=45$

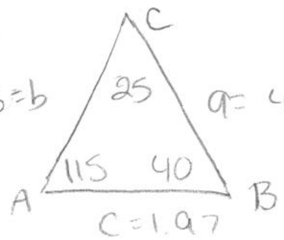
8th Initial Bearing 91°

$$\frac{\sin 90}{4.00} = \frac{\sin 45C}{c}$$

$$cx\,\frac{0.25}{0.25} = \frac{\sin 45C}{c} = 2.82$$

$$a = \frac{\sin 45}{0.25} = 2.82$$

3.



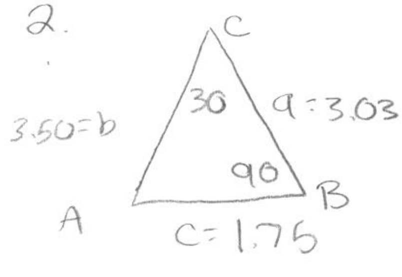
$180-40=140$
$140-25=115$

$$c = \frac{\sin 25C}{0.214262} = 1.97$$

$$a = \frac{\sin 115}{.214262} = 4.22$$

$$\frac{\sin 40}{3.00} = 0.214262$$

2.

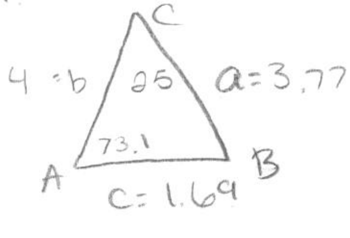


$3.03^2+3.50^2-2(3.03)(3.50)\cos 30$
$9.18+12.25-21.21\cos 30$
$21.43-18.36=3.07$
$\sqrt{3.07}=1.75$

$$\frac{0.5 \times 3.50}{1.75} = 1$$

$$\frac{\sin 30}{1.75} = \frac{0.5}{1.75} = \frac{\sin \angle B}{3.50}$$

4.



$3.77^2+4^2-2(4)(3.77)\cos 25$
$14.21+16-30.16\cos 25$
$30.21-27.33=2.88$
$\sqrt{2.88}=1.69$

$$\frac{\sin 25}{1.69} = \frac{.42}{1.69} = \frac{\sin \angle B}{4.00}$$

$$\frac{.42 \times 4}{1.69} = .99$$

$\sin^{-1}(.99) = 81.89$

PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth

Commander William R. Anderson, Captain, and his officers on the bridge of U.S.S. *Nautilus* as they approached the polar ice pack.

"*Nautilus* 90 North"

Application of Mathematical Functions

Thomas R. Beall
Captain, U. S. Navy (Ret.)

Introduction

On August 3rd, 1958, United States Ship *Nautilus* (SSN – 571), the world's first nuclear powered submarine, crossed the North Pole under the polar ice pack during a voyage from the Pacific Ocean to the Atlantic Ocean. She was the first ship in history to do this. *Nautilus* was able to complete this remarkable feat because her nuclear power plant did not require oxygen to operate on the surface or submerged. Therefore, she did not need to surface periodically to recharge her engines using oxygen as non-nuclear submarines are required to do.

Nautilus was, and still is, a remarkable feat of engineering. Since John P. Holland designed and built the first practical submarine boat (U.S.S. *Holland*, SS – 1) at his Electric Boat Company yard in Groton, Connecticut in 1900, submarine designers had always been stymied by the submarine's inability to remain underwater for extended periods of time. This was because Holland's design involved the use of two forms of engine power. On the surface, the submarine would operate and move using oxygen-breathing internal combustion engines. These engines would also charge batteries that would power the boat when submerged. When the batteries began to lose their charges, the submerged boat would need to come back to the surface to recharge using the internal combustion engines. Submarines, therefore, could only remain underwater for at most a few days.

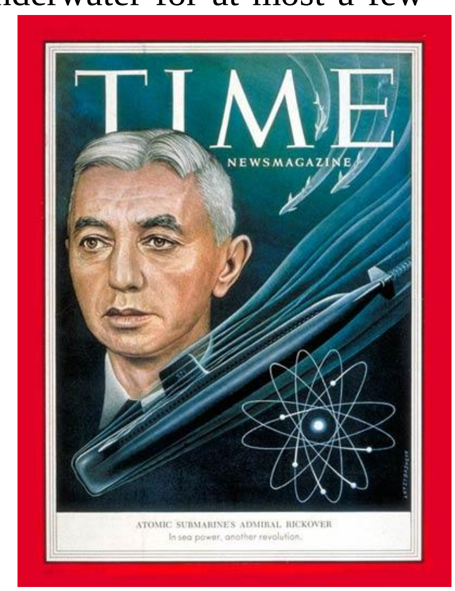


Figure 1: Admiral Rickover on the cover of TIME Magazine

In the late 1940's, naval engineers led by Captain (later Admiral) Hyman Rickover, USN, developed controlled nuclear power plants that were small enough to fit into the hulls of large submarines. A nuclear power plant produces intense heat to create steam that can power a maritime turbine engine. The nuclear power plant can do this without using oxygen. Therefore, provided a nuclear-powered submarine carries enough food and oxygen for the crew's needs, she can remain underwater indefinitely. Shortly after *Nautilus* was commissioned in 1954, she conducted a submerged transit from Groton, Connecticut to Puerto Rico in 90 hours, the longest

time a submarine had ever remained submerged. In the years to come she, and her sister nuclear powered submarines, continued to break records of time spent continuously underwater.

The idea of a submerged transit across the North Pole intrigued not only the U.S. Navy's Chief of Naval Operations, Admiral Arleigh Burke, but also President Dwight D. Eisenhower himself. As the Cold War between the Soviet Union and the United States was taking shape, both nuclear powers perceived the Arctic to be an important battle ground because it represented the shortest distance for bombers or ballistic missiles to transit in an attack. The state that controlled the Arctic, it was believed, controlled a vital avenue of strategic nuclear warfare.

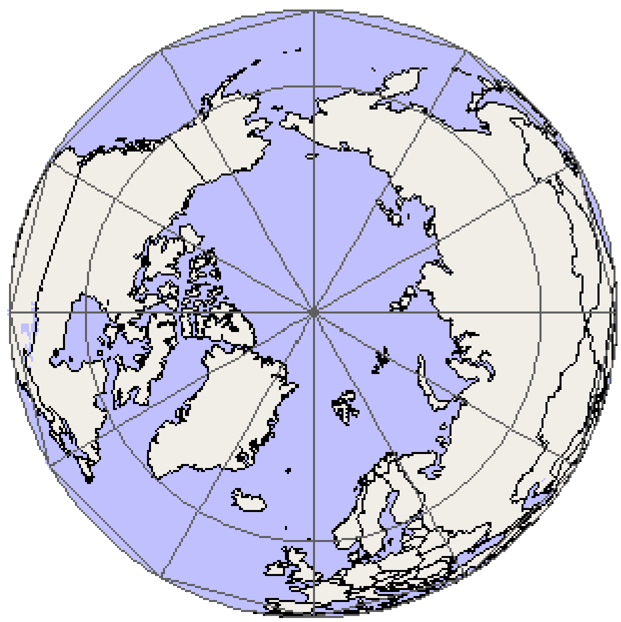


Figure 2: A Polar Projection of the Earth

Nautilus was the ideal submarine to assert United States control of the Arctic. Although not the newest submarine in the American fleet, she had the most experienced nuclear-trained crew and had compiled an impressive list of achievements. Her new Captain, Commander William R. Anderson, USN, had served in submarines in the Pacific in the Second World War and had commanded U.S.S. *Wahoo* (SS-565), one of the new *Tang*-class boats built after the war. An experienced seaman and warfighter, Anderson was an ideal choice to command *Nautilus* for this mission.

Figure 3: U.S.S. *Nautilus* (SSN – 571)

While her nuclear power plant made the submerged polar transit possible, it took an experienced, dedicated crew and improved navigation and underwater sounding techniques to make the voyage a reality. That they succeeded is a tribute to the American Sailor and the United States Navy as instruments not only of warfare but of the advancement of exploration and science. In this unit, we will follow in the footsteps of Commander Anderson and his crew as they bring their knowledge of navigation, nuclear engineering, and seamanship to bear to boldly go where no one had gone before.

Department Approved Pre- and Post-Assessment

Algebra 2 Name________________________

Functions Pre/Post Assessment Date______________

Perform the indicated operation.

1) $h(x) = 3x + 2$
$g(x) = -2x^2 + 2$
Find $(h + g)(x)$

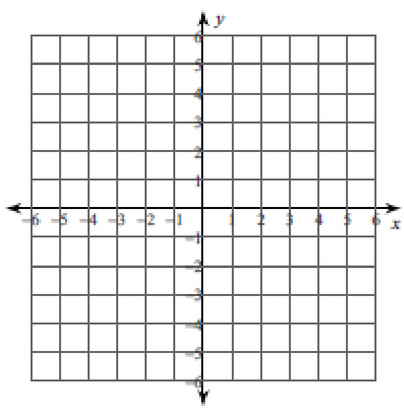
2) $h(a) = 3a - 1$
$g(a) = a^2 - 1$
Find $(h - g)(a)$

3) $h(x) = -3x - 4$
$g(x) = 4x + 5$
Find $(h \cdot g)(x)$

4) $h(x) = x^2 - 1$
$g(x) = 3x - 3$
Find $(h \circ g)(x)$

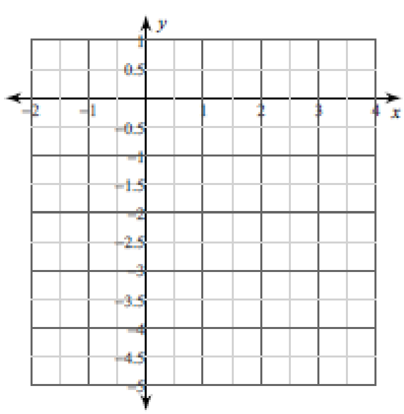
Graph each equation.

5) $y = |x + 3| - 2$



Sketch the graph of each function.

6) $y = (x - 1)^2 - 4$



Department Approved Pre- and Post-Assessment

Algebra 2 Name________________________

Functions Pre/Post Assessment

Date________________

Perform the indicated operation.

1) $h(x) = 3x + 2$
$g(x) = -2x^2 + 2$
Find $(h + g)(x)$

$-2x^2 + 3x + 4$

2) $h(a) = 3a - 1$
$g(a) = a^2 - 1$
Find $(h - g)(a)$

$-a^2 + 3a$

3) $h(x) = -3x - 4$
$g(x) = 4x + 5$
Find $(h \cdot g)(x)$

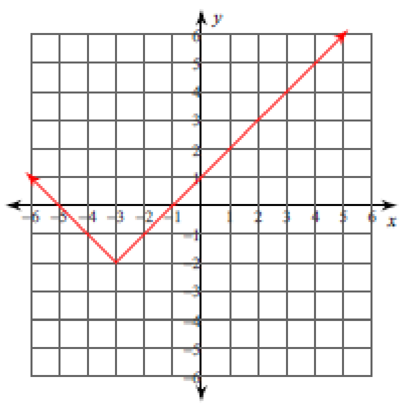
$-12x^2 - 31x - 20$

4) $h(x) = x^2 - 1$
$g(x) = 3x - 3$
Find $(h \circ g)(x)$

$9x^2 - 18x + 8$

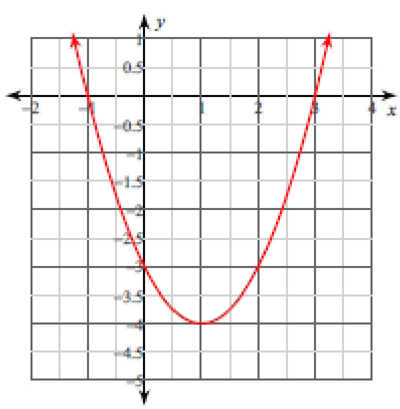
Graph each equation.

5) $y = |x + 3| - 2$



Sketch the graph of each function.

6) $y = (x - 1)^2 - 4$



Unit 1: Functions – The Basics

I. Lesson I: What is a Function?

A. **Mini-lesson 1**: Functions in a Real World Context. The word "function" is used frequently to connect inputs with outcomes. For example, "Success as a major league batter is a function of training and eye-hand coordination," or, "The mileage a car reaches on a full tank of gas is a function of the fuel efficiency of the engine." Each of these sentences, like all such statements, is constructed in the following format:

Outcome	Function Statement	Input
Success as a major league batter...	...is a function of...	...training and eye-hand coordination.
The mileage a car reaches on a full tank of gas...	...is a function of...	...the fuel efficiency of the engine.

Notice that for each input, there is a unique outcome. This is what defines the relationship as a function. We model real-world relationships as functions when **we want one and only one outcome for a given input. In other words, we want to eliminate all uncertainty as to what the outcome will be.**

Let us consider a real-world example in which uncertainty led to great loss of life and property. On January 28th, 1986, the United States Space Shuttle *Challenger* exploded shortly after launch with the loss of seven astronauts and the entire space vehicle. *Challenger* exploded because simple rubber gaskets, designed to stop the flow of explosive gases in her launch rockets failed because the outside temperature at the launch site in Florida was too cold. The shuttle's engineers had no idea how to answer the question, "At what temperature will the gaskets fail causing a Space Shuttle to explode. After the tragedy, the engineers modeled this question with a function and were able to determine the outside air temperature below which a Space Shuttle should not be launched:

$$Launch\ Decision = f(t) = temperature\ below\ which\ gaskets\ fail$$

Once this function was developed, no Space Shuttle was lost due to failing gaskets.

Now that we have identified the inputs and outcomes, we can transform these statements into mathematical functions. In the car mileage example:

E_f = the fuel efficiency of the engine.

$M = f(E_f)$ = the mileage a car reaches on a full tank of gas.

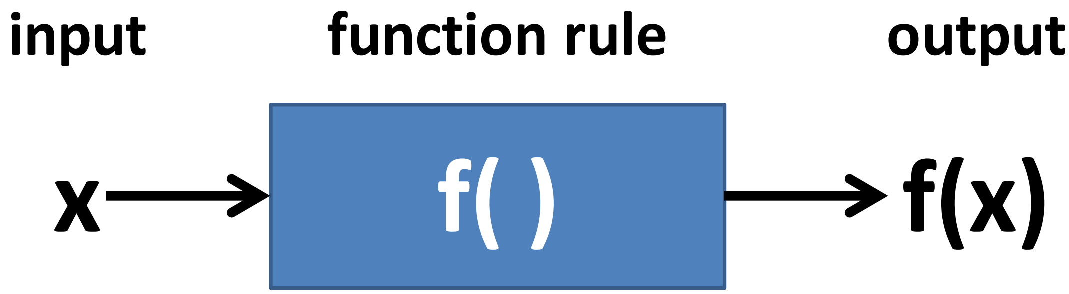
Outcome	Function Statement	Input
The mileage a car reaches on a full tank of gas...	...is a function of...	...the fuel efficiency of the engine.
$M = f(E_f)$	f( )	E_f

Now let's consider U.S.S. *Nautilus*. We want her to be able to dive deep, go fast, stay underwater a long time (endurance), possess great firepower, and be comfortable for her crew. She has a nuclear reactor power plant, a strong submarine hull, compressed air cylinders, carbon dioxide (CO_2) scrubbers, torpedoes, refrigerated storerooms, and a bunk bed for each sailor and officer.

B. **Class Work Exercise 1 (copy the problems and record your answers in your journal).** From the paragraph above, list all of the outcomes on the left and the inputs on the right of the table:

Outcome	**Function Statement**	**Input**	**Function Equation**
To dive deep...	...is a function of...	...a strong submarine hull.	D = f(strong hull)

C. **Mini-lesson 2**.[46] A function is a mathematical relationship that will take a given input, usually a number or an expression; change it by applying a function rule to it; and produce a single, unique outcome. Normally (but not always), the input is identified as x, the function as $f(\,)$, and the outcome as $f(x)$.



The following are specific examples for a function rule x^2:

$x =$	function rule $f(\,)$	$f(x) =$
2	x^2	4
4		16
6		36
8		64

Function rules can be more complicated. For example, we could establish a rule that states $f(x) = x^2$ if x is even and $f(x) = x + 5$ if x is odd.

$x =$	function rule $f(\,)$	$f(x) =$
2	x^2 if x is even $x + 5$ if x is odd	4
3		8
4		16
5		10

Functions can do more than just solve equations. For example, we could write a function rule that states, "For any given integer, find the next largest integer that begins with the same letter as the given integer."

$x =$	function rule $f(\,)$	$f(x) =$

[46] https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-relationships-functions/cc-8th-function-notation/v/what-is-a-function.

2	For any given integer, find the next largest integer that begins with the same letter as the given integer.	3
3		10
4		5
5		14

We have already dealt with many functions. For example, we could write

$$f(x) = \log(\cos(x))$$

$x =$	function rule $f(\,)$	$f(x) =$
25°	$\log(\cos(x))$	5.957276
30°		5.937531
45°		5.849485
60°		5.698970

D. **Class Work Exercise 2 (copy the problems and record your answers in your journal).** Find the outcome of the function $f(x)$ from the given input, x, and the given rule, $f(\,)$.

$x =$	function rule $f(\,)$	$f(x) =$
15	$2x + 3$	
12	$x^2 + 5$	
66°	$\cos(x)$	
5	The name of the planet which is this number from the Sun.	
13	$1/x^2$	
3	$x \times 1$ astronomical unit (au) which is equal to the distance of the Earth from the Sun or 93,000,000 miles	

E. **Mini-lesson 3**.[47] Not all mathematical relationships are functions. For example, take the equation of a circle, which we learned in Geometry is given by:

$$x^2 + y^2 = r^2$$

...where r is the radius of the circle. On the coordinate plane, this circle is depicted as shown in figure 3.

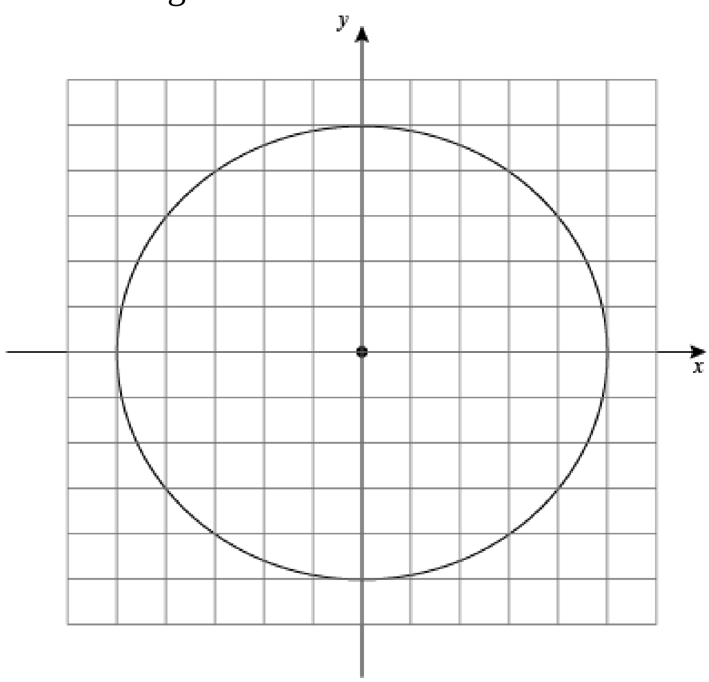


Figure 4: A Circle in the Coordinate Plane

Notice that for any given x we can find two values of y, one positive, one negative. The equation of a circle, therefore, is not a function since it does not produce one unique outcome for each input.

F. **Class Work Exercise 3 (copy the problems and record your answers in your journal).** Determine whether the given relationships are functions. If they are not, state why they are not.

[47] https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-relationships-functions/cc-8th-function-notation/v/what-is-a-function.

1. $y = x^2 + 3$
2. $y^2 = x + 3$
3. $y = \cos^{-1}(x)$
4. $y = \tan^{-1}(x)$
5. $y = \log(\csc(x))$

G. **Mini-lesson 4.**[48] It is easy to confuse the concept of an equation and the concept of a function. Both are relationships between two expressions. There are some equations that are functions and, likewise, some functions that are equations. It is also true, however, that not all equations are functions and not all functions are equations.

Think of two circles, one holding all equations in the universe, the other holding all functions in the universe. If they overlap, then the area of overlap holds all equations that are functions and all functions that are equations.

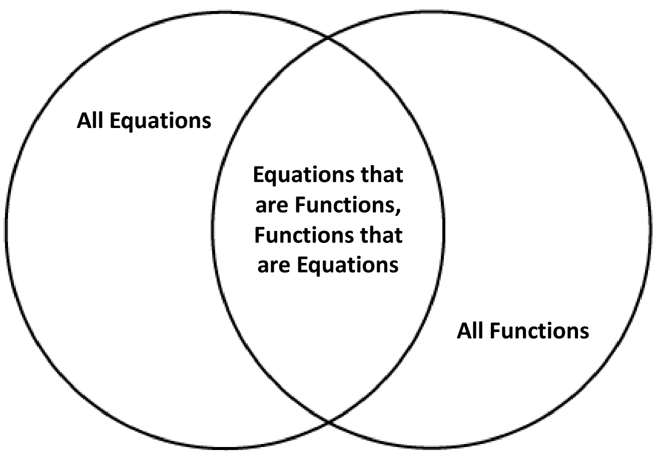


Figure 4: The Relationship Between Equations and Functions.

H. **Class Work Exercise 4 (copy the problems and record your answers in your journal).** Graph the following equations in the coordinate plane and determine if each is a function.

[48] https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-relationships-functions/cc-8th-function-notation/v/difference-between-equations-and-functions.

I. $y = 2x^2 + 3$
II. $y = 4x + 1$
III. $y = \sin(x)$
IV. $F = m \times a$, where m is a constant and a is a variable.
V. $E = m \times c$, where m is a variable and c is a constant.
VI. $Total\ Energy = Kinetic\ Energy + Potential\ Energy$

II. **Lesson II: Functions and the Design, Construction and Voyage of *Nautilus*.**

A. **Mini-lesson 1: Principles of Nuclear Submarine Operation 1:** How does a nuclear power plant work?

Figure 5 depicts the general layout of a U.S. Navy submarine nuclear power propulsion system.

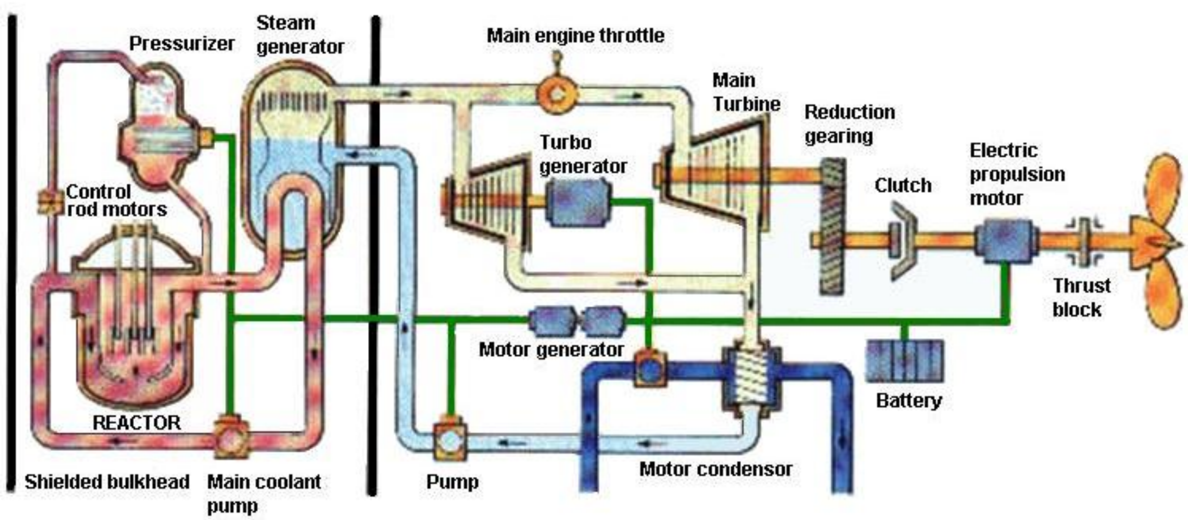


Figure 5: U.S. Navy Nuclear Propulsion System

The steps in the operation of a nuclear propulsion system, are:

1. The reactor produces heat through atomic fission.
2. Coolant water under high pressure in the primary loop, removes heat from the reactor.

3. The hot coolant water under high pressure passes through the steam generator where it transfers heat to water in the generator. That water becomes steam. By transferring heat, the coolant water becomes cooler, returns to the reactor and removes heat again.
4. Steam leaves the steam generator and flows to the main turbine, turning the turbine at high speed.
5. The main turbine turns a shaft, which is connected to the reduction gear, very fast.
6. The reduction gear slows down the rate of turn, allowing the propeller shaft to turn more slowly.
7. The propeller shaft turns the propeller which moves the ship through the water.
8. Steam from the steam generator also turns a turbine generator which produces electricity for the submarine.
9. Exhausted steam from the main turbine and the turbine generator passes through the condenser, turns back into water, and returns to the steam generator.

We could express some of the relationships between the components as functions. For example, we could say, "The heat of the reactor is a function of the amount of fission occurring as a result of the interaction of fast neutrons and radioactive isotopes in the reactor fuel." We could express this relationship as a function,

$$H_r = f(R)$$

...where:

$H_r =$ the heat of the reactor
$R =$ the rate of nuclear fission

B. **Class Work Exercise 1 (copy the problems and record your answers in your journal).** Match the outcomes with the inputs and construct a mathematical expression for each matched outcome and input pair.

1. The amount of steam produced in the steam generator 2. The speed at which the main turbine turns 3. The amount of electricity produced in the turbine generator 4. The velocity at which the submarine moves through the water 5. The amount of water that is pumped into the steam generator to make steam 6. The amount of heat produced in the nuclear reactor 7. The speed at which the propeller turns	a. The speed at which the turbine in the turbine generator turns b. The amount of fission occurring as a result of the interaction of fast neutrons and radioactive isotopes in the reactor fuel c. The rotational speed of the propeller d. The efficiency of the condenser in turning exhaust steam into water e. The reduction in main turbine shaft speed caused by the reduction gear. f. The amount of heat transferred to the steam generator from the coolant water g. The temperature of steam flowing across the main turbine

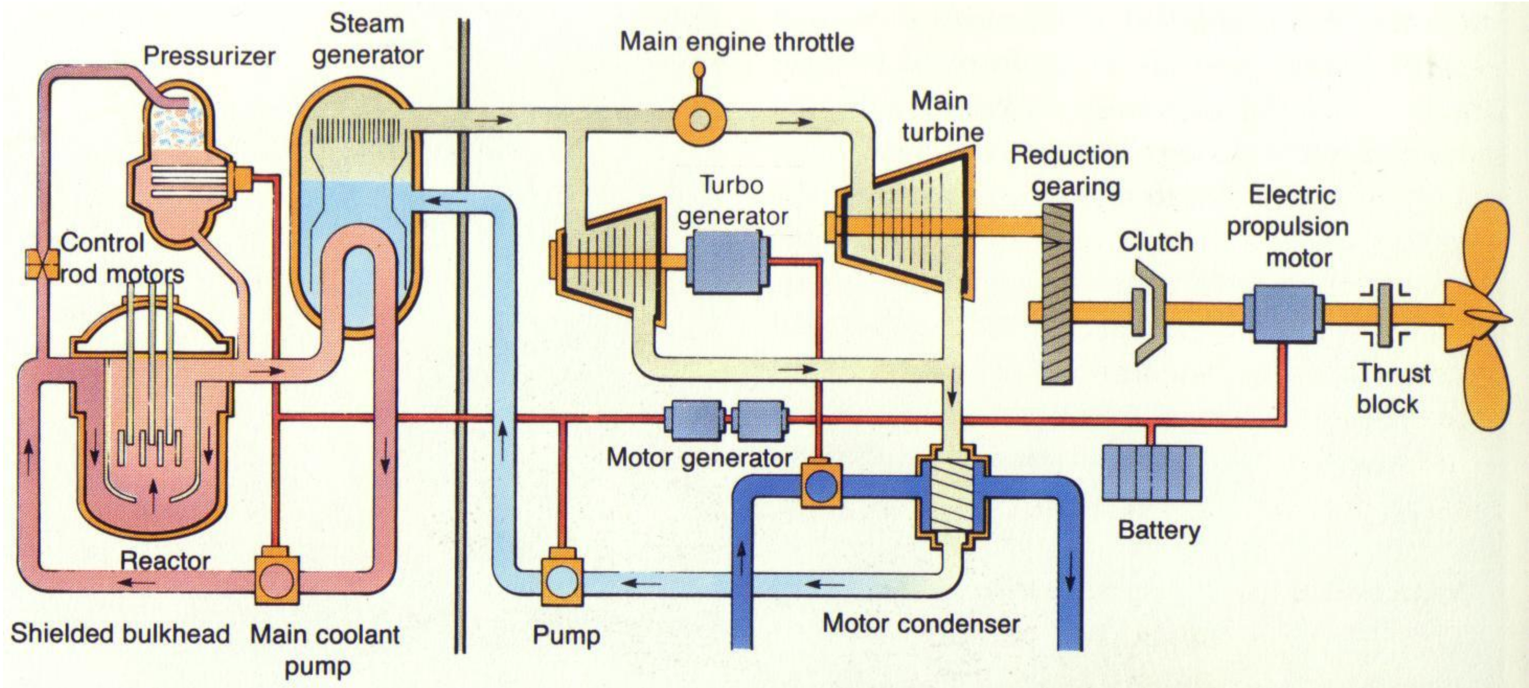
C. **Project Number 1 Part 1:**

Unit 1: Functions – The Basics

1. Reproduce this drawing approximately twice the size.
2. Search online or printed sources and define the following terms (ensure you include information on your sources and attach the definitions to the drawing):

    a. Pressurizer
    b. Steam Generator
    c. Control Rod
    d. Control Rod Motor
    e. Main Coolant Pump
    f. Main Turbine
    g. Reduction Gear
    h. Main Condenser

3. Place completed drawing and definitions in your journal.



D. **Project Number 1 Part 2:** Label your drawing with the function statements and the function equations you developed from Class Work Exercise Nr. 1.

E. **Nuclear Power Plant Schematic Rubric**

	4	**3**	**2**	**1**
Drawing is accurate, portraying all components. Components are properly labeled and in the proper position.				
Drawing is neat and professionally presented with color-coding for each sub-system.				
Function statements are complete and accurate.				

F. **Mini-lesson 2: Principles of Nuclear Submarine Operation 2**[49]**:** Propulsion Plant Efficiency.

1. Like any steam propulsion system, a nuclear propulsion system is simply a series of heat exchangers in which H_2O (water and steam), under high pressure, is the means of transferring heat and energy. The coolant water in the primary loop is kept under very high pressure so that it will **not** turn into steam as it removes heat from the reactor and transfers it to the steam generator. Coolant water leaves the reactor at a temperature of 315° Centigrade (°C).

2. Water in the steam generator is turned into steam using heat transferred from the high pressure water in the primary loop. This steam then moves to the steam turbines that propel the ship and power the electric generators.

3. The efficiency of steam turbine system is determined only by the temperature difference of the steam between the input and output of the turbine. This is called Carnot's Law. Carnot showed that the maximum efficiency a steam turbine is a function of the temperature of the steam:

$$E_f = f(T_H, T_C) = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}$$

...where:

$E_f =$ the maximum efficiency of the steam turbine system. The closer E_f gets to "1", the more efficient the system.
$T_H =$ temperature of the steam at its hottest (normally just before it enters the turbine).
$T_C =$ temperature of the steam at its coldest (normally just before it condenses into water after passing through the turbine.

[49] http://www.mpoweruk.com/steam_turbines.htm.

G. **Class Work Exercise 2 (copy the problems and record your answers in your journal).** The table below portrays the pressure (in pounds per square inch – PSI) above which, water turns into steam at the given temperature. To maintain water in a fluid state, therefore, system pressure must be greater than the indicated pressure.

℃	PSI	℃	PSI	℃	PSI	℃	PSI	℃	PSI	℃	PSI	℃	PSI	℃	PSI
38.7	1	98.7	14	123	32	144	58	158	84	181	150	239	475	270	800
52.2	2	100	14.69	125	34	145	60	158	86	189	175	242	500	272	825
60.8	3	101	15	127	36	146	62	159	88	194	200	245	525	274	850
67.2	4	102	16	129	38	147	64	160	90	200	225	247	550	276	875
72.3	5	104	17	131	40	148	66	161	92	205	250	250	575	278	900
76.7	6	106	18	132	42	149	68	162	94	210	275	252	600	281	950
80.4	7	107	19	134	44	151	70	163	96	214	300	255	625	285	1000
83.8	8	109	20	135	46	152	72	164	98	218	325	257	650		
86.8	9	112	22	137	48	153	74	164	100	222	350	260	675		
89.6	10	114	24	138	50	154	76	166	105	226	375	262	700		
92.1	11	117	26	140	52	155	78	168	110	229	400	264	725		
94.4	12	119	28	141	54	156	80	170	115	233	425	266	750		
96.6	13	121	30	142	56	157	82	172	120	236	450	268	775		

Table 1: Temperatures / Pressures at Which Water Becomes Steam

1. Enter this table into Microsoft EXCEL, create a scatter plot, and fit a curve to it.
2. Determine the equation of the curve.

   $PSI = f(°C) =$ ____

3. Using the equation, determine the minimum pressure at which water must be maintained to keep it from becoming steam at a temperature of 315^{o} C, the temperature of coolant water as it leaves the reactor.
4. Create a scatter plot of the inverse of the data.
5. Determine the equation of the curve.

   $°C = f^{-1}(PSI) =$ ____

6. Use this equation to determine if the answer you found in iii, when substituted for PSI in the inverse function is approximately 315^{o} C.

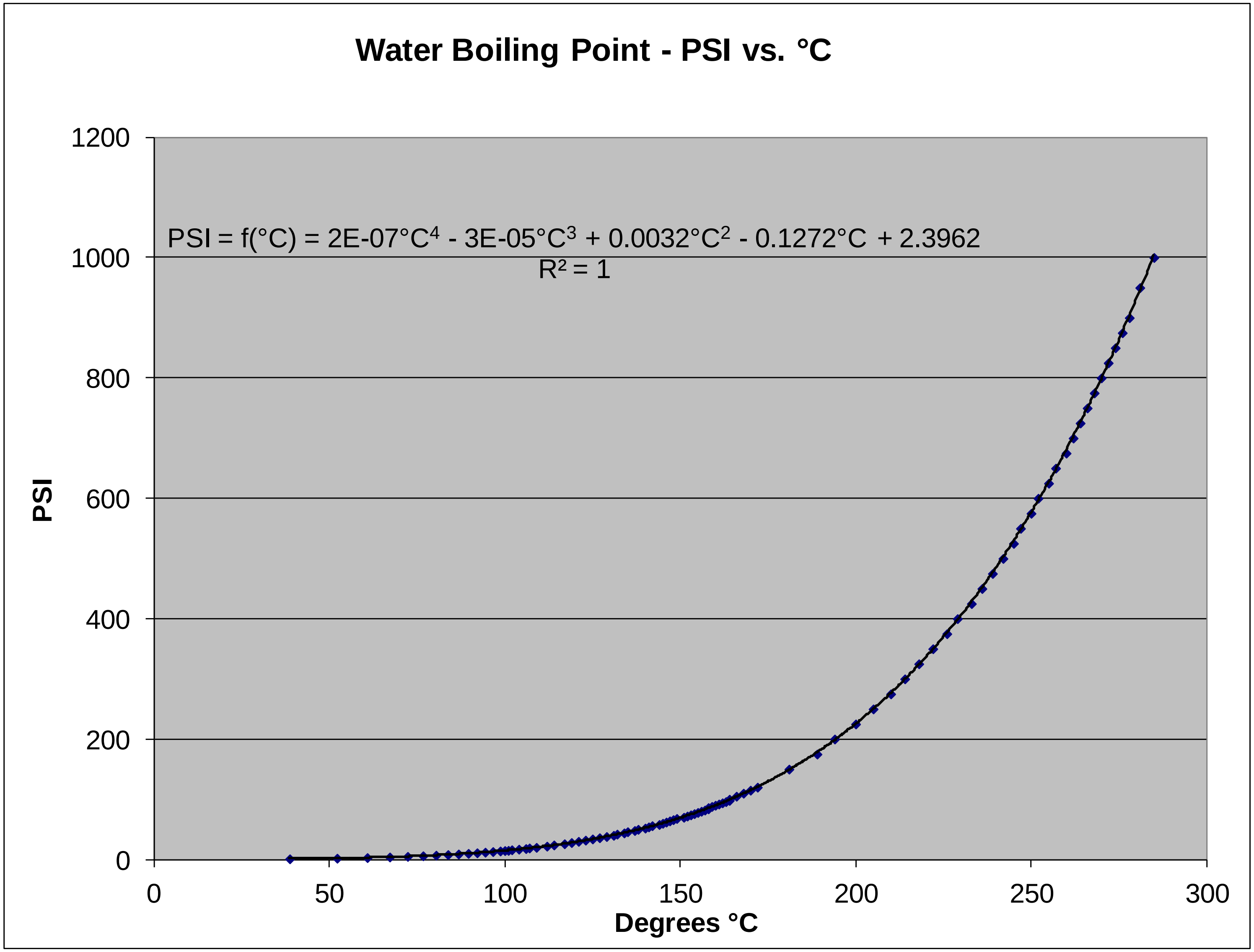


Figure 6: Temperature and Pressure of Water at Boiling Point

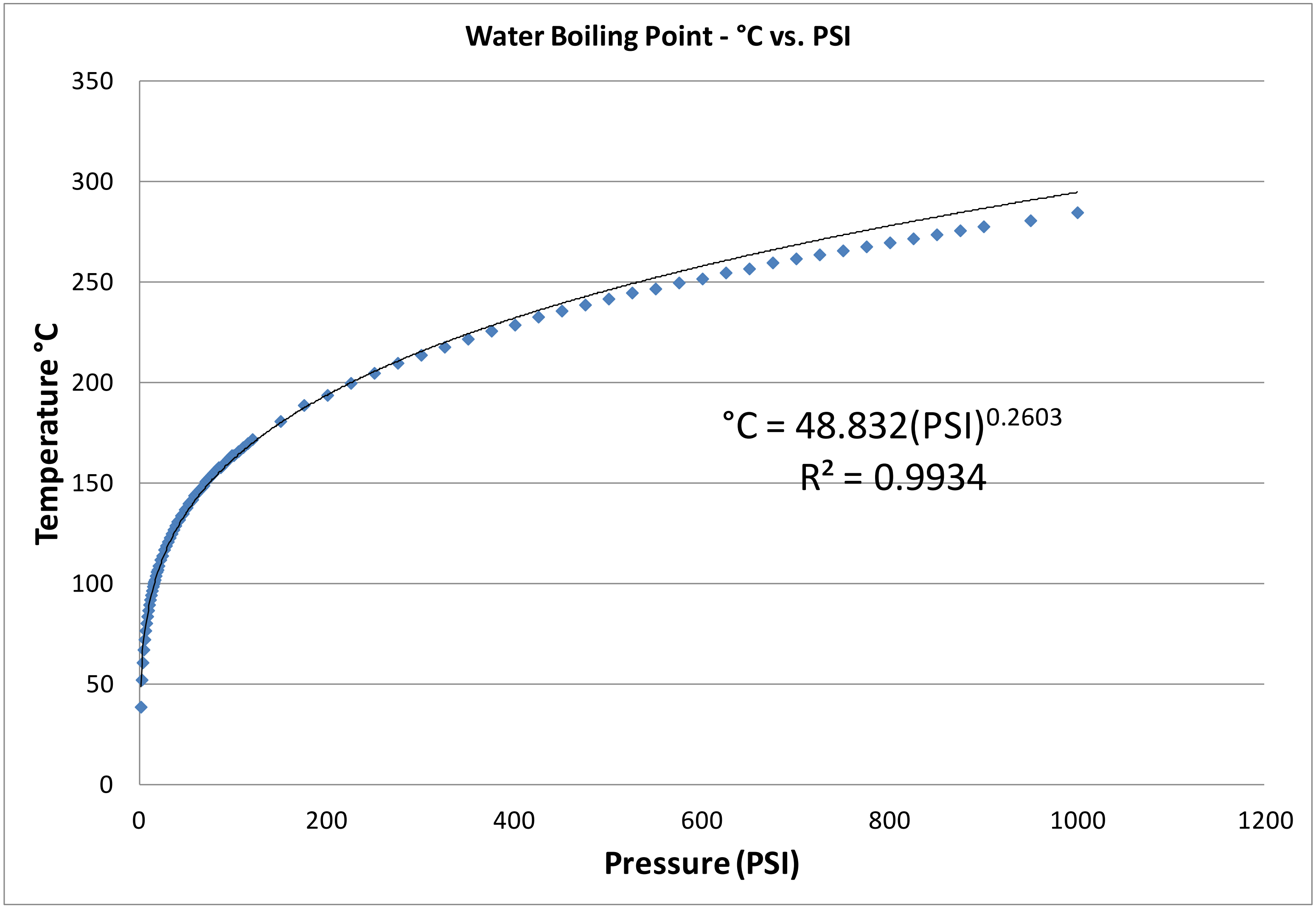


Figure 7: Pressure and Temperature of Water at Boiling Point

G. **Class Work Exercise 3 (copy the problems and record your answers in your journal).** *Nautilus*' nuclear propulsion plant produces steam at temperature of 252º C at the outlet of the steam generator and 95º C just at the outlet of the main turbine before it condenses back into water.

1. What is the pressure (PSI) differential between the outlet of the steam generator and the outlet of the main turbine.
2. Calculate E_f of the propulsion plant.
3. If the piping from the steam generator to the main propulsion turbine is 10 feet in length and the steam loses 1.25º C every 6 inches due to imperfect insulation of the piping, calculate E_f with this temperature loss.
4. How much is E_f reduced as a result of this temperature loss in the piping?

H. **Mini-lesson 3: Finding the Domain and Range of a Function, Evaluating a Function, and Determining Its Type.**[50]

1. Recall from the pressure-temperature problem above, the two functions we derived from our data analysis were:

$$PSI = f(°C) = 2E-07°C^4 - 3E-05°C^3 + 0.0032°C^2 - 0.1272°C + 2.3962$$

and,

$$f^{-1}(PSI) = °C = 48.832(PSI)^{0.2603}$$

2. The input of the first function, °C, is called the ***Domain***, and the outcome, PSI, is called the ***Range***. Formally, for any given function:

   a. **Domain:** The set of all possible inputs.
   b. **Range:** The set of all possible

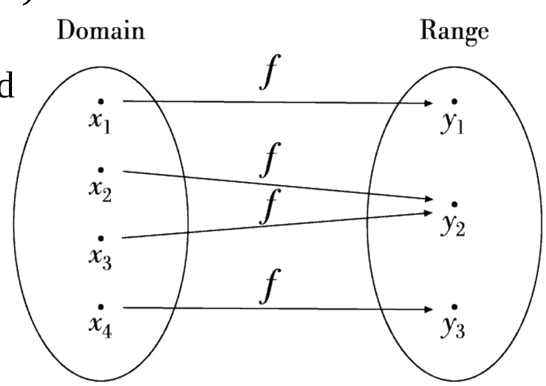


Figure 8: Domain and Range of a Function

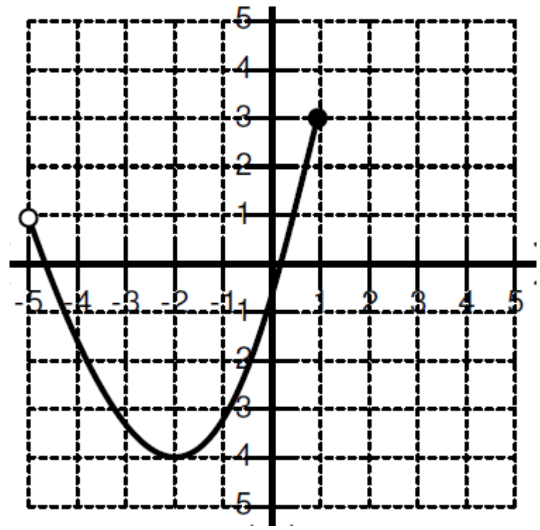
[50] https://www.khanacademy.org/math/algebra2/functions_and_graphs/domain_range/v/domain-of-a-function. The first of several topics.

outcomes.

In our pressure and temperature problem, theoretically, the domain could contain all possible temperatures and the range could contain all possible pressures. Both sets would be infinite. We would express both of them as follows:

$$[-\infty, \infty]$$

Of course, not all functions have infinite domains and ranges. Consider the following graph:



The two end points are the ordered pairs (-5, 1), which is indicated by a white dot, and (1, 3), which is indicated by a black dot. The lowest point on the graph is the point (-2, -4)

The domain of the function, all possible x's, extends from -5 to +1. The white dot means that -5 is not included in the domain. The black dot indicates that +1 is included in the domain. We would write the domain as:

$$(-5, 1]$$

The symbols (and) indicate that the number is not part of the domain. The symbols [and] indicate that the number is part of the domain.

The range of the function, all possible y's, extends from -4 to +3, with both numbers included in the range. We would write the range as:

$$[-4, 3]$$

3. Notice from figure 8 above that each member of the domain maps to only one member of the range. It is possible that some members of the range are not mapped onto by a member of the domain. If, however, each member of the domain maps onto a unique member of the range and all

members of the range have been mapped onto, then the function is said to be **onto** or **surjective**.

If a given member of the range has only one counterpart in the domain then the function is said to be **one-to-one** or **injective.**

The relationship between temperature and pressure in the problem above is:

a. **surjective** because each member of the domain maps to a unique member of the range.
b. **injective** because each member of the range maps to a unique member of the domain.
c. **bijective** or a **bijection** because it is both surjective and injective.

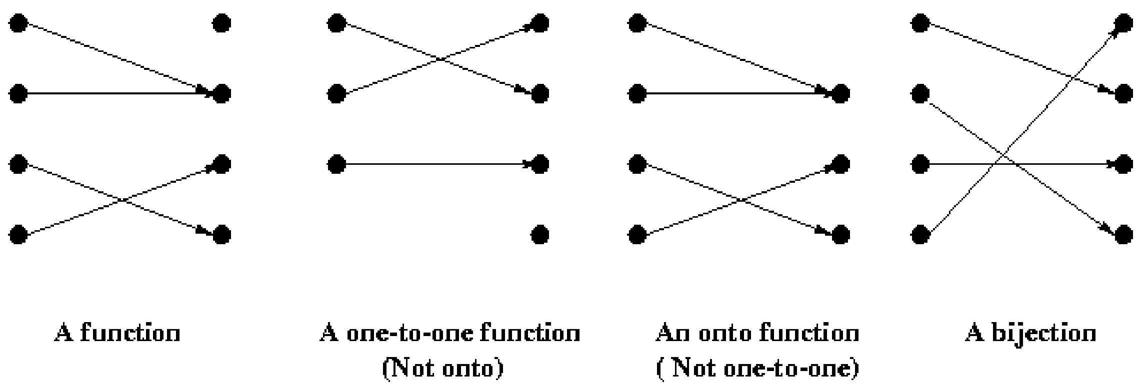


Figure 9: Types of Functions

4. To evaluate a function at a given value, we simply replace the input variable of the function with the given value. For example, to evaluate our temperature-pressure function:

$$PSI = f(°C) = 2E-07°C^4 - 3E-05°C^3 + 0.0032°C^2 - 0.1272°C + 2.3962$$

...at 315°C, we would rewrite it as below and solve:

$$2E-07*315^4 - 3E-05*315^3 + 0.0032*315^2 - 0.1272*315 + 2.3962 = 1311.292\ PSI$$

Unit 1: Project Number 2 – U.S.S. *Nautilus* Voyage Plan

I. **Project Number 2:**

1. **Background.** Every U.S. Navy ship is required to submit a voyage plan to her operational commander prior to embarking on a voyage. Such a plan normally includes positions (latitude and longitude) of various points along the ship's route, distances between points, courses to be steered, and speeds required to get to each point on schedule.

   Nautilus' chief quartermaster has begun this work for the voyage across the North Pole under the Polar Ice Cap by establishing the ship's track and the dates at which the ship must be at the most important points along the track to complete the voyage on schedule. As you can see from the table, *Nautilus* will cross the North Pole at noon local time on August 3rd.

Point	**Date (12:00 local on the given day)**	**Latitude**	**Longitude**
1	27 July	52º 30' N	175º 00' W
2		56º 30' N	173º 00' W
3		64º 00' N	172º 00' W
4		65º 00' N	168º 00' W
5		72º 30' N	168º 00' W
6	01 August	72º 15' N	155º 00' W
7	03 August	90º 00' N	Undefined
8	05 August	79º 00' N	05º 00' E

Table 2: U.S.S. *Nautilus*' Voyage Track

2. **Tasks.** In this project, you will complete the following tasks:

   a. **Task 1:** Set up a plotting sheet to represent the North Polar region.
   b. **Task 2:** Layout *Nautilus*' track by correctly plotting all point in table 2 and connecting the points.
   c. **Task 3:** Measure the distances (in nautical miles) between points.
   d. **Task 4:** Calculate the speeds (in nautical miles per hour) necessary to reach each point on schedule.

e. **Task 5:** Complete the attached voyage plan memorandum to the Chief of Naval Operations.

3. **Setting up the plotting sheet.** You will create a polar projection of the North Polar Region centered on 90° 00’ N. A polar projection is an azimuthal projection drawn to show Arctic and Antarctic areas. It is based on a plane perpendicular to the Earth’s axis in contact with the North or South Pole. Parallels of latitude are concentric circles, while meridians are radiating straight lines.

   The projection will appear as in figure 10.

4. **Completing the Voyage Plan.** Once your plot is complete, fill out the voyage plan memorandum and turn both in on January 5th as your completed project.

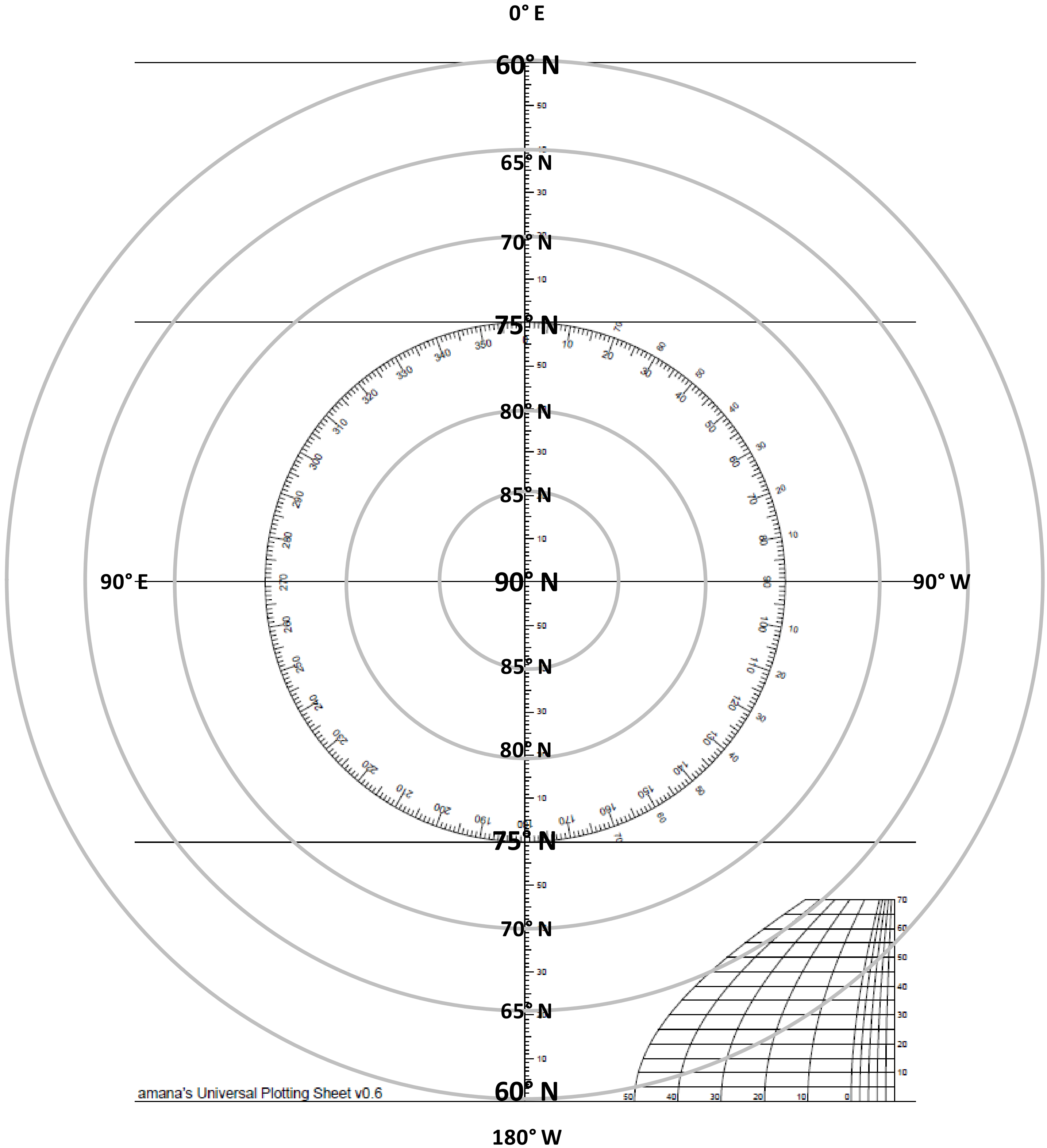


Figure 10: Polar Projection on Plotting Sheet

Name: ______________________________

Department of the Navy
U.S.S. *Nautilus* SS(N) – 571
Fleet Post Office New York

05 JAN 1958

From: Commanding Officer

To: CNO Washington, DC
CINCLANTFLT Norfolk, VA
COMSUBLANT New London, CT

Subj: U.S.S. *Nautilus* SS(N) – 571 Voyage Plan

Ref: (a) CNO Operation Order

Encl: (1) Track Chart

1. Per reference (a), the following voyage plan is submitted.

Point	**Date (12:00 local on the given day)**	**Latitude**	**Longitude**	**Distance in NM**	**Speed in Knots (NM / hr.)**
1	27 July	52º 30’ N	175º 00’ W	N/A	N/A
2		56º 30’ N	173º 00’ W		
3		64º 00’ N	172º 00’ W		
4		65º 00’ N	168º 00’ W		
5		72º 30’ N	168º 00’ W		
6	01 August	72º 15’ N	155º 00’ W		
7	03 August	90º 00’ N	Undefined		
8	05 August	79º 00’ N	05º 00’ E		

J. **Project Number 2 Solutions.**

Point	Date (12:00 local on the given day)	Latitude	Longitude	Distance in NM	Speed in Knots (NM / hr.)
1	27 July	52º 30’ N	175º 00’ W	N/A	N/A
2		56º 30’ N	173º 00’ W	250.05	12.19
3		64º 00’ N	172º 00’ W	451.30	12.19
4		65º 00’ N	168º 00’ W	119.55	12.19
5		72º 30’ N	168º 00’ W	450.32	12.19
6	01 August	72º 15’ N	155º 00’ W	236.34	12.19
7	03 August	90º 00’ N	Undefined	1065.90	22.21
8	05 August	79º 00’ N	05º 00’ E	660.37	13.76

Unit 1: Project Number 2 - U.S.S. *Nautilus* Voyage Plan Reflection

Name: ________________________________

1. Which part of the project proved most difficult (circle one):

   a. Plotting the track.
   b. Determining the distance of each leg of the tack.
   c. Calculating the speed necessary to complete each leg on time.

2. How did you overcome this challenge?

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3. Using what you have learned in Marine Science, what environmental challenges will *Nautilus* face as she proceeds along the track you have laid out for her?

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Unit 1: Project Number 2 - U.S.S. *Nautilus* Voyage Plan
Reflection

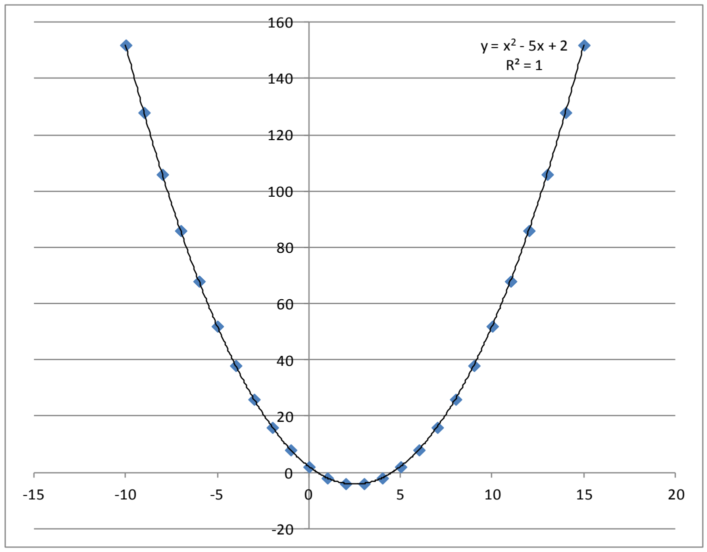
4. What environmental factors will you have to consider as *Nautilus* proceeds on her voyage to ensure you complete each leg on schedule?

Unit 1: Project Number 3 – Graphing and Evaluating Functions

Name: ______________________________

K. **Project Number 3.** Create a graph each of the following functions, determine each function's domain and range, circle the correct type of function (surjective, injective, or bijective), and evaluate each at the given input values.

1. $f(x) = x^2 - 5x + 2$

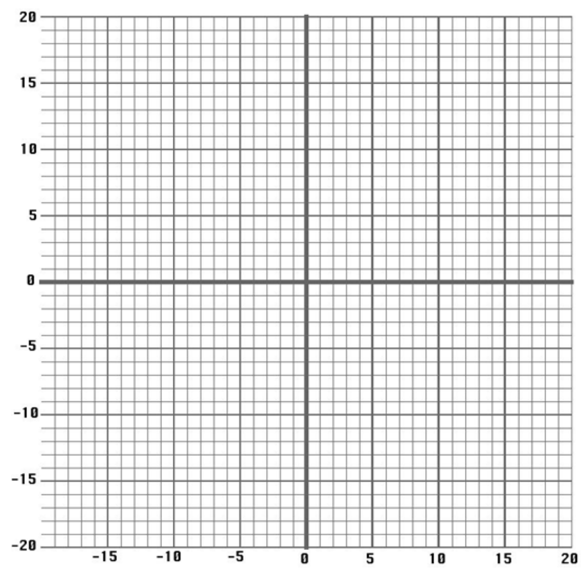


Domain: $(-\infty, \infty)$
Range: $[-4, \infty)$

surjective / injective / bijective

$f(-1) = 8$
$f(-2) = 16$
$f(0) = 2$
$f(5) = 2$

2. $f(x) = x^2 + 4x - 1$



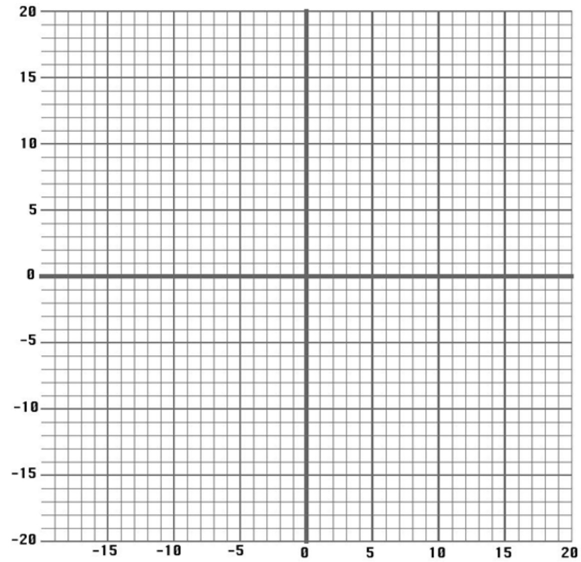
Domain: ______________
Range: ______________

surjective / injective / bijective

$f(-4) =$ ______________
$f(8) =$ ______________
$f(-1) =$ ______________
$f(1) =$ ______________

Unit 1: Project Number 3 – Graphing and Evaluating Functions

3. $f(x) = 3x^2 - 5x$

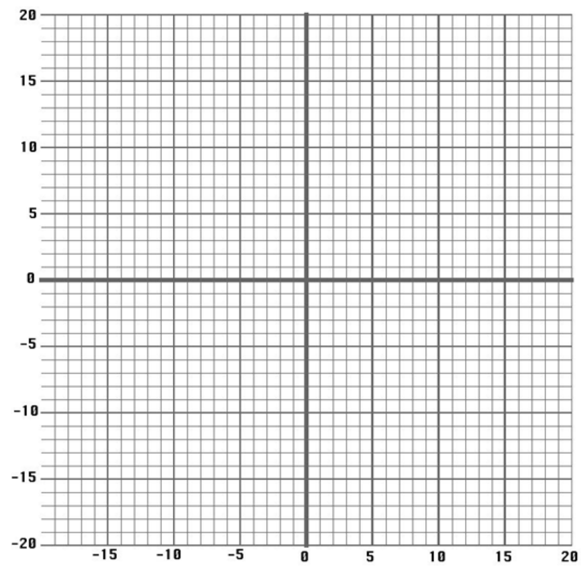


Domain: ________________
Range: ________________

surjective / injective / bijective

$f(2) =$ _______________
$f(-8) =$ _______________
$f(7) =$ _______________
$f(-1) =$ _______________

4. $f(x) = -7x + 4$

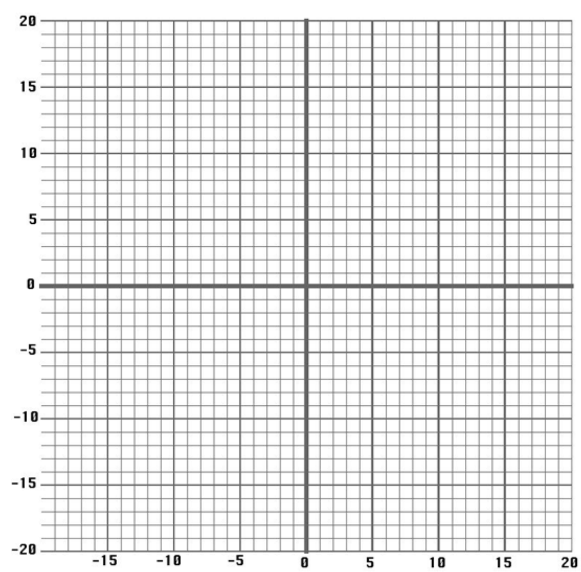


Domain: ________________
Range: ________________

surjective / injective / bijective

$f(-5) =$ _______________
$f(2) =$ _______________
$f(-1) =$ _______________
$f(0) =$ _______________

5. $f(x) = 3x^2 - 1$



Domain: ________________
Range: ________________

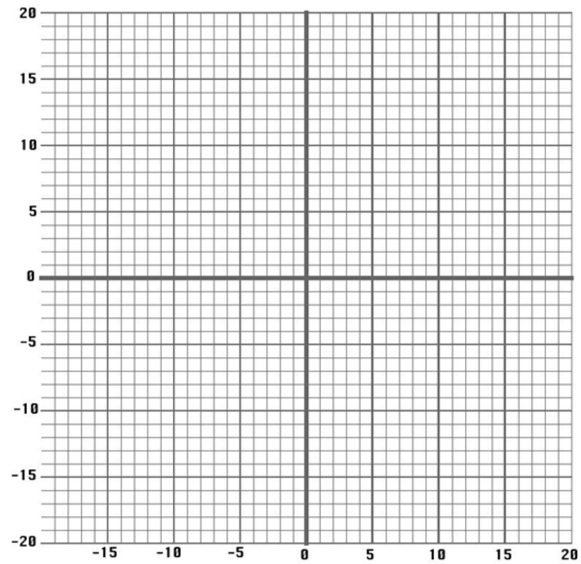
surjective / injective / bijective

$f(9) =$ _______________
$f(-2) =$ _______________

$f(1) =$ ______________

$f(0) =$ ______________

6. $f(x) = 2x^2 + 2$



Domain: _______________

Range: _______________

surjective / injective / bijective

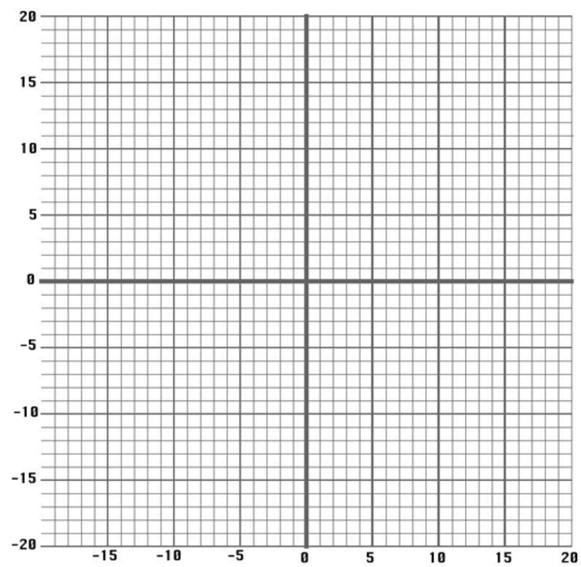
$f(-3) =$ ______________

$f(6) =$ ______________

$f(-1) =$ ______________

$f(4) =$ ______________

7. $f(x) = \sin x$



Domain: _______________

Range: _______________

surjective / injective / bijective

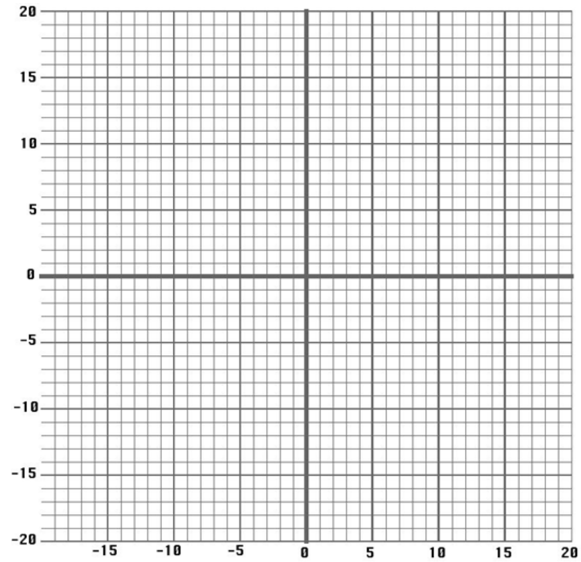
$f(0°) =$ ______________

$f(45°) =$ ______________

$f(90°) =$ ______________

$f(135°) =$ ___________

8. $f(x) = log_{10}x$



Domain: _______________

Range: _______________

surjective / injective / bijective

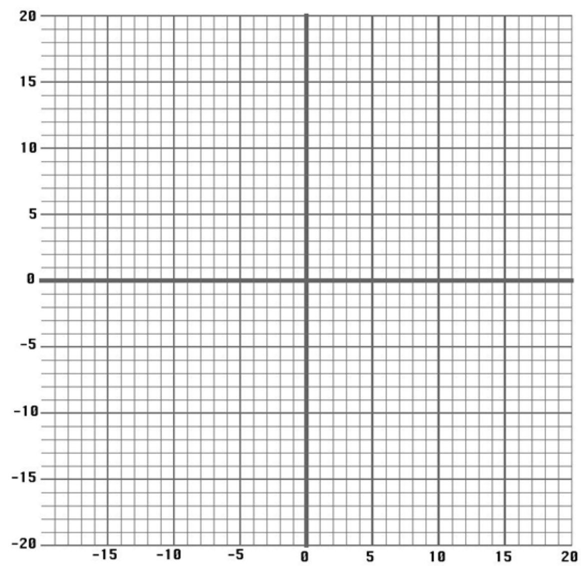
$f(1) =$ ______________

$f(10) =$ ______________

$f(100) =$ ______________

$f(250) =$ ______________

9. $\rho = f(v) = mv$



$\rho = f(v) =$ the momentum possessed by *Nautilus* at a given velocity in $kg - \frac{m}{s}$.

$m = 3.08 \times 10^6 kg.$ which is the mass of *Nautilus*

$v =$ the velocity of the *Nautilus* in meters per second.

Domain: _______________

Range: _______________

surjective / injective / bijective

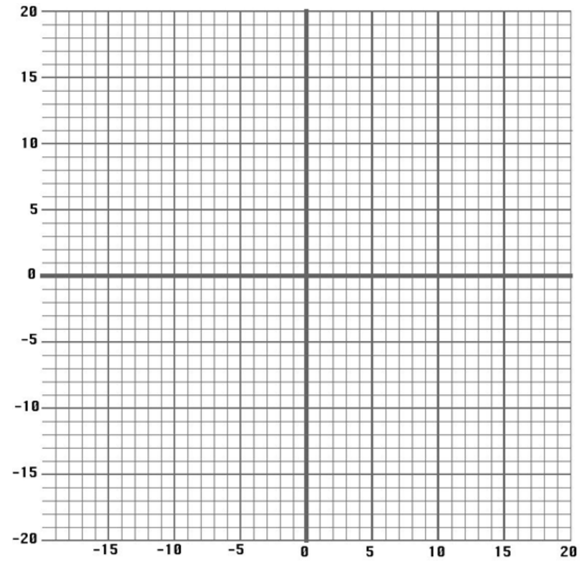
$f(15) =$ ______________

$f(8) =$ ______________

$f(12) =$ ______________

$f(5) =$ ______________

10. $F = f(v) = mv^2$



$F = f(v) =$ the force necessary to move *Nautilus* at a given velocity in Newtons (N), $\left(kg - \left(\frac{m}{s}\right)^2\right)$.

$m = 3.08 \times 10^6 kg$. which is the mass of *Nautilus*.

$v =$ the velocity of the *Nautilus* in meters per second.

Domain: ________________

Range: ________________

surjective / injective / bijective

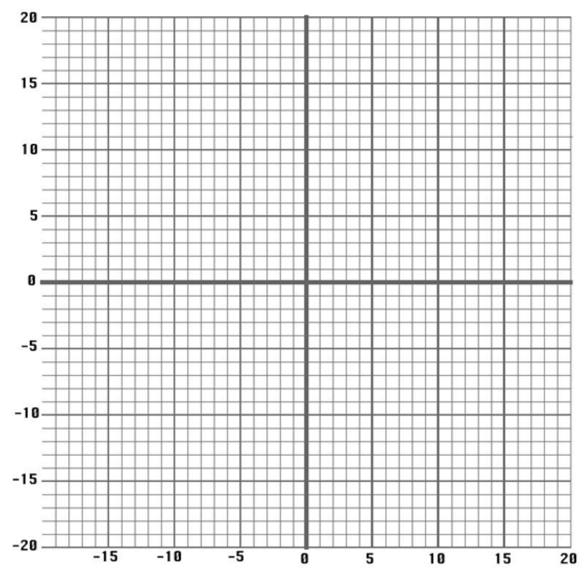
$f(15) =$ _______________

$f(8) =$ _______________

$f(12) =$ _______________

$f(5) =$ _______________

11. $SHP = g(MW) = 1341.2MW - 8.232$



$SHP = g(MW) =$ shaft horsepower of a submarine.

$MW =$ megawatts of power produced by a naval nuclear reactor.[51]

Domain: ________________

Range: ________________

surjective / injective / bijective

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[51] http://mragheb.com/Nuclear_naval_propulsion.pdf.

$f(26.1) =$ ______________
$f(104.4) =$ ______________
$f(29.8) =$ ______________
$f(11.2) =$ ______________

12. The maximum theoretical velocity *Nautilus* can achieve when submerged is a function of her hull diameter and the amount of power her reactor produces:

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}}$$

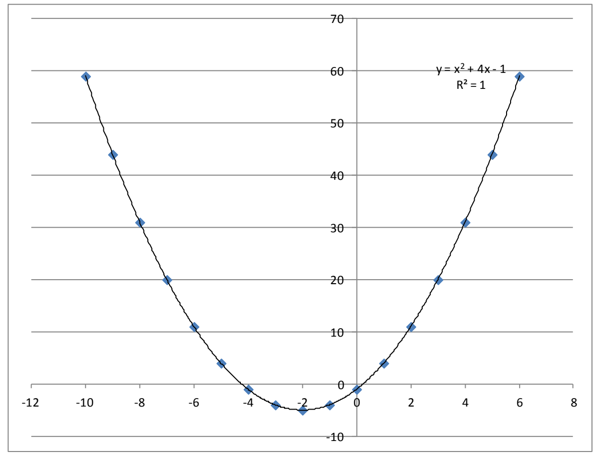
$v =$ maximum theoretical velocity in meters per second.
$\rho = 1027 \frac{kg}{m^3}$: the density of saltwater.
$P =$ power output of the reactor in Watts.
$D = 8.23\ meters$: the hull diameter of *Nautilus*.

Since *P* is the only variable, *v* is a function of *P*.[52]

a. Calculate and graph $v = f(P)$ for all values of P through $P = 45\ MW$. **Hint: the equation above produces an answer in meters per second. Your answers should be in knots (nm/hr.).**
b. Determine the Domain and Range of $v = f(P)$.
c. Is $v = f(P)$ surjective, injective, or bijective?
d. What is the maximum reactor power that will be required during your voyage under the Polar Ice Cap (from your voyage plan project)?
e. Create a graph and find the equation of the function $P = f^{-1}(v)$. Determine the reactor power in MW necessary for each leg of your voyage plan.

52 http://mragheb.com/NPRE%20402%20ME%20405%20Nuclear%20Power%20Engineering/Nuclear%20Marine%20Propulsion.pdf.

Unit 1: Project Number 3 – Graphing and Evaluating Functions (Solutions)



2. $f(x) = x^2 + 4x - 1$

Domain: $(-\infty, \infty)$

Range: $[-3, \infty)$

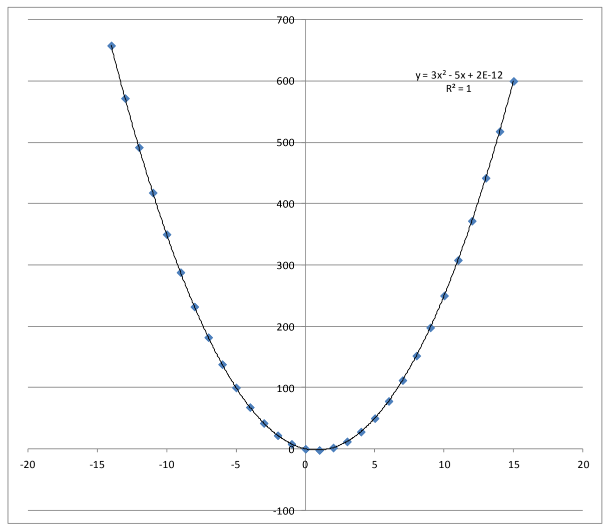
surjective / injective / bijective

$f(-4) = -1$

$f(8) = 95$

$f(-1) = -4$

$f(1) = 4$



3. $f(x) = 3x^2 - 5x$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

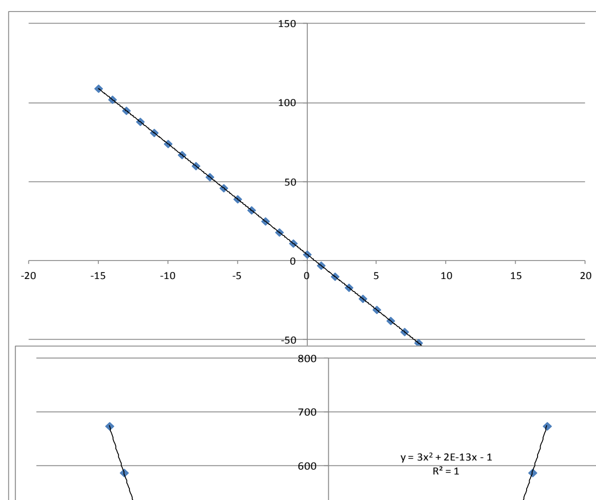
surjective / injective / bijective

$f(2) = 2$

$f(-8) = 232$

$f(7) = 112$

$f(-1) = 8$



4. $f(x) = -7x + 4$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

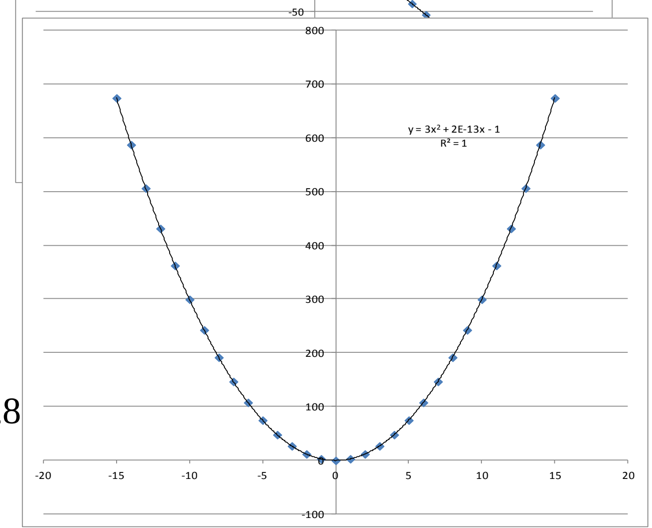
surjective / injective / bijective

$f(-5) = 39$

$f(2) = -10$

$f(-1) = 11$

$f(0) = 4$



5. $f(x) = 3x^2 - 1$

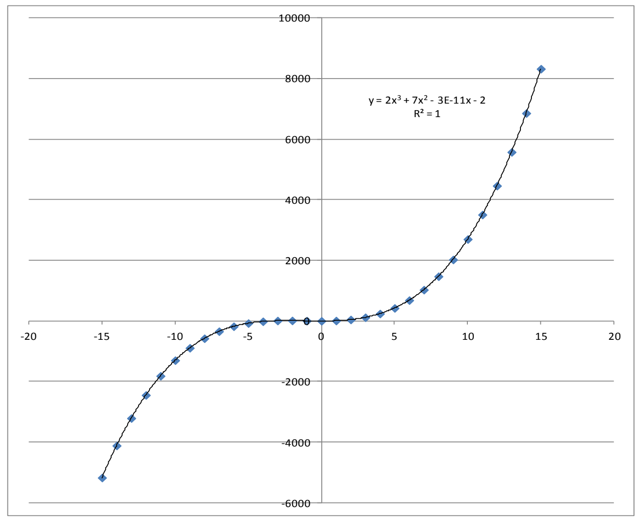
Unit 1: Project Number 3 – Graphing and Evaluating Functions (Solutions)

Domain: $(-\infty, \infty)$
Range: $[-1, \infty)$

surjective / injective / bijective

$f(9) = 242$
$f(-2) = 11$
$f(1) = 2$
$f(0) = -1$

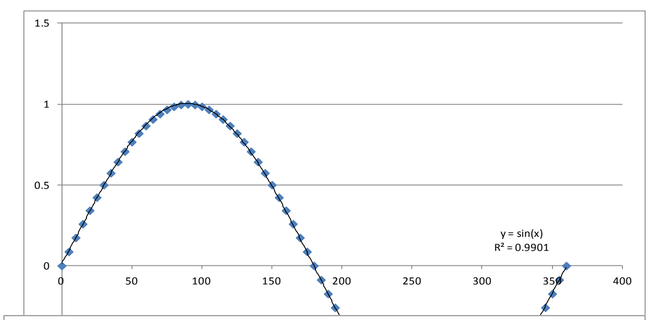
6. $f(x) = 2x^3 + 7x^2 - 2$
   Domain: $(-\infty, \infty)$
   Range: $(-\infty, \infty)$



surjective / injective / bijective

$f(-3) = 7$
$f(6) = 682$
$f(-1) = 3$
$f(4) = 238$

7. $f(x) = \sin x$

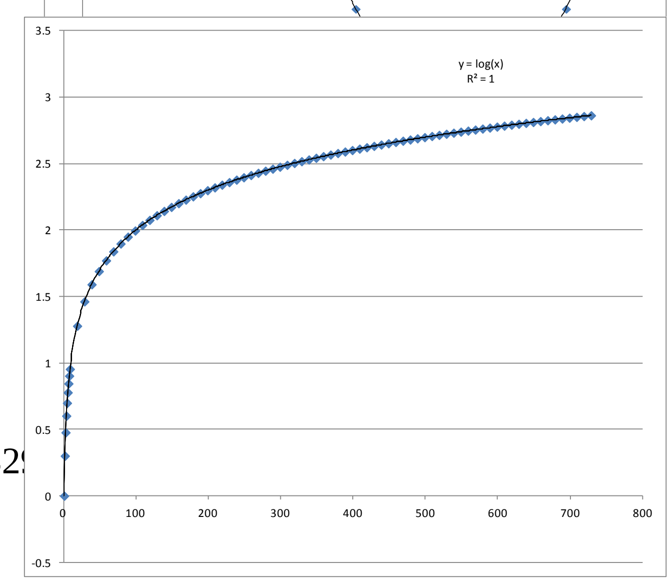


Domain: $[0°, 360°]$
Range: $[-1, 1]$

surjective / injective / bijective

$f(0°) = 0$
$f(45°) = 0.707106781$
$f(90°) = 1$
$f(135°) = 0.707106781$

8. $f(x) = log_{10}x$



Domain: $(0, \infty)$
Range: $[0, \infty)$

surjective / injective / bijective

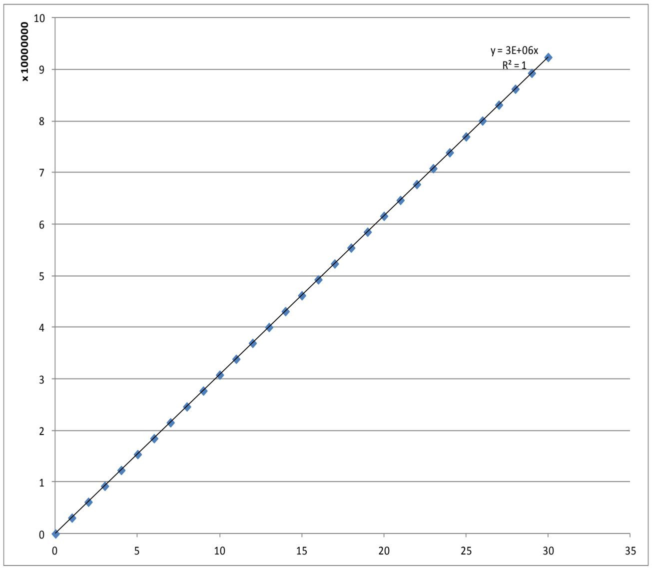
$f(1) = 0$
$f(10) = 1$
$f(100) = 2$
$f(1000) = 3$

9. $\rho = f(v) = mv$

$\rho = f(v) =$ the momentum possessed by *Nautilus* at a given velocity in $kg - \frac{m}{s}$.

$m = 3.08 \times 10^6 kg$. which is the mass of *Nautilus*

$v =$ the velocity of the *Nautilus* in meters per second.



Domain: $[0, \infty)$
Range: $[0, \infty)$

surjective / injective / bijective

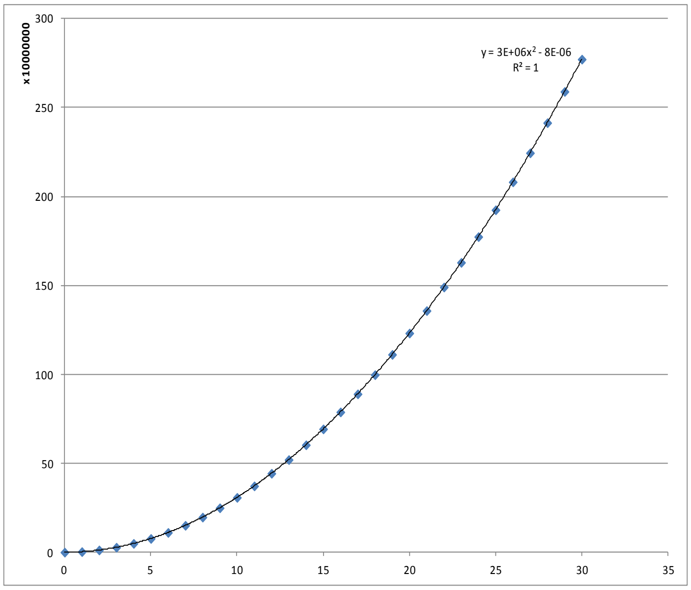
$f(15) = \quad 46200000\ kg - \frac{m}{s}$
$f(8) = \quad 24640000\ kg - \frac{m}{s}$
$f(12) = \quad 36960000\ kg - \frac{m}{s}$
$f(5) = \quad 15400000\ kg - \frac{m}{s}$

10. $F = f(v) = mv^2$

$F = f(v) =$ the force necessary to move *Nautilus* at a given velocity in Newtons (N), $(kg - \left(\frac{m}{s}\right)^2)$.

$m = 3.08 \times 10^6 kg$. which is the mass of *Nautilus*.

$v =$ the velocity of the *Nautilus* in meters per second.



Domain: $[0, \infty)$

Range: $[0, \infty)$

surjective / injective / bijective

$f(15) =$ 693000000 N

$f(8) =$ 197120000 N

$f(12) =$ 443520000 N

$f(5) =$ 77000000 N

11. $SHP = g(MW) = 1341 \times MW$

$HP = g(MW) =$ shaft horsepower of a submarine.
$MW =$ megawatts of power produced by a naval nuclear reactor.[53]

Domain: $[0, \infty)$
Range: $[0, \infty)$

surjective / injective / bijective

$f(26.1) =$ 35000 SHP
$f(104.4) =$ 140000 SHP
$f(29.8) =$ 40000 SHP
$f(11.2) =$ 15000 SHP

12. The maximum theoretical velocity *Nautilus* can achieve when submerged is a function of her hull diameter and the amount of power her reactor produces:

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}}$$

$v =$ maximum theoretical velocity in meters per second.
$\rho = 1027 \frac{kg}{m^3}$: the density of saltwater.
$P =$ power output of the reactor in Watts ($1MW = 10^6\ W$).
$D = 8.23\ meters$: the hull diameter of *Nautilus*.

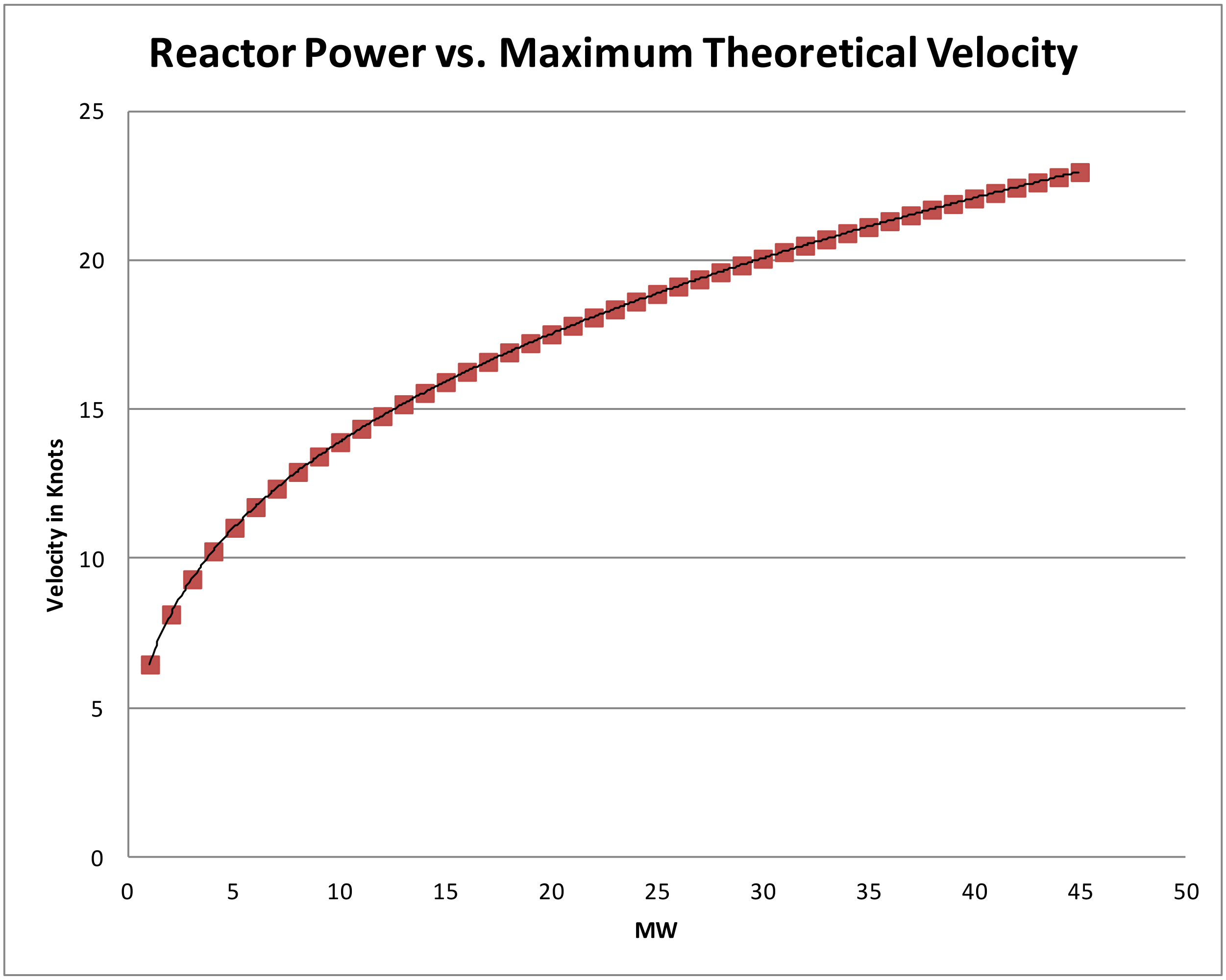
Since P is the only variable, v is a function of P.[54]

[53] http://mragheb.com/Nuclear_naval_propulsion.pdf.
[54] http://mragheb.com/NPRE%20402%20ME%20405%20Nuclear%20Power%20Engineering/Nuclear%20Marine%20Propulsion.pdf.

Unit 1: Project Number 3 – Graphing and Evaluating Functions (Solutions)

a. Calculate and graph $v = f(P)$ for all values of P through $P = 45\,MW$. **Hint: the equation above produces an answer in meters per second. Your answers should be in knots (nm/hr.).**

MW	Watts	m/s	km/s	knots
1	1000000	3.320958192	11.95545	6.455426
2	2000000	4.184145132	15.06292	8.133327
3	3000000	4.789650526	17.24274	9.310336
4	4000000	5.271692528	18.97809	10.24735
5	5000000	5.678758629	20.44353	11.03862
6	6000000	6.034581519	21.72449	11.73029
7	7000000	6.352764483	22.86995	12.34879
8	8000000	6.641916385	23.9109	12.91085
9	9000000	6.907871413	24.86834	13.42783
10	10000000	7.154787534	25.75724	13.90779
11	11000000	7.385744902	26.58868	14.35674
12	12000000	7.603096284	27.37115	14.77924
13	13000000	7.808684194	28.11126	15.17887
14	14000000	8.003981697	28.81433	15.5585
15	15000000	8.190187193	29.48467	15.92045
16	16000000	8.368290265	30.12584	16.26665
17	17000000	8.539118664	30.74083	16.59872
18	18000000	8.703372603	31.33214	16.918
19	19000000	8.861650316	31.90194	17.22567
20	20000000	9.014467422	32.45208	17.52272
21	21000000	9.162271846	32.98418	17.81003
22	22000000	9.305455471	33.49964	18.08836
23	23000000	9.444363345	33.99971	18.35837
24	24000000	9.579301052	34.48548	18.62067
25	25000000	9.710540663	34.95795	18.87578
26	26000000	9.838325588	35.41797	19.12418
27	27000000	9.962874577	35.86635	19.36628
28	28000000	10.08438502	36.30379	19.60248
29	29000000	10.20303573	36.73093	19.83311
30	30000000	10.31898925	37.14836	20.05851
31	31000000	10.43239381	37.55662	20.27895
32	32000000	10.54338506	37.95619	20.4947
33	33000000	10.65208741	38.34751	20.706
34	34000000	10.75861535	38.73102	20.91308
35	35000000	10.86307446	39.10707	21.11613
36	36000000	10.96556235	39.47602	21.31535
37	37000000	11.06616946	39.83821	21.51091
38	38000000	11.16497977	40.19393	21.70298
39	39000000	11.26207142	40.54346	21.89172
40	40000000	11.35751726	40.88706	22.07725
41	41000000	11.45138529	41.22499	22.25971
42	42000000	11.54373916	41.55746	22.43923
43	43000000	11.63463849	41.8847	22.61593
44	44000000	11.72413923	42.2069	22.7899
45	45000000	11.81229396	42.52426	22.96126



b. Determine the Domain and Range of $v = f(P)$.

The domain and range of the function are $[0, \infty)$.

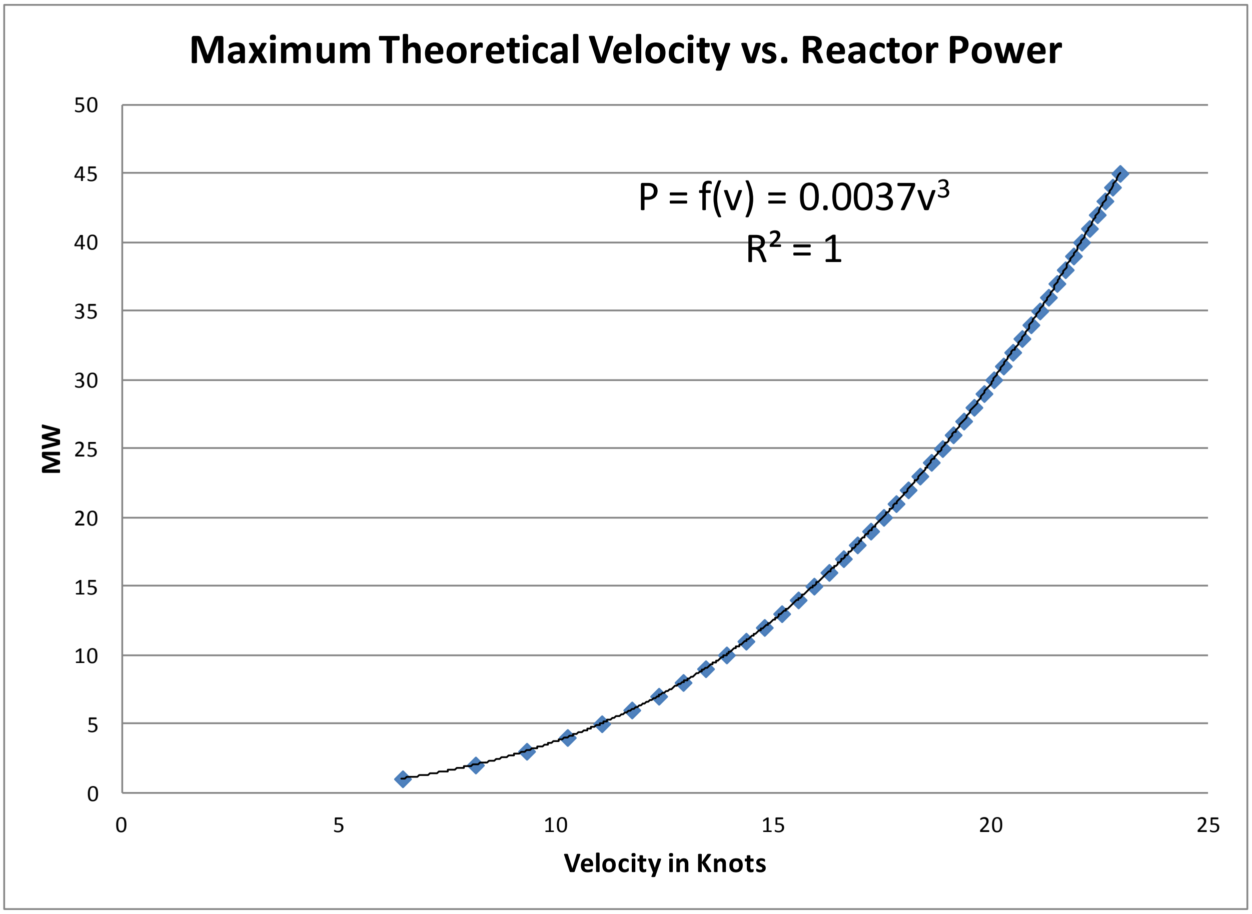
c. Is $v = f(P)$ surjective, injective, or bijective?

Bijective.

d. What is the maximum reactor power that will be required during your voyage under the Polar Ice Cap (from your voyage plan project)?

41 MW of power will be required on a continuous basis between 01 and 03 August to maintain 22.21 knots necessary to reach the North Pole on schedule.

e. Create a graph and find the equation of the function $P = f^{-1}(v)$. Determine the reactor power in MW necessary for each leg of your voyage plan.



Knots	**MW**
12.19	6.70
13.76	9.64
22.21	40.54

Unit 1 Assessment

Unit 1: Functions - The Basics
Study Guide Using the Text Shared with All Students

1. Why do we model real-world relationships with functions? (p. 11)

2. What are the three parts of a function? (p. 12)

3. Given a function, separate it into its three parts. (p. 12)

4. Match a function input with its correct outcome. (p. 19)

5. Graph a function using Microsoft EXCEL or Google Spreadsheet.

6. Define **domain**, **range**, **surjective**, **injective**, and **bijective**. (pp. 26 - 28)

7. Determine that function's type (surjective, injective, or bijective), domain, and range. (pp. 26 - 28)

8. Evaluate a real-world function for a given domain value. (p. 28)

9. On the exam,

    a. One 3 x 5 note card with notes permitted.

    b. Exam will be in computer lab.

Name: ____________________________

Unit 1 Assessment

1. Why do we model real-world relationships with functions?

   ____

   ____

2. Label the following table with the three parts of a function and separate the following statement into its three parts on the table:

   The velocity of U.S.S. Nautilus *is a function of the power output of her nuclear reactor.*

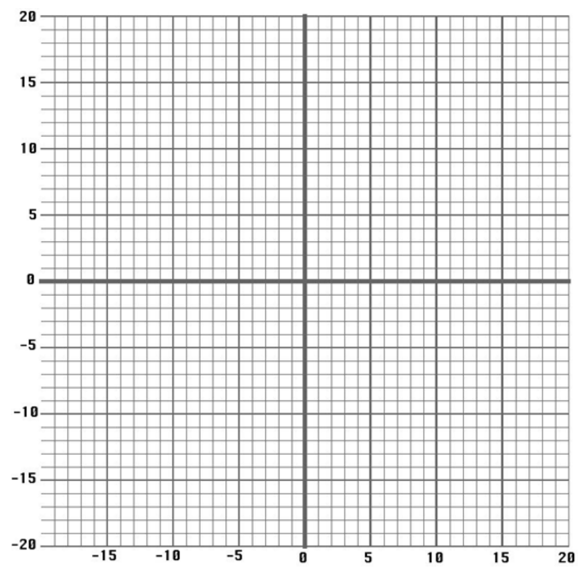
3. Given the following function: $5x^3 + 12$, find:

   a. Domain: ____________________

   b. Range: ____________________

   c. surjective / injective / bijective

   d. Sketch the graph of the function.



4. Match the function input with its correct outcome.

a. The ability of a submarine to remain on course. b. The ability of a submarine to remain underwater for a long time. c. The velocity at which a submarine travels.	a. The power output of the nuclear reactor. b. The effect of underwater currents on the submarine. c. The amount of oxygen the submarine carries for the crew to breath.

5. The maximum theoretical velocity U.S.S. *Nautilus* can achieve when submerged is a function of her hull diameter and the amount of power her reactor produces:

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}}$$

$v =$ maximum theoretical velocity in meters per second.
$\rho = 1027\frac{kg}{m^3}$: the density of saltwater.
$P =$ power output of the reactor in Watts ($1MW = 10^6\ W$).
$D = 8.23\ meters$: the hull diameter of *Nautilus.*

Determine the maximum theoretical velocity *Nautilus* can achieve at the following reactor output power levels:

$P = 12000000\ W$ ____________________

$P = 18000000\ W$ ____________________

6. If $P = f(v) = 0.0037v^3$, find the power output of *Nautilus'* reactor at 20.5 knots.

____

Unit 2: More Complicated Functions

I. Lesson I: Functions within Functions.

A. **Mini-lesson 1: The SONAR function as a Function.** Many ships use SONAR (short for ***SO***und ***N***avigation and ***R***anging) to find things under water. Scientific ships use it to map the bottom of the ocean and find objects that are very deep. Warships use SONAR to find enemy submarines. Because one cannot see underwater easily, sound is used to hear things that are underwater. U.S.S. *Nautilus* used SONAR to feel its way under the Polar Ice Cap when it was not possible to surface and visually determine her position.

Listening with SONAR is much like listening with your ears. Imagine yourself at a party where music is playing and people are talking and laughing. Your friend is standing near you talking to you. Your ability to hear him is based on many factors. These are:

6. **Detection Threshold (DT):** The outcome of the SONAR function. It is the minimum decibel (db) at which the SONAR operator can distinguish the target from all other noise.
7. **Source Level (SL):** The sound level in db coming from the active SONAR ping.
8. **Target Strength (TS):** The sound level in db at a distance of 1 meter from the target.
9. **Noise Level (NL):** The sound level in db of background noise not associated with the target.
10. **Transmission Loss (TL):** The sound level in db lost as sound travels to and from the target.

You could put these terms together in a sentence:

The sound level (DT) at which you can just barely hear your friend is a function of how loudly your friend speaks (SL) plus how well someone could hear him standing one foot from him (TS) minus twice how far away from you your friend is (2TL) minus how much noise is in the background (NL).

Now, let's translate this into an equation:

$$DT = f(SL, TS, TL, NL) = SL + TS - 2TL - NL$$

This is the basic SONAR function we will use in our operations with U.S.S. *Nautilus*.

We can write this equation in terms of each variable. For example we can rewrite the equation in terms of SL by solving for SL in the basic SONAR function above:

Source level (SL) is a function of detection threshold (DT) plus twice the transmission loss (2TL) plus noise level (NL)

$$SL = f(DT, TL, NL, TS) = DT + 2TL + NL - TS$$

Now try to rewrite the equation in terms of transmission loss (TL) and noise level (NL):

$TL =$ ____

$NL =$ ____

How SONAR Works

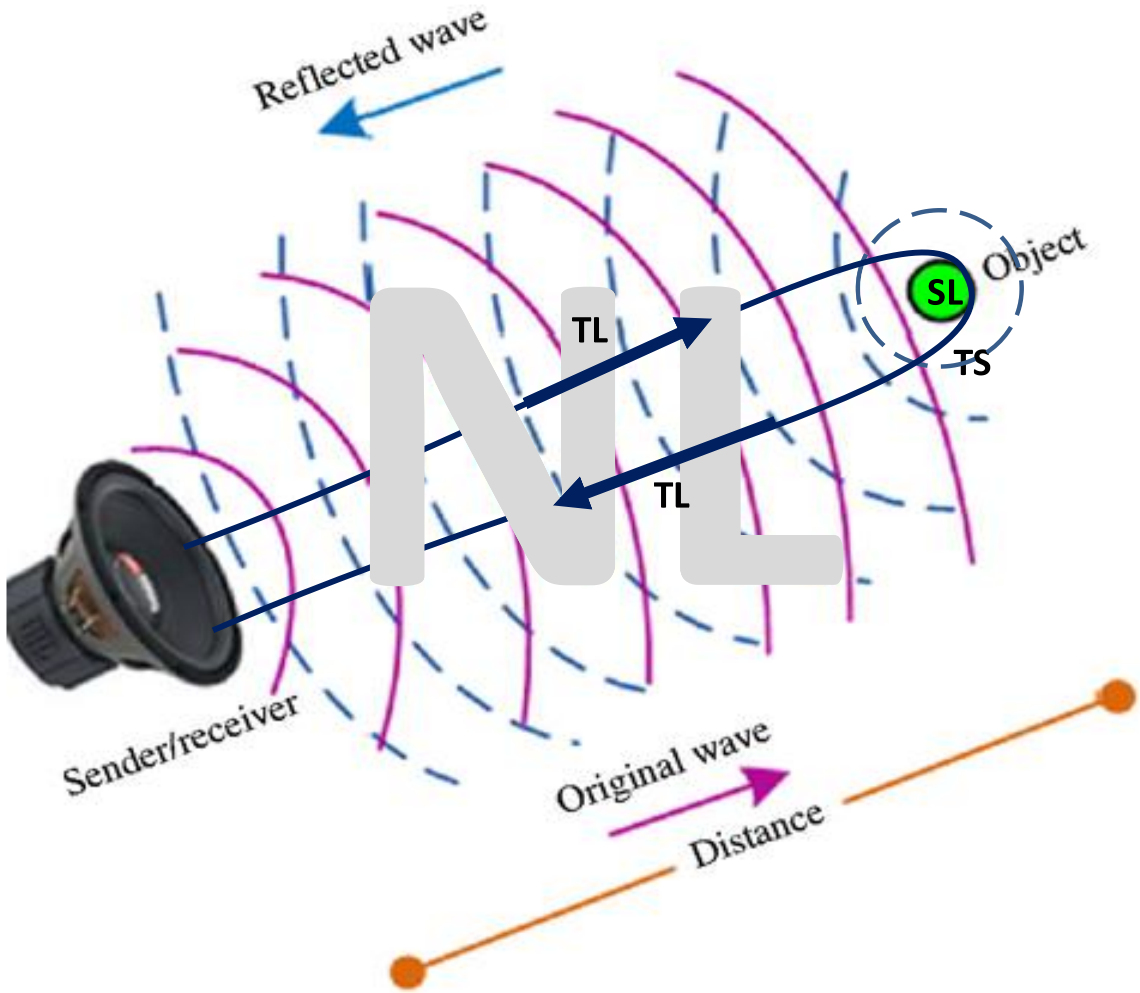


Figure 11: How SONAR Works

B. **Class Work 1:** Solving the SONAR function. All problems will use the following basic SONAR function we have already discussed:

$$DT = TS + SL - 2TL - NL$$

Variables (all variables represent sound levels in decibels [db]):

F. DT = Detection Threshold
G. TS = Target Strength
H. SL = Source Level
I. TL = Transmission Loss
J. NL = Noise Level

If the detection threshold (DT) is less than 1.0db, it will be impossible to detect the target. If DT is less than 10db, it is highly unlikely that the target will be detected.

Problems:

7. U.S.S. *Nautilus* uses pumps to cool its nuclear reactor. These pumps make a great deal of noise. A destroyer searching for *Nautilus* uses a SONAR to listen for the sound of those pumps. If the source level (SL) of the *Nautilus* is 40db, the target strength (TS) of *Nautilus* is 34db, the transmission loss (TL) is 20db and the noise level (NL) is 20db, will the destroyer detect *Nautilus*?

8. Will the destroyer detect the *Nautilus* if the noise level (NL) increases to 44db?

9. A frigate's SONAR has a detection threshold (DT) for a submarine of 10db. If the noise level (NL) is 20db, the transmission loss (TL) is 10db, and *Nautilus'* target strength (TS) is 28db, what is the source level (SL) of the submarine?

10. What will the new detection threshold (DT) be if *Nautilus'* source level (SL) increases by 5db over the level that is the answer to problem "3"?

11. *Nautilus* uses SONAR to detect underwater natural formations such as seamounts. If the SONAR has a detection threshold (DT) of 15db with a noise level (NL) of 8db, a transmission loss (TL) of 12db, and a source level (SL) of 0db, what is the target strength of the seamount?

12. What would the new detection threshold (DT) be if the noise level (NL) decreased by 4db?

C. **Class Work Exercise 3:** Solve the following equations for the variable indicated. Show all work on separate sheets of paper. All problems will use the following basic SONAR function we have already discussed:

$$DT = f(TS, SL, TL, NL) = TS + SL - 2TL - NL$$

Variables (all variables express sound levels in decibels [db]):

F. DT = Detection Threshold
G. TS = Target Strength
H. SL = Source Level
I. TL = Transmission Loss
J. NL = Noise Level

Problems: Example: Given the following equation:

$$10db + 2TS = 3TS + 4db - 2(5db) - 12db$$

1. Add given numbers on right-hand side together:

$$4db - 2(5db) - 12db =$$

$$4db - 10db - 12db = -18db$$

2. Substitute the sum for the given numbers: $10db + 2TS = 3TS - 18db$

3. Add 18db to both sides: $10db + 2TS = 3TS - 18db$

$$\underline{+18db \qquad\qquad\qquad + 18db}$$

$$28db + 2TS = 3TS \ \ - 0db$$

4. Subtract 2TS from both sides: $28db + 2TS = 3TS \ \ - 0db$

$$\underline{-2TS \ - 2TS \qquad\quad}$$

5. And the answer is..... $TS = 28db$

“2 Level” Problems	“3 Level” Problems	“4 Level” Problems
Z. $30db - 2NL = 10db - NL$ $NL =$ ______________ AA. $3TS = 4TS - 41db$ $TS =$ ______________ BB. $4SL = 44db - 3SL$ $SL =$ ______________ CC. $4TL + 87db - 2TL$ $2TL =$ ______________ $TL =$ ______________ DD. $17NL = 21db - 16NL$ $NL =$ ______________ EE. $8.2db + 12TS = 13TS - 4.1db$ $TS =$ ______________ FF. $9.7db + 4NL = 6NL + 6.9db - NL$ $NL =$ ______________ GG. $12SL = 33.3db - 11SL$	HH. $30db - 2NL = 5db - NL + 4db$ $NL =$ ______________ II. $46db + 2TS = 3TS + 65db - 41db$ $TS =$ ______________ JJ. $80db - 6SL = 37db + SL - 21db - 6SL$ $SL =$ ______________ KK. $12db - 5TL = 35db - 3TL - 15db$ $2TL =$ ______________ $TL =$ ______________ LL. $55db + 5NL = 81db - NL + 24db + 5NL$ $NL =$ ______________ MM. $8.2db - TS = 15db + 4TS - 4.1db$ $TS =$ ______________ NN. $9.7db + 3NL = 2.3db + 5NL + 6.9db - NL$ $NL =$ ______________ OO. $12SL = 33.3db + SL - 56db + 12SL$ $SL =$ ______________	PP. $30db - 6SL = 5db - NL - 5SL$ $NL =$ ______________ QQ. $46db + 2TS = TS + 65db - NL$ $TS =$ ______________ RR. $2SL + TS = TS - SL - 21db$ $SL =$ ______________ SS. $18.7TS + 12db - 6TS = 12.7TS - 2TL - 15db$ $2TL =$ ______________ $TL =$ ______________ TT. $NL) + 55db = TS + 2NL + 24db$ $TS =$ ______________ UU. $4(SL + TS) = -3SL - 3TS - 4.1db$ $SL =$ ______________ VV. $3NL) + 9.7db = 2(6SL + 6.9db) - 17NL$ $NL =$ ______________ WW. $0db) = 33.3db + SL - 21NL$

$SL =$ ______________		$SL =$ ______________

D. **Mini-lesson 2: The SONAR Function Expanded**.[55] Each of the components of the SONAR Function can be represented as individual functions. For example, **transmission loss (TL)** is a function of the distance the target is from the SONAR transmitter/receiver:

$$TL = f(d) = 20log_{10}(d)$$

$d = distance\ of\ target\ from\ the\ transmitter/receiver$

Average background **noise level (NL)** can also be expressed as a function of the force of the sound over a given area:

$$NL = f(P_N) = 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

P_N = force of the background noise in micro pascals.

The **source level (SL)** of a SONAR signal is:

$$SL = f(P_S) = 10log_{10}P_S + 171.5\ db$$

P_s = The power output of the SONAR in micro pascals.

The **target strength (TS)** of a target is a function of the noise generated by the target and the intensity of a reflected SONAR ping at a distance of one meter from the target. This can be expressed as a function:

$$TS = f(N_t, a) = N_t + 10log_{10}\left(\frac{a^2}{4}\right)$$

a^2 = the cross-section area of the target.

Taken together, the entire SONAR function can be expressed as:

[55] http://www.usna.edu/Users/physics/ejtuchol/documents/SP411/Chapter15.pdf.

$$DT = g(N_t, a) + h(P_S) - 2 \times j(d) - k(P_N)$$

$$\boldsymbol{DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\left(20log_{10}(d)\right) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)}$$

E. **Class Work Exercise 4:** Solve the SONAR function for the given values:

Submarine	a	Nt	d	Ps	Pn	DT
Nautilus	9001.64	400	400	40000	20000	f(x)
Skate	8424	x	600	50000	50000	96.89462
Skipjack	8008	250	300	60000	x	252.7041
Permit	7813	350	1000	70000	x	330.9054
Sturgeon	9344	x	1500	80000	80000	122.8361
Los Angeles	11946	101	2000	90000	90000	f(x)

Answers:

Submarine	a	Nt	d	Ps	Pn	DT
Nautilus	9001.64	400	400	40000	20000	406.504
Skate	8424	105	600	50000	50000	96.89462
Skipjack	8008	250	300	60000	60000	252.7041
Permit	7813	350	1000	70000	70000	330.9054
Sturgeon	9344	148	1500	80000	80000	122.8361
Los Angeles	11946	101	2000	90000	90000	72.46083

F. **Class Work Exercise 5:** Solve the SONAR function for the given values:

1. $DT = g(300, 7600) + h(21000) - 2 \times j(1000) - k(33000) =$ _____
2. $DT = g(245, 6789) + h(21500) - 2 \times j(1200) - k(33500) =$ _____
3. $DT = g(370, 12679) + h(22000) - 2 \times j(1400) - k(34000) =$ _____
4. $DT = g(420, 10983) + h(2250) - 2 \times j(1560) - k(34500) =$ _____
5. $DT = g(560, 9543) + h(23000) - 2 \times j(234) - k(35000) =$ _______
6. $DT = g(240, 9000) + h(23500) - 2 \times j(780) - k(36000) =$ _______
7. $DT = g(391, 6291) + h(24000) - 2 \times j(145) - k(22000) =$ _______

8. Which of nrs. 1 – 7 provides a DT with the best chance of detection of a target?

Answers:

1	*Inputs*	*300*	*7600*	*21000*	*1000*	*33000*	DT =
	Solutions	371.60		214.72	-120.00	-184.35	**281.97**
2	*Inputs*	*245*	*6789*	*21500*	*1200*	*33500*	DT =
	Solutions	315.62		214.82	-123.17	-184.48	**222.79**
3	*Inputs*	*370*	*12679*	*22000*	*1400*	*34000*	DT =
	Solutions	446.04		214.92	-125.85	-184.61	**350.51**
4	*Inputs*	*420*	*10983*	*2250*	*1560*	*34500*	DT =
	Solutions	494.79		205.02	-127.72	-184.74	**387.35**
5	*Inputs*	*560*	*9543*	*23000*	*234*	*35000*	DT =
	Solutions	633.57		215.12	-94.77	-184.86	**569.06**
6	*Inputs*	*240*	*9000*	*23500*	*780*	*36000*	DT =
	Solutions	313.06		215.21	-115.68	-185.11	**227.49**
7	*Inputs*	*391*	*6291*	*24000*	*145*	*22000*	DT =
	Solutions	460.95		215.30	-86.45	-180.83	**408.97**

G. **Mini-lesson 3: Tactical Use of the SONAR Function I.** The SONAR function has two components as illustrated below:

$$DT = \boldsymbol{TS + SL - 2TL - NL}$$

$$DT = \boldsymbol{g(N_t, a) + h(P_S) - 2 \times j(d) - k(P_N)}$$

The first component comprises the positive elements while the second component comprises the negative elements.

Recall that DT is the minimum decibel level (db) at which the SONAR operator can distinguish the target from all other noise. The higher DT is, the greater the chance that any SONAR operator, no matter how experienced, will be able to distinguish the target. So...

...as the difference between $\boldsymbol{TS + SL}$ and $\boldsymbol{-2TL - NL}$ increases in the ***positive direction***, DT increases and the chances increase that the SONAR operator will detect the target...

...as the difference between $\boldsymbol{TS + SL}$ and $\boldsymbol{-2TL - NL}$ increases in the ***negative direction***, DT decreases and the chances decrease that the SONAR operator will detect the target...

We will call the first component a function of good noise $f(N_g)$ and the second component a function of bad noise $f(N_b)$. We could then rewrite the SONAR function as:

$$DT = f(N_g) + f(N_b)$$

H. **Class Work Exercise 6:** Answer the following questions about the SONAR function:

1. If TS increases and all other parts of the SONAR function remain unchanged, what will happen to DT?

2. If TS and TL increase by the same amount and all other parts of the SONAR function remain unchanged, what will happen to DT?

3. If N_t from the complex form of the SONAR function increases and all other parts of the function remain unchanged, what will happen to DT?

4. If *d* from the complex form of the SONAR function decreases and all other parts of the function remain unchanged, what will happen to DT?

5. If *a* from the complex form of the SONAR function decreases and *d* increases while all other parts of the function remain unchanged, what will happen to DT?

Unit 2: More Complicated Functions

I. **Mini-lesson 3: Tactical Use of the SONAR Function II – Sensitivity Analysis.** Sensitivity analysis involves exploring the degree to which changing one variable while leaving the others constant impacts the overall outcome of the function. For example, given the function:

$$DT = g(300, 7600) + h(21000) - 2 \times j(1000) - k(33000) = 281.97db$$

...what would be the outcomes if we were to vary each input value in turn by a factor of 10?

Initial Function	*Inputs*	*300*	*7600*	*21000*	*1000*	*33000*	DT =
	Solutions	371.60		214.72	-120.00	-184.35	**281.97**
Vary Nt	*Inputs*	***3000***	*7600*	*21000*	*1000*	*33000*	DT =
	Solutions	3071.60		214.72	-120.00	-184.35	
Vary a	*Inputs*	*300*	***76000***	*21000*	*1000*	*33000*	DT =
	Solutions	391.60		214.72	-120.00	-184.35	
Vary Ps	*Inputs*	*300*	*7600*	***210000***	*1000*	*33000*	DT =
	Solutions	371.60		224.72	-120.00	-184.35	
Vary d	*Inputs*	*300*	*7600*	*21000*	***10000***	*33000*	DT =
	Solutions	371.60		214.72	-160.00	-184.35	
Vary Pn	*Inputs*	*300*	*7600*	*21000*	*1000*	***330000***	DT =
	Solutions	371.60		214.72	-120.00	-204.35	

1. Which input value, when changed, produces the greatest change in DT?

2. Why is this so?

Name: ______________________________

Unit 2: Assessment – Understanding a Complicated Function

Using the following table and graphs and the SONAR function, answer the following questions.

$$DT = g(N_t, a) + h(P_S) - 2 \times j(d) - k(P_N)$$

$$\boldsymbol{DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)}$$

1. *Which input value, when varied, produces the greatest change in DT?*

2. *Why is this so?*

____

____

____

____

____

____

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____

____

____

Unit 2: Assessment – Understanding a Complicated Function

Nt	g(*Nt*, a)	a	g(Nt, *a*)	Ps	h(Ps)	d	j(d)	Pn	k(Pn)	DT (Nt)	DT (a)	DT (Ps)	DT (d)	DT (Pn)
2	2	2	2.00	2	174.51	2	12.04	2	100.00	52.43	52.43	52.43	52.43	52.43
4	4	4	8.02	4	177.52	4	24.08	4	106.02	54.43	58.45	55.44	28.35	46.41
6	6	8	14.04	8	180.53	8	36.12	8	112.04	56.43	64.47	58.45	4.26	40.39
8	8	16	20.06	16	183.54	16	48.16	16	118.06	58.43	70.49	61.46	-19.82	34.37
10	10	32	26.08	32	186.55	32	60.21	32	124.08	60.43	76.51	64.47	-43.90	28.35
12	12	64	32.10	64	189.56	64	72.25	64	130.10	62.43	82.53	67.48	-67.98	22.32
14	14	128	38.12	128	192.57	128	84.29	128	136.12	64.43	88.55	70.49	-92.07	16.30
16	16	256	44.14	256	195.58	256	96.33	256	142.14	66.43	94.57	73.50	-116.15	10.28
18	18	512	50.16	512	198.59	512	108.37	512	148.16	68.43	100.59	76.51	-140.23	4.26
20	20	1024	56.19	1024	201.60	1024	120.41	1024	154.19	70.43	106.61	79.52	-164.31	-1.76
22	22	2048	62.21	2048	204.61	2048	132.45	2048	160.21	72.43	112.63	82.53	-188.40	-7.78
24	24	4096	68.23	4096	207.62	4096	144.49	4096	166.23	74.43	118.65	85.54	-212.48	-13.80
26	26	8192	74.25	8192	210.63	8192	156.54	8192	172.25	76.43	124.68	88.55	-236.56	-19.82
28	28	16384	80.27	16384	213.64	16384	168.58	16384	178.27	78.43	130.70	91.56	-260.64	-25.84
30	30	32768	86.29	32768	216.65	32768	180.62	32768	184.29	80.43	136.72	94.57	-284.73	-31.86
32	32	65536	92.31	65536	219.66	65536	192.66	65536	190.31	82.43	142.74	97.58	-308.81	-37.88
									Delta DT	30.00	90.31	45.15	-361.24	-90.31

Figure 12: EXCEL Spreadsheet Sensitivity Analysis

Unit 2: Assessment – Understanding a Complicated Function

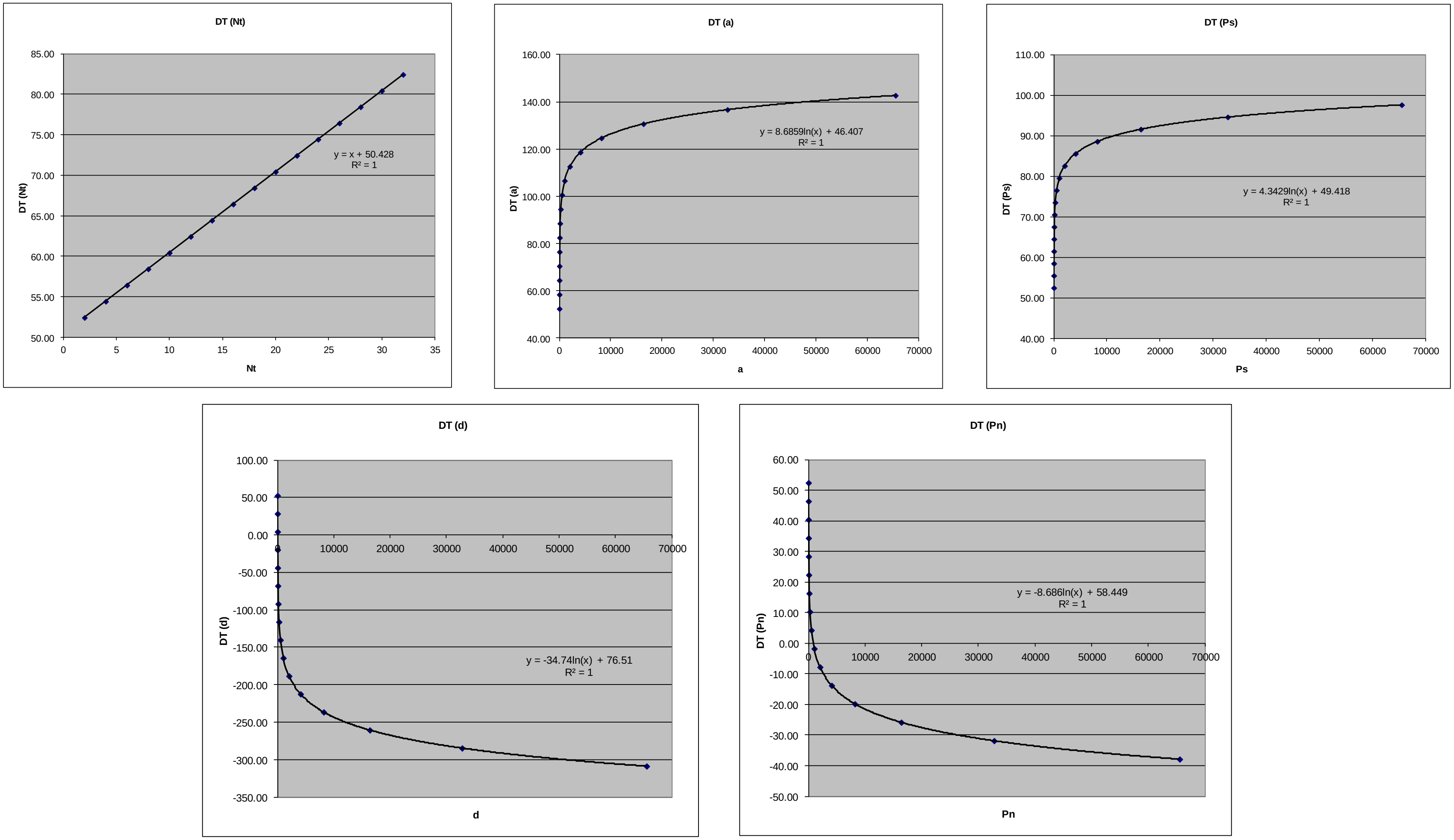

Figure 13: Graphical Answers to Sensitivity Problems

II. Lesson II: Modeling Environmental Interactions as Functions.

A. **Mini-lesson 1: Sound Velocity Under Water.**[56] Our SONAR Function modeled the transmission and reception of sound underwater without considering the ocean environment. In fact, the temperature, pressure at a given depth, and salinity (amount of salt dissolved in water) all impact the velocity (speed) and direction of sound waves moving under water.

1. On average, ocean salinity is 35 parts of salt per thousand of water (35 ppt) so underwater sound velocity is mainly a function of temperature and depth. Underwater sound velocity (C) can be expressed as the following function:

$$C(t, d, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d$$

where:

$C =$ the speed of sound underwater in meters per second.
$t =$ temperature in °Centigrade.
$d =$ depth in meters.
$s =$ salinity in parts per thousand.

2. Sound waves are bent as they pass through regions of different sound velocity (caused by differences in temperature, pressure, and/or salinity):

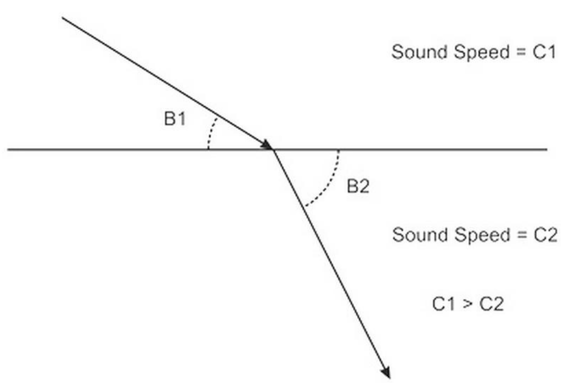

Figure 14: Sound Refraction Caused by Changes in Sound Velocity

[56] http://www.arc.id.au/UWAcoustics.html.

This phenomenon is called *refraction*. Using an equation known as Snell's Law, we can, for any given change in sound velocity, determine the degrees of refraction and, therefore, the new direction of the sound.

$$\frac{cosB_1}{cosB_2} = \frac{C_1}{C_2}$$

B. **Class Work Exercise 1 (copy the problems and show all work in your journal).**

1. Solve the Underwater Sound Velocity Function for the given input values:

$$C(t, d, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d$$

$t = 20°C, s = 15ppt, d = 10m$
$t = 15°C, s = 20ppt, d = 15m$
$t = 10°C, s = 25ppt, d = 20m$
$t = 05°C, s = 30ppt, d = 25m$
$t = 00°C, s = 35ppt, d = 30m$

2. Based upon the above results, which input variable, t, p, or s, has the greatest impact on the outcome C.

3. Using Snell's Law and given the changes in sound speed and initial angle of sound direction, find the new angle of sound direction.

$B_1 = 45°, C_1 = 1450\frac{m}{s}, C_2 = 1350\frac{m}{s}$
$B_1 = 25°, C_1 = 1350\frac{m}{s}, C_2 = 1450\frac{m}{s}$
$B_1 = 30°, C_1 = 1400\frac{m}{s}, C_2 = 1350\frac{m}{s}$
$B_1 = 65°, C_1 = 1550\frac{m}{s}, C_2 = 1250\frac{m}{s}$
$B_1 = 15°, C_1 = 1250\frac{m}{s}, C_2 = 1550\frac{m}{s}$

C. **Class Work Exercise 2 (copy the problems and show all work in your journal).**

1. A searching ship, U.S.S. *Ouellet*, has located a submarine underwater at an angle of 30 degrees at a depth of 100 meters. Water temperature is 25° and salinity is 30 ppt. If *Ouellet* sends an active SONAR ping to the submarine:

   a. What is the velocity of the SONAR ping from *Ouellet*?

   b. How long will it take the reflected sound from the target to reach *Ouellet* from the time the original SONAR ping was transmitted?

2. A searching ship, U.S.S. *Rentz*, sends a SONAR signal at an angle of 45 degrees from the surface of the ocean. The velocity of the sound from the SONAR signal, down to 50 meters is $1500\frac{m}{s}$. At 50 meters, the velocity falls to $1300\frac{m}{s}$. If a submarine bears 48° at 65 meters, will *Rentz* detect her?

D. **Mini-lesson 2: Finding Our Way Underwater.** Recall that we have considered three functions related to underwater sound:

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2\times10^{-5}}\right) \quad (1)$$

$$C(t,d,s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s-35) + 0.016d \quad (2)$$

$$\frac{cosB_1}{cosB_2} = \frac{C_1}{C_2} \quad (3)$$

All three functions are related (how?). It should, therefore, be possible to consider even more complex and ambiguous problems. For example:

U.S.S. *Nautilus* is moving underwater and under ice through the Bering Strait (the strait between Alaska and Russian Siberia). She is using her SONAR to detect underwater obstructions such as sea mountains and ice that might be blocking her way. *Nautilus*' SONAR operator is one of the best in the naval service. He has the following information available to him:

a. His detection threshold (DT) is 14 db.
b. His SONAR transmits a signal (P_s) of 10000 watts/m^2.
c. Background noise power (P_n) is 9000 µP/m^3.
d. Water temperature is 1.5° C.
e. *Nautilus*' depth is 50 m.
f. Ocean salinity is 33 ppt.

The SONAR operator, having transmitted continuous pings receives a return signal (N_t) of level equal 1.58 db 1 meter from a target. The time between SONAR transmission and reception of this return signal is 0.413 seconds.

1. How far away is this object?
2. How big is this object in terms of cross-sectional area (a)?
3. What course must *Nautilus* steer to avoid the object?

To solve problem 1, we need first to determine the speed of sound underwater in this area:

$$C(t,d,s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d$$

$$C(t,d,s) = 1449.2 + 0.055 * 1.5^2 + 4.6 * 1.5 + 1.39 * (33 - 35) + 0.016 * 50$$

Solving the function, we get a speed of sound of $C = 1454.244\frac{m}{s}$.

Since we know that the total time for the signal to be transmitted and its reflection received, we can determine the distance to the object as follows:

$$d = 1454.244\frac{m}{s} * \frac{1}{2} * 0.413\ s = 302\ m$$

To solve problem 2, we need to go back to the original SONAR function:

$$DT = 1.58 + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

$$14\ db = 1.58\ db + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}(10000\frac{W}{m^2}) + 171.5\ db - 2\big(20log_{10}(302\ m)\big) - 20log_{10}\left(\frac{9000\ \mu P}{2 \times 10^{-5}}\right)$$

Since cross-sectional area, a, is what we are looking for, our first step is to rearrange the equation as follows:

$$10log_{10}\left(\frac{a^2}{4}\right) = 14 - 1.58\ - (10 * 4) - 171.5\ + 2(20 * 2.48) + 20 * 8.65$$

Now we solve for a as follows:

$$10log_{10}\left(\frac{a^2}{4}\right) = 14\ - 1.58\ - 40\ - 171.5\ + 99.2 + 173 = 73.12$$

$$log_{10}\left(\frac{a^2}{4}\right) = \frac{73.12}{10} = 7.312$$

$$10^{7.312} =\ 20511621.78$$

$$a^2\ = 20511621.78 * 4 = 82046487.12$$

$$a = \sqrt{82046487.12} = 9057.95\ m^2$$

To solve problem 3, let us assume that the dimensions of this object are roughly equal, in other words, length = width. Therefore, we can assume that the width of the object is:

$$\sqrt{9057.95\ m^2} = 95.17\ meters$$

Looking at this situation graphically:

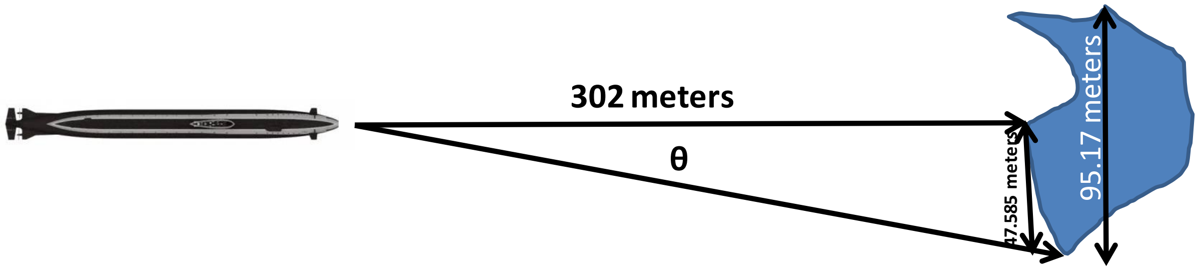

Figure 15: Course to Avoid an Underwater Object

To avoid the object, the Captain of *Nautilus* must alter course θ degrees. If her original course is 005°T, what course must the Captain order to avoid the object?

To solve the problem, recall that the tangent of an angle, $\tan\theta$, is equal to the length of the opposite side divided by the adjacent side. Therefore:

$$\tan\theta = \frac{47.585\,m}{302\,m} = 0.158$$

$$tan^{-1}\theta = tan^{-1}0.079 = 8.98° \cong 9°$$

Therefore, the Captain should order a course change to $005°T + 009T = 0.14°T$ to avoid the object.

E. **Class Work Exercise 3 (copy the problems and show all work in your journal).** Solve this problem involving U.S.S. *Nautilus* as she navigates through the Bering Strait:

U.S.S. *Nautilus* is preceding underwater and under ice through the Bering Strait (the strait between Alaska and Russian Siberia). She is using her SONAR to detect underwater obstructions such as sea mountains and ice that might be blocking her way. *Nautilus*' SONAR operator has the following information available to him:

a. His detection threshold (DT) is 11.3 db.
b. His SONAR transmits a signal (P_s) of 10000 watts/m^2.
c. Background noise power (P_n) is 7000 µP/m^3.
d. Water temperature is 3.0° C.
e. *Nautilus*' depth is 75 m.
f. Ocean salinity is 34 ppt.

The SONAR operator, having transmitted continuous pings receives a return signal (N_t) of level equal 1.70 db 1 meter from a target. The time between SONAR transmission and reception of this return signal is 0.6 seconds.

1. How far away is this object?
2. How big is this object in terms of cross-sectional area (a)?
3. *Nautilus*' initial course is 010°T, what course must *Nautilus* steer to avoid the object?

F. **Mini-lesson 3: Completing the Maneuver.**

Having avoided the underwater object, we now need to get back to our original course of 005°T from the example in mini-lesson 2 and we need to maintain our schedule. Let's assume our original speed was 12.5 knots. We need to find:

a. The original distance we would have travelled if we had not needed to change course.
b. The new course we will need to steer once we have avoided the object.
c. The new distance we will travel to avoid the object and regain our original course.
d. The new speed we will have to achieve to stay on schedule.

We already know the following information:

a) Distance to the object: $d = 302\ m$
b) Width of the object: $w = 95.17\ m$
c) Course to avoid the object: 014°T

To solve the problem, consider the following figure:

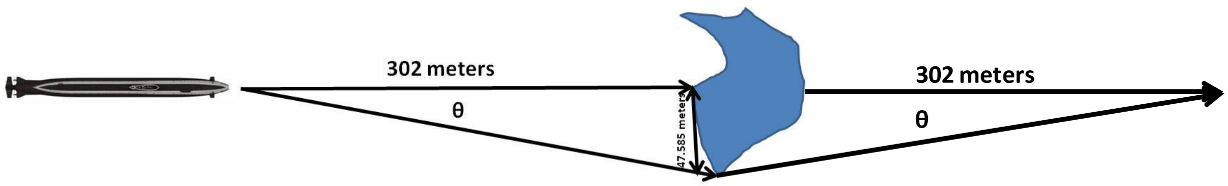

Figure 16: Course to Regain Original Track

1. The original distance is twice the distance to the object plus the width of the object:

$$2 * d + w = 2 * 302\ m + 95.17\ m = 699.17\ m$$

   Since 1 meter equals 0.000539957 nautical miles:

$$699.17m * 0.000539957 \frac{nm}{m} = 0.38\ nm$$

2. The new course we will need to steer to regain track is:

$$014°T - 2\theta = 014°T - 18° = 356°T$$

3. The new distance is two times the length of the hypotenuse of the first triangle in Figure 16 above:

$$\begin{aligned} d_1 &= 2 * \sqrt{(302 + 47.485)^2 + 47.485^2} = 2 * \sqrt{122139 + 2254.83} \\ &= 2 * 352.70 = 705.4m \\ &705.4m * 0.000539957 \frac{nm}{m} = 0.381\ nm \end{aligned}$$

4. To find the new speed, we need to find the time it would have taken to cover the original distance if we had not needed to alter course:

$$\frac{0.38\ nm}{12.5\,\frac{nm}{hr}} = 0.0304\ hrs.$$

The new speed would be given by:

$$\frac{0.381\ nm}{0.0304\ hrs} = 12.53\,\frac{nm}{hr}$$

G. **Class Work Exercise 4 (Do all problems in your journal).** For the problem in class work exercise 3, find:

1. The original distance we would have travelled if we had not needed to change course.
2. The new course we will need to steer once we have avoided the object.
3. The new distance we will travel to avoid the object and regain our original course.
4. The new speed we will have to achieve to stay on schedule.

H. **Team Exercise.** Students in each class will be organized into teams to solve problems related to U.S.S. *Nautilus*' voyage under the ice pack to the North Pole in 1958. Student roles are as follows:

1. **Captain:** This student will oversee the activities of his / her team and be held responsible for submission of a correct problem set each day.
2. **Executive Officer (XO):** This student will be responsible to the Captain for completion of all team activities. He / she will also make final recommendations for course and speed changes to the Captain and will be held accountable for incorrect recommendations.
3. **Chief Engineer / Reactor Officer:** This student will be responsible for reactor power calculations and for ensuring that recommendations regarding *Nautilus*' speed are consistent with reactor safe operation.
4. **Navigator:** This student will make recommendations to the Captain for course and speed adjustments to ensure *Nautilus* accomplishes her voyage safely and on schedule.

Unit 2: More Complicated Functions

Student Assignments for Team Exercise

	Captain	Executive Officer	Chief Engineer / RO	Navigator
Algebra II – 1				
Team 1				
Team 2				
Team 3				
Team 4				
Team 5				
Pre-Calculus				
Team 1				
Team 2				
Team 3				
Team 4				
Team 5				
Algebra II – 2				
Team 1				
Team 2				
Team 3				
Team 4				
Team 5				

I. **Team Exercise Scenario.** U.S.S. *Nautilus* (SSN – 571) is submerged at the entrance to the Bering Strait enroute to the Arctic Ocean. She is proceeding at 12.5 knots in accordance with the voyage plan submitted to the Chief of Naval Operations (CNO) on January 5th. Her speed is slightly higher than the 12.19 knots called for in the plan to ensure she does not fall behind schedule. *Nautilus* must clear the Bering Strait, cross the Arctic Ocean, and pass across the North Pole on August 3rd at 12:00 and pass through the Greenland Strait on August 5th at 12:00. *Nautilus* is using her SONAR to detect objects that might impede her safe passage through the strait into the Arctic Ocean.

J. **Exercise Requirements.** Every member of the team must participate in the exercise. It is recommended that the Captain do no work but merely supervise the work of others. The Captain and members of the team will be graded as follows:

Grade	**4**	**3**	**2**	**1**
Captain	The Captain submitted a correct solution. The Captain let his/her team members do the work without significant help but checked the work for a correct solution.	The Captain submitted a correct solution developed by team members, however, not all team members participated in finding the solution.	The Captain submitted a correct solution but did most of the work finding the solution himself/herself.	The Captain did not submit a correct solution.
Team Member	The Team Member participated in finding the correct solution and pro-actively helped other team members.	The Team Member participated in finding the correct solution.	The Team Member offered some assistance but did not really contribute to finding the correct solution.	The Team Member did nothing to assist the team.

Department of the Navy
U.S.S. *Nautilus* SS(N) – 571
Fleet Post Office New York

05 JAN 1958

From: Commanding Officer

To: CNO Washington, DC
CINCLANTFLT Norfolk, VA
COMSUBLANT New London, CT

Subj: U.S.S. *Nautilus* SS(N) – 571 Voyage Plan

Ref: (a) CNO Operation Order

Encl: (1) Track Chart

1. Per reference (a), the following voyage plan is submitted.

Point	Date (12:00 local on the given day)	Latitude	Longitude	Distance in NM	Speed in Knots (NM / hr.)
1	27 July	52º 30’ N	175º 00’ W	N/A	N/A
2	28 July	56º 30’ N	173º 00’ W	250.05	12.19
3	29 July	64º 00’ N	172º 00’ W	451.30	12.19
4	30 July	65º 00’ N	168º 00’ W	119.55	12.19
5	31 July	72º 30’ N	168º 00’ W	450.32	12.19
6	01 August	72º 15’ N	155º 00’ W	236.34	12.19
7	03 August	90º 00’ N	Undefined	1065.90	22.21
8	05 August	79º 00’ N	05º 00’ E	660.37	13.76

W. R. Anderson

U.S.S. *Nautilus* SS(N) – 571
Daily Navigation and Engineering Log

Date: 27 July 1958

Given		Find	
P_S	10000	$C(t, d_e, s) =$	
P_N	8670	$d =$	
DT	10.5	$a =$	
t	2.6	$w =$	
d_e	40	$0.5w =$	
s	33.5	*Δ Course* =	
Course	007°T	*Course to avoid* =	
Speed	12.5	$d_{original} =$	
N_t	2.98	$d_{new} =$	
time	1.4 sec.	Course to regain =	
		Speed to regain =	
		Reactor Power Necessary (MW) =	
		% Fuel Expended =	

Narrative

0830: Detected large under water object, possibly ice, directly ahead. Ordered all stop to determine distance, size, and course to avoid.
0840: Underway. Ship stopped for a total of 10 minutes.

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}} \qquad P = f(v) = 0.0037v^3$$

$$C(t, d_e, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d_e$$

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

Rank	Name	Grade
Captain:	____________	_____
XO:	____________	_____
Nav:	____________	_____
Cheng:	____________	_____

U.S.S. *Nautilus* SS(N) – 571
Daily Navigation and Engineering Log

Date: 28 July 1958

Given		Find	
P_S	10000	$C(t, d_e, s) =$	
P_N	7834	$d =$	
DT	9.6	$a =$	
t	3.1	$w =$	
d_e	75	$0.5w =$	
s	36	Δ Course =	
Course	004°T	Course to avoid =	
Speed	13.5	$d_{original} =$	
N_t	4.6	$d_{new} =$	
time	2.1	Course to regain =	
		Speed to regain =	
		Reactor Power Necessary (MW) =	
		% Fuel Expended =	

Narrative

1030: Detected large underwater object dead ahead. Captain ordered all stop.
1045: Underway, having stopped for 15 minutes. New course to avoid object.

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}} \qquad P = f(v) = 0.0037v^3$$

$$C(t, d_e, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d_e$$

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

Rank	Name	Grade
Captain:	____________	_____
XO:	____________	_____
Nav:	____________	_____
Cheng:	____________	_____

U.S.S. *Nautilus* SS(N) – 571
Daily Navigation and Engineering Log

Date: 29 July 1958

Given		**Find**		**Narrative**
P_S	10000	$C(t, d_e, s) =$		
P_N	9045	$d =$		
DT	11.2	$a =$		
t	3.4	$w =$		
d_e	90	$0.5w =$		
s	32	Δ *Course* =		
Course	004°T	*Course to avoid* =		
Speed	12.5	$d_{original} =$		
N_t	3.2	$d_{new} =$		
time	3.1	Course to regain =		
		Speed to regain =		
		Reactor Power Necessary (MW) =		
		% Fuel Expended =		

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}} \qquad P = f(v) = 0.0037v^3$$

$$C(t, d_e, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d_e$$

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

Rank	Name	Grade
Captain:	____________	_____
XO:	____________	_____
Nav:	____________	_____
Cheng:	____________	_____

U.S.S. *Nautilus* SS(N) – 571
Daily Navigation and Engineering Log

Date: 30 July 1958

Given		**Find**		**Narrative**
P_S	10000	$C(t, d_e, s) =$		
P_N	10650	$d =$		
DT	13.4	$a =$		
t	0.7	$w =$		
d_e	100	$0.5w =$		
s	35	$\Delta\ Course =$		
Course	6.5	$Course\ to\ avoid =$		
Speed	12.5	$d_{original}$ =		
N_t	8.2	d_{new} =		
time	5.2	Course to regain =		
		Speed to regain =		
		Reactor Power Necessary (MW) =		
		% Fuel Expended =		

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}} \qquad P = f(v) = 0.0037v^3$$

$$C(t, d_e, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d_e$$

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

Rank	Name	Grade
Captain:	____________	_____
XO:	____________	_____
Nav:	____________	_____
Cheng:	____________	_____

U.S.S. *Nautilus* SS(N) – 571
Daily Navigation and Engineering Log

Date: 31 July 1958

Given		Find		Narrative
P_S	10000	$C(t, d_e, s) =$		
P_N	6430	$d =$		
DT	8.5	$a =$		
t	0	$w =$		
d_e	150	$0.5w =$		
s	35	Δ *Course* =		
Course	000°T	*Course to avoid* =		
Speed	12.5	$d_{original}$ =		
N_t	1.3	d_{new} =		
time	5	Course to regain =		
		Speed to regain =		
		Reactor Power Necessary (MW) =		
		% Fuel Expended =		

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}} \qquad P = f(v) = 0.0037v^3$$

$$C(t, d_e, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d_e$$

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

Rank	Name	Grade
Captain:	____________	_____
XO:	____________	_____
Nav:	____________	_____
Cheng:	____________	_____

U.S.S. *Nautilus* SS(N) – 571
Daily Navigation and Engineering Log

Date: 01 August 1958

Given		**Find**		**Narrative**
P_S	*10000*	$C(t, d_e, s) =$		
P_N	*6378*	$d =$		
DT	*12.7*	$a =$		
t	*0*	$w =$		
d_e	*200*	$0.5w =$		
s	*31.5*	Δ *Course* =		
Course	*000°T*	*Course to avoid* =		
Speed	*12.5*	$d_{original}$ =		
N_t	*4.2*	d_{new} =		
time	*6*	Course to regain =		
		Speed to regain =		
		Reactor Power Necessary (MW) =		
		% Fuel Expended =		

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}} \qquad P = f(v) = 0.0037v^3$$

$$C(t, d_e, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d_e$$

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

Rank	Name	Grade
Captain:	____________	_____
XO:	____________	_____
Nav:	____________	_____
Cheng:	____________	_____

U.S.S. *Nautilus* SS(N) – 571
Daily Navigation and Engineering Log

Date: 02 August 1958

Given		Find		Narrative
P_S	10000	$C(t, d_e, s) =$		
P_N	5609	$d =$		
DT	9.6	$a =$		
t	0	$w =$		
d_e	250	$0.5w =$		
s	35	Δ *Course* =		
Course	000°T	*Course to avoid* =		
Speed	12.5	$d_{original} =$		
N_t	2.3	$d_{new} =$		
time	8.2	Course to regain =		
		Speed to regain =		
		Reactor Power Necessary (MW) =		
		% Fuel Expended =		

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}} \qquad P = f(v) = 0.0037v^3$$

$$C(t, d_e, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d_e$$

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\left(20log_{10}(d)\right) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

Rank	Name	Grade
Captain:	____________	_____
XO:	____________	_____
Nav:	____________	_____
Cheng:	____________	_____

Answers to Team Exercise

27-Jul-58	Ps	Pn	DT	t	depth	s	course	speed	Nt
	10000	8670	10.5	2.6	40	33.5	7	12.5	2.98
time	**C**	**d**	**a**	**w**	**0.5w**	**Δ course**	**d orig.**	**d new**	**new speed**
1.4	1460.087	1022.06076	57275.9798	239.324	119.662	6.67774	2283.446	2295.953	12.56846734
Power									
7.34596216									

28-Jul-58	Ps	Pn	DT	t	depth	s	course	speed	Nt
	10000	7834	9.6	3.1	75	36	4	13.5	4.6
time	**C**	**d**	**a**	**w**	**0.5w**	**Δ course**	**d orig.**	**d new**	**new speed**
2.1	1466.579	1539.907478	87896.7511	296.4739	148.2369	5.498551	3376.289	3389.281	13.55194726
Power									
9.2088804									

29-Jul-58	Ps	Pn	DT	t	depth	s	course	speed	Nt
	10000	9045	11.2	3.4	90	32	4	12.5	3.2
time	**C**	**d**	**a**	**w**	**0.5w**	**Δ course**	**d orig.**	**d new**	**new speed**
3.1	1462.746	2267.25599	310748.962	557.4486	278.7243	7.008471	5091.961	5122.383	12.57468342
Power									
7.35686699									

30-Jul-58	Ps	Pn	DT	t	depth	s	course	speed	Nt
	10000	10650	13.4	0.7	100	35	6.5	12.5	8.2
time	**C**	**d**	**a**	**w**	**0.5w**	**Δ course**	**d orig.**	**d new**	**new speed**
5.2	1454.047	3780.52207	736975.691	858.4729	429.2364	6.477562	8419.517	8463.17	12.56480887
Power									

7.33954916

31-Jul-58	Ps	Pn	DT	t	depth	s	course	speed	Nt
	10000	6430	8.5	0	150	35	0	12.5	1.3

time	C	d	a	w	0.5w	Δ course	d orig.	d new	new speed
5	1451.6	3629	516160.67	718.4432	359.2216	5.653087	7976.443	8008.733	12.55060206

Power
7.31468121

1-Aug-58	Ps	Pn	DT	t	depth	s	course	speed	Nt
	10000	6378	12.7	0	200	31.5	0	12.5	4.2

time	C	d	a	w	0.5w	Δ course	d orig.	d new	new speed
6	1447.535	4342.605	851501.017	922.7681	461.3841	6.064691	9607.978	9652.189	12.55751792

Power
7.32677987

2-Aug-58	Ps	Pn	DT	t	depth	s	course	speed	Nt
	10000	5609	9.6	0	250	35	0	12.5	2.3

time	C	d	a	w	0.5w	Δ course	d orig.	d new	new speed
8.2	1453.2	5958.12	1227733.01	1108.031	554.0156	5.31237	13024.27	13071.32	12.54515371

Power
7.30515919

Assessment
U.S.S. *Nautilus* SS(N) – 571
Daily Navigation and Engineering Log

Given		**Find**		**Narrative**
P_S	*1000*	$C(t, d_e, s) =$		*1200Z: Passed across North Pole. Sent following message to CNO:* *"Nautilus 90 North"*
P_N	*6690*	$d =$		
DT	*12.6*	$a =$		
t	*0*	$w =$		
d_e	*275*	$0.5w =$		
s	*35*	$\Delta\ Course =$		
Course	*000°T*	$Course\ to\ avoid =$		
Speed	*12.5*	$d_{original} =$		
N_t	*3.3*	$d_{new} =$		
time	*7.8*	**Course to regain =**		
		Speed to regain =		
		Reactor Power Necessary (MW) =		
		% Fuel Expended =		

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}} \qquad P = f(v) = 0.0037v^3$$

$$C(t, d_e, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d_e$$

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

Rank	Name	Grade
Captain:	____________	_____
XO:	____________	_____
Nav:	____________	_____
Cheng:		

Assessment
U.S.S. *Nautilus* SS(N) – 571
Daily Navigation and Engineering Log

Given		Find	
P_S	1000	$C(t, d_e, s) =$	1453.6
P_N	6690	$d =$	5669.04
DT	12.6	$a =$	1668957.05
t	0	$w =$	1291.88
d_e	275	$0.5w =$	645.94
s	35	Δ *Course* =	6.5
Course	000°T	*Course to avoid* =	007°T
Speed	12.5	$d_{original}$ =	12629.96
N_t	3.3	d_{new} =	12695.86
time	7.8	Course to regain =	353°T
		Speed to regain =	12.57
		Reactor Power Necessary (MW) =	7.34 MW
		% Fuel Expended =	

Narrative

1200Z: Passed across North Pole. Sent following message to CNO:

"Nautilus 90 North"

$$v = f(P) = \frac{2}{(\pi\rho)^{\frac{1}{3}}} \times \frac{P^{\frac{1}{3}}}{D^{\frac{2}{3}}} \qquad P = f(v) = 0.0037v^3$$

$$C(t, d_e, s) = 1449.2 + 0.055t^2 + 4.6t + 1.39 * (s - 35) + 0.016d_e$$

$$DT = N_t + 10log_{10}\left(\frac{a^2}{4}\right) + 10log_{10}P_S + 171.5\ db - 2\big(20log_{10}(d)\big) - 20log_{10}\left(\frac{P_N}{2 \times 10^{-5}}\right)$$

Rank	Name	Grade
Captain:	____________	_____
XO:	____________	_____
Nav:	____________	_____
Cheng:	____________	_____